Topic D
Division Using Units of 2 and 3

3.OA.2, 3.OA.4, 3.OA.6, 3.OA.7, 3.OA.3, 3.OA.8

Focus Standard:

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?

3.OA.6 Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Instructional Days: 3

Coherence - Links from: G2–M6 Foundations of Multiplication and Division

- Links to: G4–M3 Using Place Value Understanding and Properties of Operations to Perform Multi-Digit Multiplication and Division

In Topic D students solve two kinds of division situations—partitive (group size unknown) and measurement (number of groups unknown)—using factors of 2 and 3. The tape diagram is introduced in Lesson 11 as a tool to help students recognize and distinguish between types of division. By the end of Lessons 11 and 12 students independently draw and label tape diagrams that help them to compare and analyze problems that may use the same division sentence, but have quotients representing different things.

Lesson 13 solidifies growing understanding that the unknown can also be found from the related multiplication sentence. Students initially work through word problems using arrays and tape diagrams to practice solving the two types of division, then transition to problem solving using abstract division and multiplication equations.
### A Teaching Sequence Towards Mastery of Division Using Units of 2 and 3

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Lesson 11

Objective: Model division as the unknown factor in multiplication using arrays and tape diagrams.

Suggested Lesson Structure

- Fluency Practice (11 minutes)
- Application Problem (5 minutes)
- Concept Development (34 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (11 minutes)

- Multiply by 3 3.OA.7 (8 minutes)
- Group Counting 3.OA.1 (3 minutes)

Sprint: Multiply by 3 (8 minutes)

Materials: (S) Multiply by 3 (1–5) Sprint

Note: This activity builds fluency with multiplication facts using units of 2. It works toward students knowing from memory all products of two one-digit numbers. See Directions for Administration of Multiply By Sprint in Lesson 9.

T: (Write 3 x 5 = ____.) Skip-count by threes to find the answer. (Raise a finger for each number to track the count.)

S: 3, 6, 9, 12, 15.

T: (Circle 15 and write 3 x 5 = 15 above it. Write 3 x 4 = ____.) Skip-count up by threes to find the answer. (Track with fingers as students count.)

S: 3, 6, 9, 12.

T: Let’s count down to find the answer to 3 x 4, too. Start at 15. (Count down with your fingers as students say numbers.)

S: 15, 12.

T: Let’s practice multiplying by 3. Be sure to work left to right across the page. (Distribute Multiply by 3 Sprint.)
Lesson 11:

Model division as the unknown factor in multiplication using arrays and tape diagrams.

Date: 5/6/13

Lesson 11:
Model division as the unknown factor in multiplication using arrays and tape diagrams.

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and fours in this activity reviews multiplication with units of 2 from Topic C, and anticipates using units of 4 in Topic E.

Count by fours to 32 forward and backward. Focus on the 20 to 24 and 28 to 32 transitions.
Count by twos to 20 forward and backward.

Application Problem (5 minutes)

Rosie puts 2 lemon slices in each cup of iced tea. She uses a total of 8 slices. How many cups of iced tea does Rosie make?

Note: Students may have solved the problem as shown, or using division \(8 ÷ 2 = 4\). This problem leads into modeling with tape diagrams, which is introduced in the concept development.

\[
\text{___} \times 2 = 8
\]

Rosie makes 4 cups of iced tea.

Concept Development (34 minutes)

Materials: (S) Personal white boards

Problem 1: Relate arrays to tape diagrams, modeling division where the quotient represents the number of groups.

(Write or project a 2 by 4 array.)

T: The columns in this array show the number of lemon slices in 1 cup of Rosie’s iced tea. Reread our application problem and tell your partner what the unknown represents.

S: The unknown is the number of cups, or groups.

T: How might this array help us solve \(8 ÷ 2 = \text{___}\)?

S: We can count the number of columns to find how many cups. \(\rightarrow 2 \text{ times } 4 \text{ equals } 8\), so \(8 ÷ 2 = 4\).
Lesson 11:

Model division as the unknown factor in multiplication using arrays and tape diagrams.

Date: 5/6/13

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NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Students are familiar with tape diagrams from Grade 2. They use tape diagrams to represent the information given in a problem, and then analyze the model to help determine the unknown and solve. As you review the vocabulary tape diagram ask why the diagram might have that name. Guide students to make connections that help them remember the name.

T: (Draw a rectangle around the array.) What is the total number of lemon slices?
S: 8 lemon slices.

T: (Bracket the rectangle and label the whole 8 lemon slices.) The question asks how many cups of iced tea Rosie makes. Do the cups represent the number of groups or the number of lemon slices in each group?
S: The number of groups.

T: (Under ‘8 lemon slices’ label the unknown as ‘? cups’.)
T: Watch how I show the number of slices in one cup.
(Draw lines to divide columns and label 1 unit as ‘2 slices’.) Where do we see the cups in our diagram?
S: You made 4 glasses with the dividing lines.

T: By adding lines and labels to our array we made a tape diagram. Each boxed column shows 1 unit. 1 unit represents 1 cup, and has a value of 2 slices. Notice that I labeled the diagram with all of the known and unknown information from the problem as we solved. That made it a helpful tool for understanding the problem.
T: (Write \(8 \div 2 = \_\_\_\_\) and \(\_\_\_\_ \times 2 = 8\).) Talk to your partner about how the tape diagram helps you see the unknown in both number sentences.
S: (Discuss.)

In Problem 1, the quotient represents the number of groups. Repeat the process using the following examples, reminding students to label known and unknown information from the problem on every tape diagram.

\[
10 \div 2 = 5 \\
18 \div 3 = 6
\]

Problem 2: Use arrays to draw tape diagrams, modeling division where the quotient represents the number of objects in each group.

Write or project the following problem: Ms. Alves puts 21 papers in 7 piles. How many papers are in each pile?

T: Read the problem. What is unknown?
S: The number of objects in each group.

T: Model the problem as an array where each column represents 1 pile.
S: (Draw array, shown at right.)
T: Count to find how many papers are in each of Ms. Alves’ piles.
S: (Count to find 3 papers.)
Lesson 11: Model division as the unknown factor in multiplication using arrays and tape diagrams.

Date: 5/6/13

T: Work with a partner to model the problem as a tape diagram. Be sure to label the diagram with known and unknown information. Use your array to help.

S: (Model, tape diagram shown to the right.)

T: Use the tape diagram to write multiplication and division sentences that show the unknown.

S: (Write $7 \times ____ = 21$ and $21 \div 7 = ____$.)

In Problem 2, the quotient represents the number of objects in each group. Repeat the process using the following examples:

- $16 \div 2 = 8$
- $24 \div 3 = 8$

T: Compare models. What are the similarities and differences between arrays and tape diagrams?

S: The tape diagram is like a labeled and boxed array. → They both show the 7 piles, 3 papers in each pile, and 21 papers total. → The labels make the tape diagram a little easier to use.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Depending on your class, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Model division as the unknown factor in multiplication using arrays and tape diagrams.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.
- Compare Problem 1 and 2. What does the unknown represent in each problem?
- Compare how units are represented in tape diagrams and in arrays.
- How can each model represent both types of unknowns?
- Compare the way you solved the application problem with the tape diagram model we learned today.

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Multiply.

3 x 1 = _______ 3 x 2 = _______ 3 x 3 = _______ 3 x 4 = _______

3 x 5 = _______ 3 x 1 = _______ 3 x 2 = _______ 3 x 1 = _______

3 x 3 = _______ 3 x 1 = _______ 3 x 4 = _______ 3 x 1 = _______

3 x 5 = _______ 3 x 1 = _______ 3 x 2 = _______ 3 x 3 = _______

3 x 2 = _______ 3 x 4 = _______ 3 x 2 = _______ 3 x 5 = _______

3 x 2 = _______ 3 x 1 = _______ 3 x 2 = _______ 3 x 3 = _______

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3 x 5 = _______ 3 x 3 = _______ 3 x 2 = _______ 3 x 4 = _______

3 x 3 = _______ 3 x 5 = _______ 3 x 2 = _______ 3 x 4 = _______
1. Mrs. Prescott has 12 oranges. She puts 2 oranges in each bag. How many bags does she have?
   a. Draw an array where each column shows a bag of oranges.

   \[ \square \square \div 2 = \square \square \]

   b. Redraw the oranges in each bag as a unit in the tape diagram. The first unit is done for you. As you draw, label the diagram with known and unknown information from the problem.

2. Mrs. Prescott arranges 18 plums into 6 bags. How many plums are in each bag? Model the problem with both an array and a labeled tape diagram. Show each column as the number of plums in each bag.

   There are _______ plums in each bag.
3. Fourteen shopping baskets are stacked equally in 7 piles. How many baskets are in each pile? Model the problem with both an array and a labeled tape diagram. Show each column as the number of baskets in each pile.

4. In the back of the store, Mr. Prescott packs 24 bell peppers equally into 8 bags. How many bell peppers are in each bag? Model the problem with both an array and a labeled tape diagram. Show each column as the number of bell peppers in each bag.

5. Olga saves $2 a week to buy a toy car. The car costs $16. How many weeks will it take her to save enough to buy the toy?
Ms. McCarty has 18 stickers. She puts 2 stickers on each homework paper. How many homework papers does she have? Model the problem with both an array and a labeled tape diagram.
1. Fred has 10 pears. He puts 2 pears in each basket.
   
a. Draw an array where each column represents a basket of pears.
   
   \[ \_ \div 2 = \_ \]
   
   b. Redraw the pears in each basket as a unit in the tape diagram. Label the diagram with known and unknown information from the problem.

2. Ms. Meyer organizes 15 clipboards equally into 3 boxes. How many clipboards are in each box? Model the problem with both an array and a labeled tape diagram. Show each column as the number of clipboards in each box.

   There are \_ \_ \_ \_ clipboards in each box.
3. Sixteen action figures are arranged equally on 2 shelves. How many action figures are on each shelf? Model the problem with both an array and a labeled tape diagram. Show each column as the number of action figures on each shelf.

4. Jasmine puts 18 hats away. She puts an equal number of hats on 3 shelves. How many hats are on each shelf? Model the problem with both an array and a labeled tape diagram. Show each column as the number of hats on each shelf.

5. Corey checks out 2 books a week from the library. How many weeks will it take him to check out a total of 14 books?
Lesson 12

Objective: Interpret the quotient as the number of groups or the number of objects in each group using units of 2.

Suggested Lesson Structure

- Fluency Practice (15 minutes)
- Application Problem (5 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (15 minutes)

- Multiply by 3  3.OA.7 (8 minutes)
- Group Counting  3.OA.1 (4 minutes)
- Divide  3.OA.7 (3 minutes)

Sprint: Multiply by 3 (8 minutes)

Materials: (S) Multiply by 3 Sprint (6—10)

Note: This activity builds fluency with multiplication facts using units of 2. It works toward students knowing from memory all products of two one-digit numbers. See Directions for Administration of Multiply By in Lesson 9.

T:  (Write 3 x 6 = ____.) Let’s skip-count up by threes to solve. (Count with fingers to 6 as students count.)
S:  3, 6, 9, 12, 15, 18.
T:  Let’s skip-count down to find the answer, too. Start at 30. (Count down with fingers as students count.)
S:  30, 27, 24, 21, 18.

Repeat the process for 3 x 8 and 3 x 7.

T:  Let’s practice multiplying by 3. Be sure to work left to right across the page. (Distribute Multiply by 3 Sprint.)

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and fours in this activity reviews multiplication with units of 2 from Topic C, and anticipates using units of 4 in Topic E.
Lesson 12

T: Let’s count by fours. (Direct students to count forward and backward to 36, emphasizing the 20 to 24 and 28 to 32 transitions.)
T: Let’s count by twos. (Direct students to count forward and backward to 20.)

Divide (3 minutes)
Materials: (S) Personal white boards

Note: This activity builds fluency with multiplication and division. It works toward the goal of students knowing from memory all products of two one-digit numbers, and reviews the objective of Lesson 11.

T: (Project a 2 by 4 array of objects.) Draw an array to match my picture.
S: (Draw 2 by 4 array.)
T: Skip-count by twos to find how many total objects there are. (Point as students count.)
S: 2, 4, 6, 8.
T: How many groups of 2 are there?
S: 4.
T: Say the total as a multiplication sentence starting with the number of groups.
S: 4 x 2 = 8.
T: (Write 4 x 2 = 8. Below it, write 8 ÷ ___.) Write the division sentence. Then divide your array into 4 equal groups to find the answer.
S: (Draw lines separating array into 4 groups of 2 and write 8 ÷ 4 = 2.)
T: Erase the lines that divided the array.
S: (Erase lines.)
T: Show 8 ÷ 4 by making groups of 4.
S: (Circle 2 groups of 4.)

Repeat process for possible sequence: 9 ÷ 3, 12 ÷ 2, 12 ÷ 3.

Application Problem (5 minutes)

A chef arranges 4 rows of 3 red peppers on a tray. He adds 2 more rows of 3 yellow peppers. How many peppers are there altogether?

Note: Students might solve using an array to model the distributive property (Lesson 10) or the tape diagram (Lesson 11). If they use the latter strategy, it is likely their first use of a tape diagram to solve multiplication. The problem is a review and provides an exploratory opportunity for students to select and use appropriate tools.

Interpret the quotient as the number of groups or the number of objects in each group using units of 2.

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Concept Development (30 minutes)

Materials: (S) Personal white boards

Problem 1: Model division where the unknown represents the number of objects in each group.

T: 2 students equally share 8 crackers. How many crackers does each student get? Draw to model and solve the problem. Then explain your thinking to your partner.

S: (Draw and solve.) I gave 1 cracker to each student until I drew 8. \( \rightarrow 4 + 4 = 8 \), so I drew 4 crackers for each student. \( \rightarrow \) It’s a multiplication problem with an unknown factor.

T: Write a division sentence to represent your model.

S: (Write \( 8 \div 2 = 4 \).)

T: (Draw a rectangle.) This diagram represents the total, 8 crackers. In your mind, visualize where we would divide it to make 2 equal parts.

S: (Visualize.)

T: Say “stop” when I get to the spot you have in mind. (Move finger from left edge toward middle.)

S: Stop!

T: How does the diagram represent the students?

S: 2 students, 2 parts!

T: What is our unknown?

S: The number of crackers each student gets.

T: Watch how I label the unknown on the diagram. (Bracket and label as shown.) Tell your partner a strategy for finding the unknown using the diagram.

S: I would draw 1 cracker in each part until I drew 8. \( \rightarrow \)

Each part has to be equal. \( 4 + 4 = 8 \), so 1 part is 4. \( \rightarrow \)

I would think \( 2 \times \_\_\_ = 8 \). The question mark is 4.

T: Look at the division sentence you wrote for your first model. Does it represent this diagram too? Explain to your partner.

S: (Discuss.)

Repeat the process with the following suggested examples to model division where the quotient represents the number of objects in each group.

\[
12 \div 2 \\
18 \div 2
\]
Problem 2: Model division where the unknown represents the number of groups.

T: Let’s go back to our original problem, this time changing it a bit: There are 8 crackers, but this time each student gets 2. How many students get crackers?

T: Do we know the size of the groups or the number of groups?

S: The size of the groups.

T: We can draw 1 unit of the diagram to represent a group of 2 crackers. (Draw 1 unit of two.) What other information does the problem tell us?

S: The total.

T: (Estimate the whole and label it 8). Notice I drew a dotted line to show the whole diagram. What is our unknown?

S: The number of groups.

T: (Bracket the top part of the diagram and label with a question mark.) Let’s find the number of groups by drawing more units of two. How will we know when we’ve drawn enough units?

S: We’ll get to the total, 8.

T: Draw with me on your board. (Skip-count by two, drawing to add 3 more units.)

S: (Draw.)

T: Whisper to your partner the number of students that get crackers.

S: 4 students.

T: Write a division sentence to match the diagram.

S: (Write $8 \div 2 = 4$.)

Repeat the process with the following suggested examples to model division where the unknown represents the number of groups.

- $12 \div 2$
- $18 \div 2$

In this lesson, three division sentences are each modeled with two types of division. Use one pair of division sentences for the following reflective dialogue. (The dialogue is modeled with $8 \div 2 = 4$.)

T: The two division sentences for these diagrams are the same, but the tape diagrams are different. Turn and talk to your partner about why.

S: They use the same numbers. → The 2 and the 4 represent different things in each problem. → In the
first diagram we knew how many groups, and in the second we knew how many in each group.

T: When we divide we always know the total number of objects. We divide either to find the size of the groups like in the first problem, or the number of groups like in the second problem.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Interpret the quotient as the number of groups or the number of objects in each group using units of 2.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Ask students to describe how they labeled the tape diagram in Problem 4. The number 2 appears in the problem; ask students where they see it in the diagram.
- Analyze Problems 1 and 2 on the Problem Set to compare different unknowns. (There are 2 birds in each cage in Problem 1, and 2 fish in each bowl in Problem 2.)
- How does what the quotient represents affect the way a tape diagram is drawn?
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Multiply.

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Lesson 12 Problem Set

Name ____________________________________________ Date ________________________

1. There are 8 birds at the pet store. 2 birds are in each cage. Circle to show how many cages there are.

8 ÷ 2 = __________

There are _______ cages of birds.

2. The pet store sells 10 fish. They equally divide the fish into 5 bowls. Draw fish to find the number in each bowl.

_______ × 5 = 10

10 ÷ 5 = ________

There are ______ fish in each bowl.

3. Match.

10 ÷ 2
16 ÷ 2
18 ÷ 2
14 ÷ 2
12 ÷ 2

8
5
9
6
7
4. Laina buys 14 meters of ribbon. She cuts her ribbon into 2 equal pieces. How many meters long is each piece? Label the tape diagram to represent the problem, including the unknown.

Each piece is _______ meters long.

5. Roy eats 2 cereal bars every morning. Each box has a total of 12 bars. How many days will it take Roy to finish 1 box?

6. Sarah and Esther equally share the cost of a present. The present costs $18. How much does Sarah pay?
Lesson 12 Exit Ticket

Name _______________________________ Date __________________

There are 14 mints in 1 box. Cecilia eats 2 mints each day. How many days does it take Cecilia to eat 1 box of mints? Draw and label a tape diagram to solve.

It takes Cecilia ________ days to eat 1 box of mints.
1. 10 people wait in line for the roller coaster. 2 people sit in each car. Find the total number of cars needed.

\[ 10 ÷ 2 = \quad \] 

There are _______ cars needed.

2. Mr. Ramirez divides 12 frogs equally into 6 groups for students to study. How many frogs are in each group? Label known and unknown information on the tape diagram to help you solve.

There are_______ frogs in each group.

3. Match.

- \[ 10 ÷ 2 = \quad 7 \text{ frogs} \]
- \[ 16 ÷ 2 = \quad 8 \text{ frogs} \]
- \[ 18 ÷ 2 = \quad 9 \text{ frogs} \]
- \[ 14 ÷ 2 = \quad 8 \text{ frogs} \]
4. Betsy pours 16 cups of water to equally fill 2 bottles. How many cups of water are in each bottle? Label the tape diagram to represent the problem, including the unknown.

There are __________ cups of water in each bottle.

5. An earthworm tunnels 2 cm into the ground each day. The earthworm tunnels at about the same pace every day. How many days will it take the earthworm to tunnel 14 cm?

6. Sebastian and Teshawn go to the movies. The tickets cost $16 in total. The boys share the cost equally. How much does Teshawn pay?
Lesson 13

Objective: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

Suggested Lesson Structure

- Fluency Practice (15 minutes)
- Application Problem (5 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (15 minutes)

- Divide by 2 3.OA.7 (10 minutes)
- Group Counting 3.OA.1 (3 minutes)
- Divide 3.OA.7 (2 minutes)

Sprint: Divide by 2 (10 minutes)

Materials: (S) Divide by 2 Sprint

Note: This activity builds fluency with division using units of 2. It works toward students’ ability to divide fluently within 100. See Directions for Administration of Sprints in Lesson 2.

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by threes and fours in this activity reviews multiplication with units of 3 from Topic C, and anticipates using units of 4 in Topic E.

Count by threes to 30 forward and backward.
Count by fours to 40 forward and backward. Focus on the 20 to 24, 28 to 32, and 36 to 40 transitions.

Divide (2 minutes)

Materials: (S) Personal white boards

Note: This activity builds fluency with multiplication and division. It works toward students knowing from memory all products of two one-digit numbers.

T: (Write 2 x 3 = ___.) Say the multiplication sentence.
Lesson 13: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

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S: \(2 \times 3 = 6\).

T: (Write \(2 \times 3 = 6\). Directly below it, write \(\frac{6}{3} = 2\).) On your board, write the number that completes the division sentence.

S: (Write \(6 \div 3 = 2\).)

Repeat process for possible sequence: \(3 \times 3\), \(5 \times 3\), and \(9 \times 3\).

**Application Problem (5 minutes)**

Mark spends $16 on 2 video games. Each game costs the same amount. Find the cost of each game.

Notes: This problem reviews equal groups division from Lesson 12 where the unknown is the number of objects in each group.

**Concept Development (30 minutes)**

Materials: (S) Personal white boards

**Pictorial:** Draw and analyze tape diagrams to determine the unknown.

Write or project the following story and the tape diagram drawn below: 3 students equally share a pack of 12 pencils.
Lesson 13:
Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

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T: What information do we know from reading the story?
S: The total pencils and the number of students.
T: How does the tape diagram show the story?
S: The whole diagram represents 12 pencils, and it’s divided into 3 parts. Those are the students. We don’t know how many pencils each student gets, that’s what the question mark represents.

T: Write a division sentence to find how many pencils each student gets.
S: (Write 12 ÷ 3 = ?_.)

T: Draw my tape diagram on your personal board. Then draw to share the 12 pencils equally among the 3 students. Complete your division sentence.
S: (Draw 4 in each unit on the tape diagram. Write 12 ÷ 3 = 4.)
(Students can check their work by writing a multiplication sentence.)

Write or project the following problem and the first tape diagram drawn below: A school buys 12 boxes of pencils. Each class gets 3 boxes. How many classes get boxes of pencils?

T: What information do we know from the problem?
S: The total boxes and the number of boxes each class gets.
T: The box drawn with a solid line represents the number of boxes 1 class gets. I used the dotted line to estimate the total boxes. How should I label the unknown on this diagram?
S: It’s the number of classes that get boxes.
T: Where can I record my question mark?
T: (Label the unknown.) On your personal board, skip-count by three to draw more units in the tape diagram. How will you know when to stop?
S: We stop when we get to 12. (Draw and count 6, 9, 12.)
T: Use the tape diagram to write and solve a division sentence that represents the problem.
S: (Write 12 ÷ 3 = 4.) It’s the same division problem as before.
T: What does the 4 represent in this problem?
S: It’s the number of classes that get boxes of pencils. → It’s the number of groups.

Repeat the process showing division with both types of unknowns using the following suggested examples:

18 ÷ 3
21 ÷ 3
Lesson 13: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

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Abstract: Interpret tape diagrams to determine the unknown and write division problems.

(Draw or project the following tape diagrams. Students work in pairs.)

Directions:

- Write division problems to represent each diagram. (Division sentences should be the same for both diagrams.)
- Label each tape diagram, including the unknown.
- The tape diagrams and division sentences show solutions. Write a word problem to match each solution.
- Save the word problems to compare with other groups during the student debrief.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They
should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Describe how the model in 2(a) helped for drawing a tape diagram in 2(b).
- How does the application problem connect the work we did yesterday and today?
- Share work for Problem 5. The language “some friends” rather than a number may have presented a challenge.
- Compare Problems 4 and 5. How did your approach to drawing the tape diagram change? Why?
- Share word problems from the Abstract activity in the concept development. The class may solve, or simply discuss which is the unknown factor. (Guide students to notice how different the contexts are, but that each pair of problems always shows the same two unknowns.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
### Lesson 13: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

#### Date: 5/6/13

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<tr>
<td>4</td>
<td>5 x 2 =</td>
<td>26 20 ÷ 2 =</td>
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<td>4 ÷ 2 =</td>
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<td>31 __ x 2 = 12</td>
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<td>12</td>
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<td>34 __ x 2 = 16</td>
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<td>35 14 ÷ 2 =</td>
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<td>12 ÷ 2 =</td>
<td>41 12 x 2 =</td>
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<td>20 ÷ 2 =</td>
<td>42 24 ÷ 2 =</td>
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<td>21</td>
<td>__ x 2 = 10</td>
<td>43 14 x 2 =</td>
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<td>22</td>
<td>__ x 2 = 2</td>
<td>44 28 ÷ 2 =</td>
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### Lesson 13 Sprint

**Lesson 13:** Interpret the quotient as the number of groups or the number of objects in each group using units of 3.

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<td>____ x 2 = 2</td>
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<tr>
<td>22</td>
<td>____ x 2 = 10</td>
<td>44 26 ÷ 2 =</td>
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Lesson 13: Interpret the quotient as the number of groups or the number of objects in each group using units of 3. 

5/6/13
Lesson 13 Problem Set

Name ____________________________ Date ________________

1. Complete the related expressions.

\[ \begin{align*}
1 \times 3 &= 3 \\
3 \div 3 &= 1 \\
2 \times 3 &= 6 \\
6 \div 3 &= 2 \\
3 \times 3 &= 9 \\
9 \div 3 &= 3 \\
4 \times 3 &= 12 \\
10 \div 3 &= 3 \text{ R } 1 \\
5 \times 3 &= 15 \\
15 \div 3 &= 5 \\
6 \times 3 &= 18 \\
18 \div 3 &= 6 \\
7 \times 3 &= 21 \\
21 \div 3 &= 7 \\
8 \times 3 &= 24 \\
24 \div 3 &= 8 \\
9 \times 3 &= 27 \\
27 \div 3 &= 9 \\
10 \times 3 &= 30 \\
30 \div 3 &= 10 \\
\end{align*} \]

2. Mr. Lawton picks tomatoes from his garden. He divides the tomatoes into bags of 3.

a. Circle to show how many bags he packs. Then skip-count to show the total number of tomatoes.

![Tomatoes](image)

b. Draw and label a tape diagram to represent the problem.

\[ \_ \_ \_ \_ \_ \_ \_ \div 3 = \_ \_ \_ \_ \_ \_ \_ \]

Mr. Lawton packs ______ bags of tomatoes.
3. Camille buys a sheet of stamps that measures 15 centimeters long. Each stamp is 3 centimeters long. How many stamps does Camille buy? Draw and label a tape diagram to solve.

Camille buys _________ stamps.

4. Thirty third-graders go on a field trip. They are equally divided into 3 vans. How many students are in each van?

5. Some friends spend $24 altogether on frozen yogurt. Each person pays $3. How many people buy frozen yogurt?
1. Andrea has 21 apple slices. She uses 3 apple slices to decorate 1 pie. How many pies does Andrea make? Draw and label a tape diagram to solve.

2. There are 24 soccer players on the field. They form 3 equal teams. How many players are on each team?
1. Complete the related expressions.

   \[
   \begin{align*}
   2 \times 3 & = 6 \\
   6 \div 3 & = \_\_\_ \\
   1 \times 3 & = \_\_\_ \\
   3 \div 3 & = \_\_\_ \\
   7 \times 3 & = \_\_\_ \\
   \_\_\_ \div 3 & = 7 \\
   9 \times 3 & = \_\_\_ \\
   \_\_\_ \div 3 & = 9
   \end{align*}
   \]

2. Ms. Jones’ pet fish are shown below. She keeps 3 fish in each tank.
   a. Circle to show how many fish tanks she has. Then skip-count to find the total number of fish.

   b. Draw and label a tape diagram to represent the problem.

   \[
   \_\_\_\_\_ \div 3 = \_\_\_\_\_ \\
   \]

   Ms. Jones has \_\_\_ fish tanks.
3. Juan buys 18 meters of wire. He cuts the wire into pieces that are each 3 meters long. How many pieces of wire does he cut?

4. A teacher has 24 pencils. They are divided equally among 3 students. How many pencils does each student get?

5. There are 27 third graders working in groups of 3. How many groups of third graders are there?