Topic C
Analyzing Arrays to Multiply Using Units of 2 and 3

3.OA.1, 3.OA.5, 3.OA.3, 3.OA.4

Focus Standards:
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

3.OA.5 Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

In Topic C, students begin building fluency with facts of 2 and 3 using the array model and familiar skip-counting strategies.

Lessons 7 and 8 introduce the new complexity of manipulating arrays to study the commutative property. Students learn to distinguish rows from columns as they rotate arrays 90 degrees, noticing that the meaning of the factors changes depending on the orientation of the array. Students write two different multiplication sentences to interpret the same array. These lessons emphasize the equivalence of facts by demonstrating, for example, that 2 groups of 8 and 8 groups of 2 are the same. Students observe the pattern and begin to recognize commutativity as a strategy for solving twice as many facts.

Lessons 9 and 10 introduce the distributive property as a strategy for multiplication. In Lesson 9, students use arrays to decompose unknown facts as the sum or difference of two known facts. For example, they analyze an array to see that 7 × 3 can be decomposed as 2 rows of three + 5 rows of three. In Lesson 10 students learn to write the decomposition as (5 × 3) + (2 × 3) = 21. They explain each step of the solving process in anticipation of the work they are expected to complete independently on the mid-module assessment.

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A Teaching Sequence Towards Mastery of Analyzing Arrays to Multiply Using Units of 2 and 3

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Lesson 7

Objective: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

Suggested Lesson Structure

- Fluency Practice (13 minutes)
- Application Problem (5 minutes)
- Concept Development (32 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

**Fluency Practice (13 minutes)**

- Group Counting 3.OA.1 (4 minutes)
- Divide Equal Groups 3.OA.2 (5 minutes)
- Multiply with Twos 3.OA.7 (4 minutes)

**Group Counting (4 minutes)**

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity reviews work with those factors from Topic C.

T: Let’s count by twos. (Direct students to count forward and backward to 20, emphasizing the 8 to 10, 10 to 12, and 18 to 20 transitions.)

T: Let’s count by threes. (Direct students to count forward and backward to 30, periodically changing directions. Emphasize the 9 to 12, 18 to 21, and 27 to 30 transitions.)

**Divide Equal Groups (5 minutes)**

Materials: (S) Personal white boards

Note: Students directly relate repeated addition to division. They interpret the unknown in division. This activity reviews Lesson 6.

T: (Project a picture array with 2 groups of 4 circled.) Say the total as a repeated addition sentence.

S: $4 + 4 = 8$.

T: Write a division sentence for 8 divided into 2 equal groups.

S: (Write $8 \div 2 = 4$.)
Lesson 7

Lesson 7: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

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Lesson 7

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T: Below that division sentence write a division sentence dividing 8 into 4 equal groups.
S: (Write 8 ÷ 4 = 2.)

Continue with possible sequence: 5 groups of 3, 3 groups of 4, and 6 groups of 2.

Multiply with Twos (4 minutes)

Materials: (S) Personal white boards, Twos Array Template, blank paper

Note: Students unit count objects in an array and write multiplication sentences that match the count-by in anticipation of this lesson’s objective.

T: Slip your Twos Array Template into your personal white board.
T: Turn your board so that it’s vertical. Use your blank paper to cover all but the first row of dots.
T: How many twos show?
S: 1 two.
T: Say the multiplication sentence to represent the shown array and solve.
S: 1 × 2 = 2.
T: Uncover another row.

Continue this sequence having students uncover twos for 2 × 2, 3 × 2, 10 × 2, 5 × 2, 6 × 2, 7 × 2, 9 × 2, and 8 × 2.

Application Problem (5 minutes)

Anna picks 24 flowers. She makes equal bundles of flowers and gives 1 bundle to each of her 7 friends. She keeps a bundle for herself too. How many flowers does Anna put in each bundle?

Note: This problem reviews equal groups division from Lesson 5 where the unknown represents the size of the group. The problem’s complexity is in understanding that the flowers are divided equally into 8 bundles, not 7, since they need to count Anna. Students may choose to solve by drawing a division array learned in Lesson 6 or a number bond learned in Lesson 3.
Concept Development (32 minutes)

Materials: (S) Personal white boards

Problem 1: Rotate arrays 90 degrees.

T: Turn your personal white board so it’s horizontal. Draw a line down the middle to make 2 sides. On the left side skip-count by two 4 times and write each number.
S: (Write 2, 4, 6, 8.)
T: On the right side skip-count by four 2 times.
S: (Write 4, 8.)
T: How are the count-bys related?
S: The first one is 4 twos and the second one is 2 fours.
T: Under each count-by draw an array to match it.
S: (Draw arrays shown below.)

T: What do you notice about the arrays? Do they both have 4 groups of 2?
S: (Discuss.)
T: Do they both have 2 groups of 4?
S: Yes. The one on the right has 2 rows of 4. If you turn it sideways then the one on the left does too. Or you can just see that it has 2 vertical rows of 4.
T: It’s the same array turned different ways. We have a special name for rows when they are vertical. We call them columns.

Prompt students to write and solve multiplication sentences to show the total objects in each array. Continue with the following possible examples:
2 × 5 array
7 × 2 array

As you circulate, guide them to notice that factors switch places and help them to relate the change to the rotated array. Write, for example, 4 × 2 = 2 × 4 and ask students to discuss how they know it is true.

T: Depending on how we look at an array, columns or rows can be the number of groups. Discuss with your partner how you know that’s true.
S: (Discuss.)
Problem 2: **Interpreting rows and columns in rotated arrays.**

T: Turn your board so it’s vertical. Draw an array that shows 8 equal groups of two. How many rows of 2 did you draw?
S: 8 rows.
T: How many columns of 8 did you draw?
S: 2 columns.
T: Write a multiplication sentence to match the array. Don’t solve it yet.
S: (Write $8 \times 2 = \_\_\_$.)
T: Rotate your board so that it’s horizontal. How many rows of 8 do you have now?
S: 2 rows.
T: How many columns of 2?
S: 8 columns.
T: Write a multiplication sentence to match the array. Don’t solve it yet.
S: (Write $2 \times 8 = \_\_\_$.)
T: Explain to your partner using the words *columns* and *rows* why your multiplication sentence changed.
S: When the array turned the columns and rows switched. → Columns became rows and rows became columns. They both represent equal groups. It depends on how you look at the array.
T: Will $8 \times 2$ and $2 \times 8$ have the same total?
S: Yes!
T: How do you know?
S: They have the same array. → 2 groups of 8 and 8 groups of 2 are the same.

(Prompt students to skip-count to find the totals of the array in both positions.)

Work through the following examples to build vocabulary and understanding of commutativity.

\[
\begin{align*}
6 \times 2 \\
2 \times 9
\end{align*}
\]

T: When we multiply, changing the order of the factors doesn’t change the total. We say the factors are *commutative*. That means they can switch around. Tell your partner what commutative means.

S: It means numbers can switch around. → The factors change places in a multiplication sentence, but the total doesn’t change. → Addition works the same way.

T: What we’ve explored today is called the *commutative property*.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students need not master the words *commutative* or *commutative property*. However, they will need to be familiar with the vocabulary moving forward in this module.
Lesson 7

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to de brief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Discuss the usefulness of unit counting to solve multiplication facts.
- Build fluency by having students demonstrate unit counting to find the answer to the following facts without the help of an array. They can keep track of the count using fingers.
  - 3 twos, 2 threes
  - 4 twos, 2 fours
  - 2 eights, 8 twos
  - 2 tens, 10 twos
- Review the vocabulary columns in contrast with rows.
- Discuss the commutativity of multiplication and how it relates to equal groups, columns, rows and arrays.
- Relate the *commutative property* of multiplication to the commutative property of addition to help students recognize it in their prior learning.

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. a. Count by 2 six times.  
   _______,_______,_______,_______,_______,_______  
   b. Draw an array that matches your count-by.  
   c. Write a multiplication sentence that represents the total number of objects in your array.  
   _______ × _______ = _______  

2. a. Count by 6 two times.  
   _______,_______  
   b. Draw an array that matches your count-by.  
   c. Write a multiplication sentence that represents the total number of objects in your array.  
   _______ × _______ = _______  

3. a. Compare your work in Problems 1 and 2. Turn your paper as you study the arrays to look at them in different ways.  
   b. Why are the factors in your multiplication sentences in a different order?  

4. Count by the unit (the number in word form) the number of times indicated. Write the multiplication sentence that matches your count by. The first one is done for you.  
   a. 6 twos: 6 × 2 = 12  
   b. 2 sixes: _______  
   c. 7 twos: _______  
   d. 2 sevens: _______  
   e. 9 twos: _______  
   f. 2 nines: _______  
   g. 11 twos: _______  
   h. 2 twelves: _______  
   Bonus Questions:
5. Write and solve a different multiplication sentence to describe each array.

6. Ms. Nenadal writes \(2 \times 7 = 7 \times 2\) on the board. Do you agree or disagree? Draw arrays to help explain your thinking.

7. Find the missing factor to make each number sentence true.

8. Jada gets 2 new packs of erasers. Each pack has 6 erasers in it.
   a. Draw an array to show how many erasers Jada has altogether.
   b. Write and solve a multiplication sentence to describe the array.
   c. Use the commutative property to write and solve a different multiplication sentence for the array.
Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.
Lesson 7: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

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1. a. Count by 2 seven times.

   ____, ____, ____, ____, ____, ____, _____

   b. Draw an array that matches your count-by.

   c. Write a multiplication sentence that represents the total number of objects in your array.

      ______ x ______ = ______

2. a. Count by 7 two times.

   ____, _____

   b. Draw an array that matches your count-by.

   c. Write a multiplication sentence that represents the total number of objects in your array.

      ______ x ______ = ______

3. a. Compare your work in Problems 1 and 2. Turn your paper as you study the arrays to look at them in different ways.

   b. Why are the factors in your multiplication sentences in a different order?

4. Count by the unit (the number in word form) the number of times indicated. Write the multiplication sentence that matches your count-by. The first one is done for you.

   a. 2 twos: 2 x 2 = 4
   b. 3 twos: 3 x 3 = __________
   c. 2 threes: 2 x 3 = __________
   d. 2 fours: 2 x 4 = __________
   e. 4 twos: 4 x 2 = __________
   f. 5 twos: 5 x 2 = __________
   g. 2 fives: 2 x 5 = __________
   h. 2 sixes: 2 x 6 = __________
5. Write and solve a different multiplication sentence to describe each array.

6. Angel writes $2 \times 8 = 8 \times 2$ in his notebook. Do you agree or disagree? Draw arrays to help explain your thinking.

7. Find the missing factor to make each number sentence true.

8. Tamia buys 2 bags of candy. Each bag has 7 pieces of candy in it.
   a. Draw an array to show how many pieces of candy Tamia has altogether.
   b. Write and solve a multiplication sentence to describe the array.
   c. Use the commutative property to write and solve a different multiplication sentence for the array.
Lesson 7

Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

Date: 5/6/13
Lesson 8

Objective: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

Suggested Lesson Structure

- Fluency Practice (6 minutes)
- Application Problem (10 minutes)
- Concept Development (34 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)

Fluency Practice (6 minutes)

- Group Counting 3.OA.1 (3 minutes)
- Commutative Multiplying 3.OA.5 (3 minutes)

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos, threes and fours in this activity supports work with units of 2 and 3 in this topic, and anticipates work using units of 4 in Topic E.

T: Let’s count by twos to 20. Whisper then speak the numbers.
T: Let’s count by twos to 20 again. This time, hum the first number and then speak. As you hum, think of the number.
T: Let’s count by twos to 20. This time, instead of humming, think every other number.
T: What did we just count by?
S: Fours.
T: Let’s count by fours. (Direct students to count forward and backward to 20, periodically changing directions.)
T: Let’s count by threes. (Direct students to count forward and backward to 30, periodically changing directions. Emphasize the 9 to 12, 18 to 21, and 27 to 30 transitions.)
Lesson 8

Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

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Lesson 8

Commutative Multiplying (3 minutes)

Materials: (S) Personal white boards

Note: Practicing this topic that was taught in Lesson 7 helps students build confidence and automaticity with this concept.

T: (Project a picture of a 3 by 2 array.) How many groups of 2 do you see?
S: 3 groups of 2.
T: Write two different multiplication sentences for the picture.
S: (Write $3 \times 2 = 6$ and $2 \times 3 = 6$.)
Continue with possible sequence 3 by 5 and 4 by 3.
T: (Write $4 \times 2 = 2 \times ___$.) On your boards, fill in the blank.
S: (Students write $4 \times 2 = 2 \times 4$.)
Repeat process for $9 \times 5 = 5 \times ___$ and $3 \times 6 = 6 \times ___$.

Application Problem (10 minutes)

Children sit in 2 rows of 9 on the carpet for math time. Erin says, “We make 2 equal groups.” Vittesh says, “We make 9 equal groups.” Who is correct? Explain how you know using models, numbers, and words.

Note: This problem reviews the commutativity of multiplication introduced in Lesson 7 and prepares students for day 2 on the same concept lesson.

Concept Development (34 minutes)

Materials: (S) Personal white boards

Problem 1: Rotate arrays 90 degrees.

T: Turn your personal white board so it’s vertical. Skip-count by threes 4 times and write each number.
S: 3, 6, 9, 12.
T: Draw an array to match your count. How many rows and columns does your array show? Why?
S: (Draw a $4 \times 3$ array. Discuss that there are 4 rows and 3 columns because there are 4 groups of 3 in the count.)
Lesson 8

Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

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T: Turn your board so it's horizontal. How many rows and columns does it show now?
S: (Turn boards 90 degrees.) 3 rows and 4 columns.
T: What is the difference between the vertical and horizontal arrays?
S: In the first array the 4 threes were rows, and in the second they were columns. → Same with the 3 fours. First they were columns then rows, but they still show equal groups.
T: Did the total number of dots change?
S: No.
T: So the total and the factors stay the same, the factors just switch places. Yesterday we learned a special name for that. It's called...
S: Commutative! → The commutative property!
T: Use the commutative property to write 2 multiplication sentences for the array.
S: (Write 4 × 3 = 12 and 3 × 4 = 12.)
T: To make the array, you skip-counted by threes 4 times. Look at your array horizontally. Tell your partner the directions for another count-by to make the horizontal array. Check your directions by writing out the count-by.
S: Skip-count by fours 3 times. (Write 4, 8, 12.)

Students practice with partners using the following examples. Partner A gives skip-counting directions. Partner B writes the count, draws the array, and writes multiplication sentences. Then roles switch.

Skip-count by twos 3 times
Skip-count by threes 6 times

Problem 2: Interpreting rows and columns in rotated arrays.

T: Work with your partner to draw an array that shows 5 rows and 3 columns.
S: (One possible process.) Let's draw 5 circles going down to show the start of each row. → Then we can draw 3 circles to show the columns across the top. → Wait, we already drew 1 column when we made the rows, so we can just draw 2 more columns.
T: Write a multiplication sentence to match your array. Don't solve it yet.
S: (Write 5 × 3 = ____.)
T: I'm going to change the problem slightly. Listen carefully and rotate your array to match: 3 rows and 5 columns.
S: (Turn boards 90 degrees.)
T: Write the multiplication sentence for the new array. Don't solve it yet.

NOTES ON DRAWING ROWS AND COLUMNS:

Students may not immediately recognize that they do not need to redraw the corner circle to make 3 columns. After drawing rows they already have 1 column, and for this problem only need to add 2 more. If they make a mistake, help them recognize it by encouraging them to recount their total columns.
S: (Write $3 \times 5 = \underline{\phantom{0000}}$.)
T: Explain the difference between these problems to your partner.
S: The array turned and the factors switched places. → When the array turns the 3 represents the rows instead of the columns, and the 5 represents columns instead of rows.
T: When we turn the array, columns become rows and rows become columns. We call that the commutative property.
T: Solve each of your multiplication sentences by skip-counting, and write each number as you say it.
S: (Write 3, 6, 9, 12, 15 and 5, 10, 15.)

Continue with the following possible examples:
- 7 rows and 2 columns
- 3 rows and 9 columns

T: (After students have worked through the problem, write the final example in groups language: 3 groups of 9 and 9 groups of 3.) Are these statements equal? Use your array to discuss with your partner how you know.

**Problem Set (10 minutes)**

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

**Student Debrief (10 minutes)**

**Lesson Objective:** Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the
Lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Discuss the usefulness of skip-counting to solve multiplication facts.
- Build fluency by having students skip-count to find the answer to the following facts without the help of an array. They can keep track of their count using fingers.
  - 3 sixes, 6 threes
  - 3 eights, 8 threes
  - 5 threes, 3 fives
- Discuss the meaning of the commutative property and how it relates to equal groups, columns, rows and arrays.
- You may want to quickly give students practice drawing an array from a question similar to the apricot problem in the concept development.

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 8 Problem Set

Name ____________________________ Date ________________

1. a. Count by 3 five times.
   b. Draw an array that matches your count-by.

2. a. Count by 5 three times.
   b. Draw an array that matches your count-by.

3. Write multiplication expressions below to represent your arrays in Problems 1 and 2. Use the commutative property to make the equation true.

   _______ × _______ = _______ × _______
   Problem 1

   _______ × _______ = _______ × _______
   Problem 2

4. Count by the unit (the number in word form) the number of times indicated. Write the multiplication sentence that matches your count by. The first one is done for you.

   a. 2 threes: 2 × 3 = 6
   b. 3 twos: _______
   c. 3 fours: _______
   d. 4 threes: _______
   e. 3 sevens: _______
   f. 7 threes: _______
   g. 3 nines: _______
   h. 9 threes: _______
   i. 10 threes: _______

5. Find the unknowns that make the number sentences true. Then draw a line to match facts that are related.

   a. 3 + 3 + 3 + 3 + 3 = _______
   b. 3 × 9 = _______
   c. 7 threes + 1 three = _______
   d. 3 × 8 = _______
   e. _______ = 5 × 3
   f. 27 = 9 × _______
6. Isaac picks 3 tangerines from his tree every day for 7 days.
   a. Use circles to draw an array that represents the tangerines Isaac picks.

   b. How many tangerines does Isaac pick in 7 days? Write and solve a multiplication sentence.

   c. Isaac decides to pick 3 tangerines every day for 3 more days. Draw ‘x’s to show the new tangerines on the array in part A.

   d. Write and solve a multiplication sentence to find the total number of tangerines Isaac picks.

   a. How much money does Sarah spend if she buys 3 bottles of soap?

      __________ × __________ = $________

   b. How much money does she spend if she buys 6 bottles of soap?

      __________ × __________ = $_____
Name ____________________________________________ Date ______________________

1. Mary Beth organizes stickers on a page in her sticker book. She arranges them in 3 rows and 4 columns. Draw an array to show Mary Beth’s stickers.

   a. Use your array to write a multiplication sentence to find Mary Beth’s total number of stickers.

   b. Label your array to show how you skip-count to solve your multiplication sentence.

   c. Use what you know about the commutative property to write a different multiplication sentence for your array.
Lesson 8: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

Date: 5/6/13

Name ____________________________________________ Date ______________________

1. a. Count by 3 six times.
   
   ________, ________, ________, ________, ________, ________

   b. Draw an array that matches your count-by.

2. a. Count by 6 three times.
   
   ________, ________, ________

   b. Draw an array that matches your count-by.

2. Write multiplication expressions below to represent your arrays in Problems 1 and 2. Use the commutative property to make the equation true.

   ________ × ________ = ________ × ________

   Problem 1                Problem 2

3. Count by the unit (the number in word form) the number of times indicated. Write the multiplication sentence that matches your count by. The first one is done for you.

   a. 5 threes: 5 × 3 = 15
   
   d. 3 sixes: ____________

   b. 3 fives: _____________

   e. 7 threes: ____________

   c. 6 threes: ____________

   f. 3 sevens: ____________

   g. 8 threes: ____________

   h. 3 nines: ____________

   i. 10 threes: ____________

4. Find the unknowns that make the number sentences true. Then draw a line between related facts.

   a. 3 + 3 + 3 + 3 + 3 + 3 = ____________

   d. 3 × 9 = ____________

   b. 3 × 5 = ____________

   e. ____________ = 6 × 3

   c. 8 threes + 1 three = ____________

   f. 15 = 5 × ____________
5. Fernando puts 3 pictures on each page of his photo album. He puts pictures on 8 pages.
   a. Use circles to draw an array that represents the total number of pictures in Fernando’s photo album.

   b. Use your array to write and solve a multiplication sentence to find Fernando’s total number of pictures.

   c. Fernando adds 2 more pages to his book. He puts 3 pictures on each new page. Draw x’s to show the new pictures on the array in Part A.

   d. Write and solve a multiplication sentence to find the new total number of pictures in Fernando’s album.

6. Ivania recycles. She gets 3 cents for every can she recycles.
   a. How much money does Ivania make if she recycles 4 cans?

      \[ \underline{\text{________}_\times \underline{\text{________}} = \underline{\text{_______}}} \text{ cents} \]

   b. How much money does she make if she recycles 7 cans?

      \[ \underline{\text{________}_\times \underline{\text{________}} = \underline{\text{_______}}} \text{ cents} \]
Lesson 9

Objective: Find related multiplication facts by adding and subtracting equal groups in array models.

Suggested Lesson Structure

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<th>Student Debrief</th>
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<td>(15 minutes)</td>
<td>(35 minutes)</td>
<td>(10 minutes)</td>
</tr>
<tr>
<td>Total Time</td>
<td>(60 minutes)</td>
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Fluency Practice (15 minutes)

- Multiply By 2 3.OA.7 (7 minutes)
- Group Counting 3.OA.1 (4 minutes)
- Forms of Multiplication 3.OA.1 (4 minutes)

Sprint: Multiply by 2 (7 minutes)

Materials: (S) Multiply by 2 (1–5) Sprint

Note: This activity builds fluency with multiplication facts using units of 2. It works toward students knowing from memory all products of two one-digit numbers.

T: (Write $2 \times 5 = \underline{\hspace{2cm}}$.) Let’s skip-count by twos to find the answer. (Count with fingers to 5 as students count.)
S: 2, 4, 6, 8, 10.
T: (Circle 10 and write $2 \times 5 = 10$ above it. Write $2 \times 3 = \underline{\hspace{2cm}}$.) Let’s skip-count up by twos again. (Count with fingers to 3 as students count.)
S: 2, 4, 6.
T: Let’s see how we can skip-count down to find the answer, too. Start at 10 with 5 fingers, 1 for each two. (Count down with your fingers as students say numbers.)
S: 10 (five fingers), 8 (4 fingers), 6 (3 fingers).

Repeat the process for $2 \times 4$.

T: Let’s practice multiplying by 2.
Directions for Administration of Multiply By Sprint

- Distribute Multiply By Sprint.
- Allow a maximum of 2 minutes for students to complete as many problems as possible.
- Direct students to work left to right across the page.
- Encourage skip-counting strategies to solve unknown facts.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by threes and fours in this activity supports work with units of 3 in this topic, and anticipates work using units of 4 in Topic E.

T: Let’s count by fours. (Direct students to count forward and backward to 24, emphasizing the 16 to 20 transition.)
T: Let’s count by threes. (Direct students to count forward and backward to 30, emphasizing transition from 18 to 21.)

Forms of Multiplication (4 minutes)

Materials: (S) Personal white boards

Note: Students directly relate repeated addition to multiplication in preparation for using the distributive property in this Lesson.

T: (Project a 3 by 5 picture array.) Represent this array as a repeated addition sentence using 5 as the size of the groups.
S: (Write 5 + 5 + 5 = 15.)
T: (Project a 3 by 4 array. Write _____ fours = _____.) Complete the expression on your personal board.
S: (Write 3 fours = 12.)
T: (Project a 7 by 2 array.) Write 2 multiplication sentences for 7 groups of 2.
S: (Write 7 × 2 = 14 and 2 × 7 = 14.)
T: (Project a 6 by 3 array.) Write 18 = 6 × _____.) Complete the expression on your personal board.
S: (Write 18 = 6 × 3.)
T: (Project a 5 by 3 array. Write 5 threes = _____.) Complete the expression on your personal board.
S: (Write 5 threes = 15.)
T: (Add one more group of 3 to the array. Write 5 threes + 1 three = _____ threes = ____ ones.)
S: (Write 5 threes + 1 three = 6 threes = 18 ones.)
Concept Development (35 minutes)

Materials: (S) Personal white board, Threes Array No Fill Template (pictured at right), blank paper

Problem 1
Add 2 known smaller facts to solve an unknown larger fact.

T: Slip the template into your personal white board. Cover part of the array with blank paper to show 5 rows of 3. Draw a box around the uncovered array. Write and solve a multiplication sentence to describe it.

S: (Cover, then box array and write $5 \times 3 = 15$.)

T: Move the paper so the array shows $7 \times 3$. Shade the rows you added.

S: (Shade 2 rows.)

T: Write and solve a multiplication sentence to describe the shaded part of your array.

S: (Write $2 \times 3 = 6$.)

T: How many threes are in $5 \times 3$?

S: 5 threes.

T: How many threes did you add to $5 \times 3$ to make the array show $7 \times 3$?

S: 2 threes.

T: (Write $7 \text{ threes} = 5 \text{ threes} + 2 \text{ threes}$.) So $7 \text{ threes}$ equals $5 \text{ threes}$ plus $2 \text{ threes}$. (Write $7 \times 3 = 5 \times 3 + 2 \times 3$ as shown to the right.) Do you agree or disagree?

S: I agree. That’s just adding the 2 parts of the array together. $\rightarrow 7$ rows of three is the same as $5$ rows of three plus $2$ rows of three.

T: We already wrote totals for the 2 parts of our array. Let’s add those to find the total for the whole array. What is the total of $5 \times 3$?

S: 15.

T: (Write $15 +$ on the board.) What is the total of $2 \times 3$?

S: 6.

T: (Add to the board so the equation reads $\_ \_ \_ = 15 + 6$.) Say the total at the signal. (Signal.)

S: 21.

Provide students with another example. Have them use the template to add the totals of $4 \times 3$ and $4 \times 3$ to find the answer to $8 \times 3$. Teach them to double the total for $4 \times 3$. 

NOTES ON MULTIPLE MEANS FOR REPRESENTATION:

Decomposing this way naturally relates to the part-whole relationship that students studied in grades K-2. The vignette implies the relationship, but you may want to make a more formal connection to prior knowledge.

Sample teacher board

$$7 \text{ threes} = 5 \text{ threes} + 2 \text{ threes}$$

$$7 \times 3 = 5 \times 3 + 2 \times 3$$

$$21 = 15 + 6$$
T: Explain how we added to find $7 \times 3 = 21$ and $8 \times 3 = 24$.
S: We added the totals of smaller facts together to find the whole.  We used 2 facts we already knew to find one we didn’t know.

**Problem 2**

Subtract 2 known smaller facts to solve an unknown larger fact.

T: Draw a box around an array that shows $9 \times 3$. Notice that $9 \times 3$ is very close to $10 \times 3$. $10 \times 3$ is easier to solve because we can count by tens to get the total. Let’s do that now.
S: 10, 20, 30.
T: Let’s use $10 \times 3 = 30$ to help us solve $9 \times 3$.
T: What should we subtract to show 9 threes instead?
S: 1 three!

T: (Write 10 threes – 1 three = _____ on the board.) 10 threes equals?
S: 30.
T: 30 – 3 equals?
S: 27.

Provide another example. Have students subtract to find the answer to $8 \times 3$. $10 \times 3$ is the basic fact, so the subtraction to find $8 \times 3$ is $30 – 6$.

T: Tell your partner how we used $10 \times 3$ to help us find the answer to $9 \times 3$ and $8 \times 3$.
S: (Discuss.)

**Problem Set (10 minutes)**

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models.

Student Debrief (10 minutes)

Lesson Objective: Find related multiplication facts by adding and subtracting equal groups in array models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the ideas below to lead the discussion.

- Review the strategy of adding and subtracting the totals of known “easy” facts for solving unknown facts
- Differentiate between when to apply addition or subtraction through analysis of the example $8 \times 3$ from concept lesson. (Students solved $8 \times 3$ using both addition and subtraction.) You may then ask students to apply the strategy to solve $8 \times 4$.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models.

Multiply.

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Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models.

Date: 5/6/13

Name __________________________ Date __________________

1. The team organizes soccer balls into 2 rows of 5. The coach adds 3 rows of 5 soccer balls. Complete the number sentences to describe the total array.

   \[ (5 + 5) + (5 + 5 + 5) = \underline{\ \ \ \ \ \ } \]
   \[ 2 \text{ fives} + \underline{\ \ \ } \text{fives} = \underline{\ \ \ \ \ } \text{fives} \]
   \[ \underline{\ \ \ } \times 5 = \underline{\ \ \ \ \ } \]

2. \[ 7 \times 2 = \underline{\ \ \ \ } \]
   \[ 5 \times 2 = \underline{\ \ \ } \]
   \[ 2 \times 2 = \underline{\ \ \ } \]

3. \[ 9 \times 2 = \underline{\ \ \ \ } \]
   \[ 10 \times 2 = \underline{\ \ \ } \]
   \[ 1 \times 2 = \underline{\ \ \ } \]

1. The team organizes soccer balls into 2 rows of 5. The coach adds 3 rows of 5 soccer balls. Complete the number sentences to describe the total array.

   \[ (5 + 5) + (5 + 5 + 5) = \underline{\ \ \ \ \ \ } \]
   \[ 2 \text{ fives} + \underline{\ \ \ } \text{fives} = \underline{\ \ \ \ \ } \text{fives} \]
   \[ \underline{\ \ \ } \times 5 = \underline{\ \ \ \ \ } \]

2. \[ 7 \times 2 = \underline{\ \ \ \ } \]
   \[ 5 \times 2 = \underline{\ \ \ } \]
   \[ 2 \times 2 = \underline{\ \ \ } \]

3. \[ 9 \times 2 = \underline{\ \ \ \ } \]
   \[ 10 \times 2 = \underline{\ \ \ } \]
   \[ 1 \times 2 = \underline{\ \ \ } \]
   a. Draw an array that represents Matthew’s cards using an x to show each card.

   b. Solve the multiplication sentence to find Matthew’s total number of cards.  \( 4 \times 3 = \) ______

5. Matthew adds 2 more rows. Use circles to show his new cards on the array in part 4a.
   a. Write and solve a multiplication sentence to represent the circles you added to the array.

   \[
   _____ \times 3 = _____
   \]

   b. Add the totals from the multiplication facts in 4b and 5a to find Matthew’s total cards.

   \[
   _____ + _____ = 18
   \]

   c. Write the multiplication sentence that shows Matthew’s total number of cards.

   \[
   _____ \times _____ = 18
   \]
Lesson 9 Exit Ticket

1. Mrs. Stern roasts cloves of garlic. She places 10 rows of two cloves on a baking sheet.
   Write a multiplication sentence to describe the number of cloves Mrs. Stern bakes.
   \[ \underline{\quad} \times \underline{\quad} = \underline{\quad} \]

2. When the garlic is roasted, Mrs. Stern uses some for a recipe, leaving 2 rows of two garlic cloves on the pan.
   a. Complete the number sentence below to show how many garlic cloves she uses.
      \[ \underline{\quad} \text{twos} - \underline{\quad} \text{twos} = \underline{\quad} \text{twos} \]
   b. \[ 20 - \underline{\quad} = 16 \]
   c. Write a multiplication sentence to describe the number of garlic cloves she uses.
      \[ \underline{\quad} \times 2 = \underline{\quad} \]
Name ____________________________ Date __________________

1. Dan organizes his star stickers into 3 rows of 4. Irene adds 2 more rows of stickers. Complete the number sentences to describe the total number of stickers in the array.

\[
\begin{array}{c}
\text{Dan:} \\
\text{Irene:} \\
\text{Total:}
\end{array}
\]

a. \((4 + 4 + 4) + (4 + 4) = \underline{\hspace{1cm}}\)
b. \(3 \text{ fours} + \underline{\hspace{1cm}} \text{ fours} = \underline{\hspace{1cm}} \text{ fours}\)
c. \(\underline{\hspace{1cm}} \times 5 = \underline{\hspace{1cm}}\)

2. \(7 \times 2 = \underline{\hspace{1cm}}\)

\[
\begin{array}{c}
\text{Array:} \\
\text{6 rows:} \\
\text{1 row:} \\
\text{Total:}
\end{array}
\]

\(6 \times 2 = \underline{\hspace{1cm}}\)

\(1 \times 2 = \underline{\hspace{1cm}}\)

\(12 + 2 = \underline{\hspace{1cm}}\)

\(\underline{\hspace{1cm}} \times 2 = 14\)

3. \(9 \times 3 = \underline{\hspace{1cm}}\)

\[
\begin{array}{c}
\text{Array:} \\
\text{10 rows:} \\
\text{1 row:} \\
\text{Total:}
\end{array}
\]

\(10 \times 3 = \underline{\hspace{1cm}}\)

\(1 \times 3 = \underline{\hspace{1cm}}\)

\(30 - \underline{\hspace{1cm}} = 27\)

\(\underline{\hspace{1cm}} \times 3 = 27\)
4. Franklin collects stickers. He organizes his stickers in 5 rows of 4 on his table.

Draw an array that represents Franklin’s stickers using an x to show each sticker.

\[ 5 \times 4 = \ \ \ \ \ \] 

5. Franklin adds 2 more rows. Use circles to show his new stickers on the array in part 3a.

a. Write and solve a multiplication sentence to represent the circles you added to the array.

\[ \ \ \ \ \times 4 = \ \ \ \ ] 

b. Complete the addition sentence to show how you added the totals of 2 multiplication facts to find Franklin’s total number of stickers.

\[ \ \ \ \ + \ \ \ \ = \ \ \ \ ] 

c. Complete the unknown to show Franklin’s total number of stickers.

\[ \ \ \ \times 4 = \ \ \ ]
Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models.

Date: 5/6/13
Lesson 10

Objective: Model the distributive property with arrays to decompose units as a strategy to multiply.

Suggested Lesson Structure

- **Fluency Practice** (12 minutes)
- **Application Problem** (5 minutes)
- **Concept Development** (33 minutes)
- **Student Debrief** (10 minutes)
- **Total Time** (60 minutes)

**Fluency Practice (12 minutes)**

- Multiply By 2 3.OA.7 (8 minutes)
- Group Counting 3.OA.1 (4 minutes)

**Sprint: Multiply by 2 (8 minutes)**

Materials: (S) Multiply by 2 (6–10) Sprint

Note: This activity builds fluency with multiplication facts using units of 2. It works toward students knowing from memory all products of two one-digit numbers. See Directions for Administration of Multiply By in Lesson 9.

T: (Write $2 \times 7 = \_\_\_\_$.) Let’s skip-count up by twos. (Count with fingers to 7 as students count.)

S: 2, 4, 6, 8, 10, 12, 14.

T: Let’s skip count by twos starting at 10.

S: (Show 5 fingers) 10, 12, 14. (Count with fingers to 7 as students count.)

T: Let’s see how we can skip-count down to find the answer, too. (Show 10 fingers) Start at 20. (Count down with your fingers as student say numbers.)

S: 20, 18, 16, 14.

Repeat the process for $2 \times 9$ and $2 \times 8$.

T: (Distribute Multiply by 2 Sprint.) Let’s get some practice multiplying by 2. Be sure to work left to right across the page.
Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by threes and fours in this activity supports work with units of 3 in this topic, and anticipates work using units of 4 in Topic E.

T: Let’s count by fours. (Direct students to count forward and backward to 24, emphasizing the 16 to 20 transition.)
T: Let’s count by threes. (Direct students to count forward and backward to 30, emphasizing transition from 18 to 21.)

Application Problem (5 minutes)

A guitar has 6 strings. How many strings are there on 3 guitars?
Write a multiplication sentence to solve.

Note: This problem leads into today’s concept development. Students will compare their multiplication equation with the new equations presented in the lesson.

Concept Development (33 minutes)

Materials: (S) Personal white boards, 1 sheet of blank paper per student

T: Draw an array to represent the total number of guitar strings. Let the number of strings on 1 guitar be one row.
S: (Draw a 3 by 6 array, shown to the right.)
T: Make a dotted line below the first row to show just 1 guitar.
T: Write and solve a multiplication sentence to describe each part of your array.
S: (Write $1 \times 6 = 6$ and $2 \times 6 = 12$, as shown to the right.)
T: (Write $6 + 12 = 3$ sixes.) Why is this true?
S: 1 six is 6, 2 sixes are 12. When I add 6 and 12, I get 18, which is 3 sixes.
T: (Write $(1 \times 6) + (2 \times 6) = 3$ sixes on the board as shown to the right.) How do you know the 2 number sentences on the board are equal?
S: $1 \times 6$ is the same as 6, and $2 \times 6$ is the same as 12. You just rewrote 6 and 12 as multiplication facts.
T: (Write $(1 \times 6) + (2 \times 6) = 6 + \_\_\_\_\_\_\_\_$.) With your partner discuss what number completes the equation.
S: $1 \times 6$ equals 6. That’s how the teacher got 6. To get the
other number we do $2 \times 6$. That's 12. → I know its 12 because you need the same amount on each side of the equal sign. On the left the value is $6 + 12$ if you solve the multiplication. That’s what it should be on the right too.

T: (Write 12 to complete the equation.)

T: Notice the symbols around my multiplication expressions. They are called parentheses. Let’s say that word together.

S: Parentheses.

T: (Write $(1 \times 6) + (2 \times 6) = ___$ and $(1 + 2) \times 6 = ___$ below it as shown to the right.) My parentheses show how I make groups. How did I rearrange the groups?

S: You added the number of rows. Then you multiplied by 6.

T: Look back at the array you drew. Do the 1 and 2 represent the number of groups or the size of groups?

S: The number of groups.

T: What does the 6 represent?

S: The size of the groups.

T: Use that language—the number of groups and the size of groups—to tell your partner about my second equation.

S: The teacher added the number of groups first. That’s $1 + 2$. Then she multiplied the number of groups times the size of the groups, which is 6.

T: $1 + 2$ equals?

S: 3.

T: (Write $3 \times 6 = _____$ under the second equation.) Look back at the work you did on today’s application problem. How does this equation compare with what you did?

S: It’s the same! → It’s the number of groups times the size of groups, just like we did.

T: Rewrite each equation on your board and solve them. What is the answer to all 3 equations?

S: 18.

T: (Complete the equations on the board.) Think back to the problem we’re solving. 18 what?

S: 18 strings.

T: (Write $(1 \times 6) + (2 \times 6) = 3 \times 6$ on the board.) True or false?

S: True.

T: In your own words, tell your partner how we got $3 \times 6$ and why it’s equal to $(1 \times 6) + (2 \times 6)$. Use the 3 equations you just solved to help you explain.

S: (Students retell the steps using the 3 equations and solutions to guide them.)
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Model the distributive property with arrays to decompose units as a strategy to multiply.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- In Problems 1 and 2, why might breaking an array into 2 parts to multiply, add, then solve be easier than just multiplying the total number of groups times their size?
- Check Problem 3(a) by drawing and writing on the board as students give you verbal directions for how to create the page in Ruby’s photo album. Then invite several students to share their work on 3(b).
- Understand that 5×3 is the result of the number of groups added together and then multiplied by the size of groups in (2×3) + (3×3).
- Understand that 6 and 9 are the products of each multiplication expression.
- Relate the factors in 5×3 to the number of groups and size of groups in the array.
- Recognize that both sides of the equation 5×3 = 6 + 9 have a value of 15.
Exit Ticket  (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

NOTES ON VOCABULARY:

In this lesson students are not responsible for the vocabulary *distributive property*. Students revisit the distributive property as a strategy for multiplication and division in Topics E and F. In those lessons they name the strategy with precise vocabulary as they use it.
Lesson 10: Model the distributive property with arrays to decompose units as a strategy to multiply.

Date: 5/6/13

Multiply.

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Lesson 10: Model the distributive property with arrays to decompose units as a strategy to multiply.

1. \(7 \times 3 = (5 \times 3) + (2 \times 3) = \) 

\[
\begin{array}{l}
\text{(5 \times 3) = 15} \\
\text{(2 \times 3) = } \\
(5 \times 3) + (2 \times 3) = 15 + \\
15 + \text{______ = ________}
\end{array}
\]

2. \(8 \times 3 = (4 \times 3) + (4 \times 3) = \) 

\[
\begin{array}{l}
\text{(______ \times 3) = _____} \\
\text{(______ \times 3) = _____} \\
(4 \times 3) + (4 \times 3) = \text{_______ + _________} \\
\text{_______ \times 3 = _______}
\end{array}
\]
3. Ruby is making a photo album. She puts 3 pictures in each row.

   a. Use the multiplication sentences on the left. Draw arrays to show the photos on the upper and lower parts of Ruby’s album page.

   b. Ruby calculates the total number of pictures as shown below. Use the array you drew to help explain her calculation.

   $5 \times 3 = 6 + 9 = 15$
Lesson 10 Exit Ticket

1. \(6 \times 3 = \) ______
   \(\{\) (4 \(\times 3) = \) ______
   \(\{\) (2 \(\times 3) = \) ______
   \((4 \times 3) + (2 \times 3) = \) ______+ ______
   \(6 \times 3 = \) ______+ ______
   
   ______ \(\times 3 = \) ______

2. \(7 \times 3 = \) ______
   \(\{\) (_____ \(\times 3) = \) ______
   \(\{\) (_____ \(\times 3) = \) ______
   \((5 \times 3) + (2 \times 3) = \) ______+ ______
   \(7 \times 3 = \) ______+ ______
   
   ______ \(\times 3 = \) ______
1. $6 \times 3 = \underline{18}$

   \[
   \begin{array}{c}
   \{ \text{(4 \times 3) = 12} \\
   \{ \text{(2 \times 3) = } \underline{6} \} \\
   \underline{12 + \underline{6} = \underline{18}} \\
   \end{array}
   \]

   $6 \times 3 = \underline{18}$

2. $8 \times 2 = \underline{16}$

   \[
   \begin{array}{c}
   \{ \text{(4 \times 2) = } \underline{8} \} \\
   \{ \text{(4 \times 2) = } \underline{8} \} \\
   \underline{4 \times 2 + 4 \times 2} = \underline{8} + \underline{8} \\
   \underline{\text{____ \times 2 = } \underline{16}} \\
   \end{array}
   \]
3. Adriana is organizing her books on shelves. She puts 3 books in each row.

   a. Use the multiplication sentences on the right to draw arrays to show the books on Adriana’s top and bottom shelves.

      \[
      \begin{align*}
      \square \times 3 &= 15 \\
      \square \times 3 &= 3
      \end{align*}
      \]

   b. Adriana calculates the total number of books as shown below. Use the array you drew to help explain her calculation.

      \[6 \times 3 = 15 + 3 = 18\]