In Topic E, students synthesize their Grade 3 knowledge of division types (group size unknown and number of groups unknown) with their new, deeper understanding of place value.

Students focus on interpreting the remainder within division problems both in word problems and within long division (4.OA.3). A remainder of one, as exemplified below, represents a left over flower in the first situation and a remainder of 1 ten in the second situation.¹

¹ Note that care must be taken in the interpretation of remainders. Consider the fact that 7 ÷ 3 is not equal to 5 ÷ 2 because the remainder of 1 is in reference to a different whole amount (2 ⅓ is not equal to 2 ½).
While we have no reason to subdivide a remaining flower, there are good reasons to subdivide a remaining ten. Students apply this simple idea to divide two-digit numbers unit by unit: dividing the tens units first, finding the remainder (the number of tens unable to be divided), and decomposing remaining tens into ones to then be divided.

Lesson 14 begins Topic E by having students solve division word problems involving remainders. In Lesson 15, students deepen their understanding of division by solving problems with remainders using both arrays and the area model. Students practice dividing two-digit dividends with a remainder in the ones place using number disks in Lesson 16 and continue that modeling in Lesson 17 in which the remainder in the tens place is decomposed into ones. The long division algorithm is introduced in Lesson 16 by directly relating the steps of the algorithm to the steps involved when dividing using number disks. Introducing the algorithm in this manner helps students to understand how place value plays a role in the steps of the algorithm. The same process of relating the standard algorithm to the concrete representation of division continues in Lesson 17. Lesson 18 moves students to the abstract level by requiring them to solve division problems numerically without drawing. In Lesson 19, students explain the successive remainders of the algorithm by using place value understanding and number disks. Finally, in Lessons 20 and 21, students use the area model to solve division problems and then compare the standard algorithm to the area model (4.NBT.6). Lesson 20 focuses on division problems without remainders, while Lesson 21 involves remainders.

Quotients and remainders are independent of each other, but must both be included to give a complete response. A quotient and a remainder cannot be recorded after an equal sign because the symbol R or the words with a remainder of are invalid in an equation. Therefore, a quotient and a remainder can be written as a statement such as seven divided by two is three with a remainder of one, or the quotient is three and the remainder is one. It is mathematically correct to record the quotient and the remainder together at the top of the long division algorithm.

Students become fluent with the standard division algorithm in Grade 6 (6.NS.2). For adequate practice in reaching fluency, students are introduced to, but not assessed on, the division algorithm in Grade 4 as a general method for solving division problems.
### A Teaching Sequence Towards Mastery of Division of Tens and Ones with Successive Remainders

<table>
<thead>
<tr>
<th>Objective</th>
<th>Description</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objective 1:</strong></td>
<td>Solve division word problems with remainders.</td>
<td>14</td>
</tr>
<tr>
<td><strong>Objective 2:</strong></td>
<td>Understand and solve division problems with a remainder using the array and area models.</td>
<td>15</td>
</tr>
<tr>
<td><strong>Objective 3:</strong></td>
<td>Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.</td>
<td>16</td>
</tr>
<tr>
<td><strong>Objective 4:</strong></td>
<td>Represent and solve division problems requiring decomposing a remainder in the tens.</td>
<td>17</td>
</tr>
<tr>
<td><strong>Objective 5:</strong></td>
<td>Find whole number quotients and remainders.</td>
<td>18</td>
</tr>
<tr>
<td><strong>Objective 6:</strong></td>
<td>Explain remainders by using place value understanding and models.</td>
<td>19</td>
</tr>
<tr>
<td><strong>Objective 7:</strong></td>
<td>Solve division problems without remainders using the area model.</td>
<td>20</td>
</tr>
<tr>
<td><strong>Objective 8:</strong></td>
<td>Solve division problems with remainders using the area model.</td>
<td>21</td>
</tr>
</tbody>
</table>
Lesson 14

Objective: Solve division word problems with remainders.

Suggested Lesson Structure

- Application Problem (8 minutes)
- Fluency Practice (12 minutes)
- Concept Development (32 minutes)
- Student Debrief (8 minutes)

Total Time (60 minutes)

Application Problem (8 minutes)

Tyler planted potatoes, oats, and corn. There were 23 acres planted with potatoes. There were 3 times as many acres planted with oats as potatoes and 4 times as many acres planted with corn as oats. How many acres did he plant with potatoes, oats, and corn in all?

Note: This Application Problem relates to the objective of Lesson 13: Use multiplication, addition, or subtraction to solve multi-step word problems.

Fluency Practice (12 minutes)

- Group Count to Divide 4.OA.1 (4 minutes)
- Number Sentences in an Array 4.NBT.5 (4 minutes)
- Divide with Remainders 4.NBT.6 (4 minutes)
Group Count to Divide (4 minutes)

Note: This drill prepares students to divide with remainders during this lesson’s Concept Development.

T: (Write 8 ÷ 2 = ___.) Let’s find the quotient counting by twos. Show a finger for each multiple you count by.
S: 2 (show 1 finger), 4 (show 2 fingers), 6 (show 3 fingers), 8 (show 4 fingers).
T: What’s 8 ÷ 2?
S: 8 ÷ 2 = 4.

Continue with the following possible sequence: 12 ÷ 2, 18 ÷ 2, 14 ÷ 2, 15 ÷ 5, 25 ÷ 5, 40 ÷ 5, 30 ÷ 5, 9 ÷ 3, 15 ÷ 3, 27 ÷ 3, 21 ÷ 3, 16 ÷ 4, 24 ÷ 4, 32 ÷ 4, and 36 ÷ 4.

Number Sentences in an Array (4 minutes)

Materials: (S) Personal white boards

Note: This fluency drill prepares students for G4–M3–Lesson 15’s Concept Development.

T: (Project a 3 × 4 grid.) How many boxes do you see altogether?
S: 12.
T: Let’s count by threes to check. (Point at columns as students count.)
S: 3, 6, 9, 12.
T: Let’s count by fours to check. (Point at rows as students count.)
S: 4, 8, 12.
T: On your boards, write two multiplication sentences to show how many boxes are in this array.
S: (Write 3 × 4 = 12 and 4 × 3 = 12.)
T: (Write 12 ÷ __ = __, 12 ÷ __ = __.) Write two division sentences for this array.
Students write 12 ÷ 3 = 4 and 12 ÷ 4 = 3.

Continue with the following possible sequence: 5 × 2 and 7 × 3 array.

Divide with Remainders (4 minutes)

Note: This fluency drill prepares students for this lesson’s Concept Development.

T: How many groups of 2 are in 10?
S: 5.
T: Let’s prove it by counting by twos. Use your fingers as you count.
S: (Showing one finger for each multiple.) 2, 4, 6, 8, 10.
T: Show and say how many groups of 2 are in 10.
S: (Showing 5 fingers.) 5.
T: (Write 11 ÷ 2.) Let’s find out how many groups of 2 are in 11. Count with me.
S: (Showing one finger for each multiple.) 2, 4, 6, 8, 10.
T: How many groups?
Lesson 14: Solve division word problems with remainders.

Date: 8/28/13

S: 5.
T: How many left?
S: 1.

Continue with the following possible sequence: 8 ÷ 4 and 9 ÷ 4, 12 ÷ 3 and 13 ÷ 3, 15 ÷ 5 and 17 ÷ 5, 20 ÷ 4 and 23 ÷ 4, 50 ÷ 10 and 55 ÷ 10.

Concept Development (32 minutes)

Materials: (S) Personal white boards

Problem 1: Divide a two-digit number by a one-digit number modeled with an array.

There are 12 students in PE class separated into 4 teams. How many students are on each team?

T: Read the problem and draw an array to represent the division.
S: (Draw an array as pictured to the right.)
T: Tell me a division expression that matches the situation.
S: 12 ÷ 4.
T: What is the quotient?
S: The quotient is 3.
T: How many students are on each team?
S: There are 3 students on each team.
T: How can you check to make sure your division was correct?
S: I can count by three 4 times to get 12. → I can multiply 4 times 3 to get 12.
T: Does this quotient tell us the size of the group or the number of groups?
S: The size of the group.
T: Let’s revise the story a bit. Again there are 12 students in PE class but now 3 students are needed on each team. How many teams can be made? (Point to the same array.) What is the division expression for this new story?
S: 12 ÷ 3.
T: Does the quotient tell us the size of the group or the number of groups?
S: The number of groups.
T: The same array can represent a situation with the group size unknown or number of groups unknown.
Problem 2: Divide a two-digit number by a one-digit number with a remainder modeled with an array.

13 ÷ 4

T: One more student joined the class described at the beginning of Problem 1. There are now 13 students to be divided into 4 teams. Draw an array to find how many students are on each team.

S: I can represent 13 in four groups. ➔ Four groups of 3 make 12, but I have 1 left over. ➔ One student won’t be on a team.

T: Tell me an expression to represent this problem.

S: 13 ÷ 4.

T: When we divide a number into equal groups sometimes there is an amount leftover. We call the number that we have left a remainder.

T: What is the quotient?

S: The quotient is 3.

T: What is the amount left over, the remainder?

S: 1.

T: We state our answer by saying the quotient and then the remainder. The quotient is 3. The remainder is 1. We can also say or write “the quotient is 3 with a remainder of 1.”

T: Discuss with your partner how you can use multiplication to check your work for this answer.

S: Four threes is 12. That doesn’t prove our answer is right. ➔ We can add the remainder to the product. Four times 3 is 12. Add 1 to get 13.

T: Let’s return again to a second story. There are 13 students in PE class. Three students are needed on each team. How many teams can be made?

T: Tell me the new expression.

S: 13 ÷ 3.

T: State the quotient and remainder.

T: The quotient is 4 and the remainder is 1.

T: Talk to your partner. What do the quotient and the remainder mean in the second story?

S: Four teams can be made and there is one extra person.

Compare the number bond with the quotient and the remainder. Notice the part on the left represents the equal groups and the part on the right is the remainder.
Problem 3: Divide a two-digit number by a one-digit number with a remainder modeled with a tape diagram.

Kristy bought 13 roses. If she puts 6 roses in each vase, how many vases will she use? Will there be any roses left over?

S: I can’t because 13 is an odd number and $6 + 6 = 12$. An even number plus an even number won’t give you an odd number. → You can divide by 6, but there will be 1 extra flower left over. → I can fill 2 vases and have 1 flower left over.
T: Tell your partner a statement that tells the quotient and remainder for this problem.
S: The quotient is 2 and the remainder is 1.
T: Describe to your partner what that statement tells us.
S: We started with 13 and made groups of 6. We made 2 groups, with 1 rose remaining. → Kristy can fill 2 vases. She will have 1 rose left over.
T: Again, let’s revise our story a bit. Now Kristy bought 13 roses and wants to put them in 2 vases. How many roses will be in each vase? Is this the same array?
S: Yes.
T: Talk to your partner. How has our interpretation of the array changed?
S: In the first story, we didn’t know the number of vases. In the second story, we didn’t know the number in each vase. → We changed the story from finding the number of groups to finding the size of the group.
T: How can we check our work for both situations?
S: We can draw a number bond to show 2 groups of 6, and then 1 more. → Six times 2 is 12, and 12 plus 1 is 13.
T: Let’s turn our array into a tape diagram to show 13 in 2 groups of 6 with a remainder of 1 (demonstrate).
T: Using the array, draw a rectangle around the flowers. Erase the flowers and label the diagram.
S: You should divide the bar into two parts. I know each part is worth 6, but 6 plus 6 isn’t 13.
T: Our tape diagram must have a third part to represent the remainder. Let’s separate the bar into two equal parts and make a very small third part, and shade to show the remaining flower (demonstrate).
T: With your partner, draw a tape diagram to show 13 roses divided equally into 4 vases.
Students draw a tape diagram, dividing it into four parts. Using their basic facts, they know 13 can’t be divided into four equal parts. They shade a fifth part of the tape diagram to show the remainder.

S: The quotient is 3. The remainder is 1. → We can check our work by drawing a number bond and adding the parts or multiplying 4 times 3 and adding 1. Whatever method we use, we get back to the original total when our quotient and remainder are correct.

T: Look at your tape diagram. Is the model the same when we don’t know the number of groups, when we know that there are 3 flowers in each vase but we don’t know the number of vases?

S: Yes!

Problem 4: Divide a two-digit number by a one-digit number, interpreting the remainder.

Allison has 22 meters of fabric to sew dresses. She uses 3 meters of fabric for each dress. After how many dresses will Allison need to buy more fabric?

T: Let’s represent this problem using a tape diagram together. (Model for the students as you talk.) We don’t know the number of groups, or the number of dresses she will make. We know each dress uses 3 meters so let’s draw one group and label it as fabric for 1 dress. We don’t know how many dresses she can make, or how many threes there are, so we label that with a question mark. We do know there will be a remainder because we know our facts of 3, and 22 isn’t a multiple of 3. Solve this problem.

S: Twenty-two divided by 3 is 7 with a remainder of 1.

T: With your partner, discuss your answer to the question. After how many dresses will Allison need to buy more fabric?

S: Well she can make 7 dresses. I guess she’ll have only 1 meter to make her next dress. → No, the problem says she must have 3 meters of fabric for each dress, so after 7 dresses she will have to buy more fabric. → She can make 7 dresses, but to make an eighth dress, she will need to buy 2 more meters of fabric. I can prove that my tape diagram is right with an array. See, 7 threes means 7 dresses, and 1 left over means to make the eighth dress she will need 2 more meters.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Lesson Objective: Solve division word problems with remainders.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 3, there are 2 extra chairs. How can the remainder help you to find how many more chairs are needed to set up 1 more complete table?
- In Problem 4, how many full days of baking can be done? How much more flour is needed to bake on the sixth day?
- In Problem 6, 45 ÷ 7 equals 6 with a remainder of 3. What do the quotient and remainder represent in this problem? If 6 vans are full with 3 people remaining, why do we need 7 vans? Does the quotient always give the final answer? Why is it important to think carefully about the remainder? How would a model support your answer of 7 vans?
- How does an array help you to determine a remainder? Use the problems 12 ÷ 3, 13 ÷ 3, and 13 ÷ 2 in your conversation. How do the arrays with the whole 12 and 13 differ?
- What complications are there in modeling a division problem with a remainder using a tape diagram?
- What new math vocabulary did we use today to communicate precisely?
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Name ____________________________ Date ________________

Solve the following problems. Use the RDW process.

1. There are 19 identical socks. How many pairs of socks are there? Will there be any socks without a match? If so, how many?

2. If it takes 8 inches of ribbon to make a bow, how many bows can be made from 3 feet of ribbon (1 foot = 12 inches)? Will any ribbon be left over? If so, how much?

3. The library has 27 chairs and 5 tables. If the same number of chairs is placed at each table, how many chairs can be placed at each table? Will there be any extra chairs? If so, how many?
4. The baker has 42 kilograms of flour. She uses 8 kilograms each day. After how many days will she need to buy more flour?

5. Caleb has 76 apples. He wants to bake as many pies as he can. If it takes 8 apples to make each pie, how many apples will he use? How many apples will not be used?

6. Forty-five people are going to the beach. Seven people can ride in each van. How many vans will be required to get everyone to the beach?
Solve the following problem. Use the RDW process.

1. Fifty-three students are going on a field trip to the zoo. Before the trip, a teacher forms groups of students and assigns a chaperone to each group. As much as she can, the teacher divides the students into groups of 6. How many groups of students will there be? Will each group have 6 students? How many total chaperones are needed?
Lesson 14 Homework

Solve the following problems. Use the RDW process.

1. Linda makes booklets using 2 sheets of paper. She has 17 sheets of paper. How many of these booklets can she make? Will she have any extra paper? How many sheets?

2. Linda uses thread to sew the booklets together. She cuts 6 inches of thread for each booklet. How many booklets can she stitch with 50 inches of thread? Will she have any unused thread after stitching up the booklets? If so, how much?

3. Ms. Rochelle wants to put her 29 students into groups of 6. How many groups of 6 can she make? If she puts any remaining students in a smaller group, how many students will be in that group?
4. A trainer gives his horse, Caballo, 7 gallons of water every day from a 57-gallon container. How many days will Caballo receive his full portion of water from the container? On which number day will the trainer need to refill the container of water?

5. Meliza has 43 toy soldiers. She lines them up in rows of 5 to fight imaginary zombies. How many of these rows can she make? After making as many rows of 5 as she can, she puts the remaining soldiers in the last row. How many soldiers are in that row?

6. Seventy-eight students are separated into groups of 8 for a field trip. How many groups are there? The remaining students form a smaller group of how many students?
Lesson 15

Objective: Understand and solve division problems with a remainder using the array and area models.

Suggested Lesson Structure

<table>
<thead>
<tr>
<th>Fluency Practice</th>
<th>(12 minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Application Problem</td>
<td>(5 minutes)</td>
</tr>
<tr>
<td>Concept Development</td>
<td>(33 minutes)</td>
</tr>
<tr>
<td>Student Debrief</td>
<td>(10 minutes)</td>
</tr>
<tr>
<td><strong>Total Time</strong></td>
<td><strong>(60 minutes)</strong></td>
</tr>
</tbody>
</table>

Fluency Practice (12 minutes)

- Show Values with Number Disks 4.NBT.1
- Divide with Remainders 4.NBT.6
- Number Sentences in an Array 4.NBT.

Show Values with Number Disks (4 minutes)

Materials: (S) Personal white boards

Note: This fluency drill prepares students for G4–M3–Lesson 16’s Concept Development.

T: (Project a place value chart with 2 tens disks and 4 ones disks.) On your boards, write the number in standard form.
S: (Write 24.)

Repeat process for 5 tens and 3 ones; 4 tens and 1 one; 3 tens and 11 ones; and 3 tens and 17 ones.

T: (Write 32.) Say the number.
S: 32.

T: Show 32 using number disks.
S: (Draw disks for 3 tens and 2 ones.)

Continue with the following possible sequence: 21 and 43.
Divide with Remainders (4 minutes)

Note: This fluency drill prepares students for this lesson’s Concept Development.

Repeat the process from G4–M3–Lesson 14 for the following possible sequence: 6 ÷ 2 and 8 ÷ 2; 24 ÷ 3 and 25 ÷ 3; 12 ÷ 4 and 15 ÷ 4; 18 ÷ 6 and 21 ÷ 6; and 45 ÷ 5 and 49 ÷ 5.

Number Sentences in an Array (4 minutes)

Materials: (S) Personal white boards

Note: This fluency drill prepares students for this lesson’s Concept Development.

Repeat Lesson 14’s drill using the following possible sequence: 5 × 3 + 1, 3 × 6 + 1, and 3 × 4 + 2.

Application Problem (5 minutes)

Chandra printed 38 photos to put into her scrapbook. If she can fit 4 photos on each page, how many pages will she use for her photos?

Note: This Application Problem relates to the objective of Lesson 14 in that students solve a division word problem with a remainder. Here, students interpret the remainder to determine the total number of scrapbook pages needed. This anticipates the last problem in this lesson.

Modeling the array (rather than the tape diagram) may give students a clearer picture of the solution to the Application Problem. Encourage students to use the labels photo and page, if beneficial. Discuss how the equation informs the solution, yet the picture reveals the solution.
Concept Development (33 minutes)

Materials: (T/S) Graph paper

Problem 1: Solve a division problem with and without a remainder using the area model.

Display 10 ÷ 2.

T: Draw an array to represent 10 ÷ 2. Explain to your partner how you solved.

S: (Draw.) I drew 2 circles and placed 10 dots evenly among the circles. → I drew 10 dots as 2 rows of 5 dots.

T: Let’s use graph paper to draw a rectangle with the area of 10 square centimeters and one side length of 2 centimeters. Tell your partner how we can find the unknown side length.

S: The total is 10, so we know it is 5. → If the width is 2 centimeters, that means the length is 5 centimeters, and 2 centimeters times 5 centimeters gives an area of 10 square centimeters. → We can count and mark off by twos until we get to 10.

T: Discuss with your partner how the length of 5 centimeters is represented in the area model.

S: The length is 5, and the quotient is 5. → The length of the area model represents the quotient of this division problem.

Display 11 ÷ 2.

T: With your partner, discuss how you would draw an area model for 11 ÷ 2.

S: Two can be the length or the width. → I can’t just draw 2 rows of square units because of the remainder. → If I mark off 2 squares at a time, I count 2, 4, 6, 8, 10. I can’t do another group of 2 because it would be 12. There aren’t enough.

T: Eleven square centimeters is the total area. Let’s draw a rectangle starting with a width of 2 centimeters. We’ll continue lengthening it until we get as close to 11 square centimeters as we can.

S: A length of 5 centimeters and width of 2 centimeters is as close as we can get to 11 square centimeters. → We can’t do 2 × 6 because that’s 12 square centimeters and the total area is 11 square centimeters.

T: We can show a total area of 11 square centimeters by modeling 1 more square centimeter. The remainder of 1 represents 1 more square centimeter.
Lesson 15

Lesson 15: Understand and solve division problems with a remainder using the array and area models.

Date: 8/28/13

Repeat for 16 ÷ 3 and 23 ÷ 4.

Problem 2: Solve a division problem using an array and the area model.

Display 38 ÷ 4

T: In the Application Problem, you drew an array (pictured to the right) to solve. Represent the same problem using the area model on graph paper. (Allow two minutes to work.)

T: What do you notice about the array compared to the area model on graph paper?

S: The area model is faster to draw. Thirty-eight dots is a lot to draw. → There are the same number of dots and squares when I used graph paper. → Both get us the same answer of a quotient 9 with a remainder of 2.

T: Let’s represent 38 ÷ 4 even more efficiently without graph paper since it’s hard to come by graph paper every time you want to solve a problem.

T: (Give students one minute to draw.) Talk to your partner about how the array and graph paper models supported you in drawing the rectangle with a given structure.

S: I knew the length was a little more than twice the width. → I knew that the remainder was half a column. → I knew that there was a remainder. It was really obvious with the array and graph paper.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Notes on Multiple Means of Representation:
Help English language learners distinguish between terms used for division: division, divisor, quotient, and whole. Label a division equation and post for future reference. Make a word web of synonyms for division that students can interchange, if desired. Encourage students to speak these words as they participate in the Debrief.
Lesson 15: Understand and solve division problems with a remainder using the array and area models.

Lesson Objective: Understand and solve division problems with a remainder using the array and area models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- What does the quotient represent in the area model?
- When does the area model present a challenge in representing division problems?
- Explain to your partner how Problem 1(a) and Problem 1(b) are similar. How are they different?
- How can Problem 3 and Problem 4 have the same remainder?
- How could you change the 43 in Problem 5 so that there would be the same quotient but with no remainder?
- The quotient represents a side length. The remainder consists of square units. Why?
- How is the whole represented in an area model?
- What new math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today’s lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 15 Problem Set

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Show division using an array.</th>
<th>Show division using an area model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $18 \div 6$</td>
<td></td>
</tr>
<tr>
<td>Quotient = _________</td>
<td>Can you show $18 \div 6$ with one rectangle? _____</td>
</tr>
<tr>
<td>Remainder = _________</td>
<td></td>
</tr>
</tbody>
</table>

| 2. $19 \div 6$ | |
| Quotient = _________ | Can you show $19 \div 6$ with one rectangle? _____ |
| Remainder = _________ | Explain how you showed the remainder: |
Solve using an array and an area model. The first one is done for you.

Example: 25 ÷ 2

a. .................................
   Quotient = 12  Remainder = 1

b. 2

3. 29 ÷ 3

   a. 
   b. 

4. 22 ÷ 5

   a. 
   b. 

5. 43 ÷ 4

   a. 
   b. 

6. 59 ÷ 7

   a. 
   b.
Lesson 15 Exit Ticket

Name __________________________________________ Date _________________________

Solve using an array and area model.

1. $27 \div 5$
   a. __________________________________________ b. __________________________________________

2. $32 \div 6$
   a. __________________________________________ b. __________________________________________
<table>
<thead>
<tr>
<th>Show division using an array.</th>
<th>Show division using an area model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 24 ÷ 4</td>
<td>Quotient = __________</td>
</tr>
<tr>
<td></td>
<td>Remainder = _________</td>
</tr>
<tr>
<td></td>
<td>Can you show 24 ÷ 4 with one rectangle? ______</td>
</tr>
</tbody>
</table>

| 2. 25 ÷ 4                   | Quotient = __________             |
|                             | Remainder = _________            |
|                             | Can you show 25 ÷ 4 with one rectangle? ______ |
|                             | Explain how you showed the remainder: |
Lesson 15 Homework

Solve using an array and area model. The first one is done for you.

Example:  $25 \div 3$

a. 

![Array and Area Model](image) 

b. 

Quotient = 8   Remainder = 1

3.  $44 \div 7$

a. 

b. 

4.  $34 \div 6$

a. 

b. 

5.  $37 \div 6$

a. 

b. 

6.  $46 \div 8$

a. 

b.
Lesson 16

Objective: Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

Suggested Lesson Structure

- Fluency Practice (8 minutes)
- Concept Development (42 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (8 minutes)

- Group Count 4.OA.1 (4 minutes)
- Divide with Remainders 4.NBT.6 (4 minutes)

Group Count (4 minutes)

Note: This drill prepares students to divide with remainders during the Concept Development.

Direct students to count forward and backward, occasionally changing the direction of the count.

- Twos to 20
- Threes to 30
- Fours to 40
- Fives to 50

Divide with Remainders (4 minutes)

Note: This fluency drill prepares students for the Concept Development.

Repeat the process from G4–M3–Lessons 14 and 15 using the following possible sequence: $6 \div 2$, $20 \div 5$, $16 \div 4$, $18 \div 3$, $15 \div 2$, $18 \div 5$, $11 \div 3$, $13 \div 4$, and $33 \div 4$.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Since learners differ in their physical abilities, provide options for modeling and crossing out small dots, such as concrete number disks in an enlarged place value chart, drawing larger circles, tick marks, or using fingerprints. Adjust response time accordingly.
Concept Development (42 minutes)

Materials: (S) Personal white boards

Problem 1

6 ones ÷ 3
3 tens 6 ones ÷ 3

Display 6 ÷ 3 on the board.

T: 6 ones represents what?
S: The whole. → The total. → What you are dividing.
T: Show 6 using number disks. What is the number we are dividing by?
S: 3.
T: Let’s assume it’s telling us how many groups to make, and draw 3 groups below. Can we distribute 6 ones into 3 groups? Think of it like dealing cards evenly among 3 players. First, put one in each group. Cross off the ones one at a time as you distribute them evenly. Next, put another one in each group if you are able. Continue this until all of the ones are distributed.

S: We can put 2 ones in each group.
T: Are there any ones left over?
S: No.
T: How many ones are in each of our 3 groups?
S: 2 ones.
T: What is 6 ones ÷ 3? Give me the number sentence.
S: 6 ones ÷ 3 equals 2 ones.
T: Let’s represent 6 ÷ 3 in a new way. Let’s record the whole and the divisor (record with long division symbol as shown above). Look back to your model. 6 ones divided by 3 is?
S: 2 ones.
T: (Record 2 ones.)
T: (Point to the place value chart.) You distributed 2 ones, 3 times. 2 ones times 3 is?
S: 6 ones.
T: (Refer to the numbers carefully, pointing to 2 ones, the divisor, and recording 6 ones.)
T: (Point to the place value chart.) We divided 6 ones, and have no ones remaining. Six minus six equals zero. (Write the subtraction line.) What does this zero mean?
S: There is no remainder. → All the ones were divided with no left overs. → We subtracted the total number divided from the total number of ones.
Lesson 16: Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

Date: 8/28/13

T: We can see the 3 groups of 2 both in our model and in our numbers and know our answer is correct since 3 times 2 equals 6.

Display 36 ÷ 3 on the board.

T: 3 tens and 6 ones represents what?
S: The whole.
T: Show 36 using number disks. What is the number we are dividing by?
S: 3.
T: Make room for 3 groups below. Let’s start dividing with the largest units. What is the largest unit?
S: The tens.
T: 3 tens divided by 3 is?
S: 1 ten.
T: Distribute the 3 tens and cross them off to show they are now divided equally into the 3 groups.
T: Are there any tens left over?
S: No.
T: 6 ones divided by 3 is?
S: 2 ones. We did that in the last problem. We distribute the ones evenly, one at a time, into each group. We cross off the ones, one at a time, as we distribute them.
T: Are there any ones left over?
S: No.
T: How many tens and ones are in each of our 3 groups?
S: 1 ten and 2 ones.
T: What is 36 ÷ 3?
S: 36 ÷ 3 is 12.
T: Let’s represent 36 ÷ 3 using numbers. Record the whole and the divisor.
T: Look back to your model. 3 tens divided by 3 is?
S: 1 ten.
T: (Record 1 ten. Point to the place value chart.) You distributed 1 ten, 3 times. Give a multiplication sentence that says that.
S: 1 ten times 3 equals 3 tens. (As they speak, refer to the algorithm.)
T: (Point to the place value chart.) How many tens are remaining to be distributed?
S: None. → Zero.
S: 3 tens minus 3 tens equals 0 tens. (Refer to the algorithm.)
T: What of our whole amount remains to be divided?
Lesson 16: Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

Have the students notice the 3 groups of 12 and relate that to the checking equation of 3 twelves or 3 times 12.

**Problem 2**

5 ones ÷ 4
4 tens 5 ones ÷ 4

Display 5 ÷ 4 on the board.

- **T:** With your partner, represent the whole and the divisor, 4, on the place value chart.
- **S:** (Draw 5 ones and draw 4 groups below in the place value chart, and record the algorithm.)
- **T:** 5 ones divided by 4 equals?
- **S:** It doesn’t divide evenly. \( \rightarrow \) I can place 1 one in each group, but I will have 1 one left over.
- **T:** Distribute as many ones as you can, crossing off the ones you use. What is the quotient for 5 ones divided by 4?
- **S:** 1 one.
- **T:** Record your quotient numerically. Say a multiplication sentence for how many ones were distributed.
- **S:** 1 one times 4 equals 4 ones.
- **T:** Record 4 ones numerically and subtract.
- **S:** 5 ones minus 4 ones is 1 one.
- **T:** Record 1 one numerically. How many ones are remaining in the place value chart?
- **S:** 1 one.
- **T:** Circle 1 one. Tell your partner why 1 one is a remainder.
- **S:** It is what is left over after we made our groups. \( \rightarrow \) Our groups must be equal. If we put this 1 one into a group, the groups will not be equal.
Lesson 16

Lesson 16: Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

T: Watch as I record the remainder numerically using R1.

Display 45 ÷ 4 on the board.

T: Represent 45 using number disks. Prepare to represent it numerically.

T: 4 tens divided by 4 equals?

S: 1 ten.

T: Cross off and place your tens below in each of the 4 groups. Record 1 ten in the tens column. Tell your partner the next numerical steps.

S: 1 ten times 4 is 4 tens. We subtract 4 tens from 4 tens and get 0 tens. We have 5 ones remaining, so we record those next to the 0 tens.

T: 5 ones divided by 4 equals?

S: 1 one. ➔ We can place 1 one in each group. But we will have 1 one remaining.

T: Distribute the disks, crossing off the 4 you use. Then tell your partner how to record that using numbers.

S: 5 ones divided by 4 is 1 one. 1 one times 4 is 4 ones. 5 ones minus 4 ones is 1 one. Hey, we have 1 one left in the place value chart!

T: Correct. Circle that 1 one. It is your remainder. Show your partner how to record the remainder.

T: What is 45 ÷ 4?

S: 11 with a remainder of 1.

T: What do you notice about using the algorithm and number disks?

S: Both help us get to the same answer. ➔ In the place value chart, we can see the remainder of 1. Then we can write out all of the steps we did with the disks and still show the quotient of 1 and the remainder of 1. ➔ We started with the largest units and went to the smallest with the disks and the numbers.

Problem 3

8 ones ÷ 3

6 tens 8 ones ÷ 3

Display 8 ÷ 3 on the board.

T: Solve for 8 ÷ 3 by using number disks and represent the problem using numbers, using long division, with your partner.

Circulate listening for students using place value as they divide, multiply, and subtract.
Lesson 16

Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

Date: 8/28/13

S: The quotient is 2 and the remainder is 2.
T: How do we use multiplication to check our quotient and remainder in division?
S: Two times 3 is 6. Six plus 2 is 8. We multiply the quotient times the divisor and add the remainder. We multiply the number in each group by the number of groups and then add the remainder.

Display 68 ÷ 3 on the board.

T: Solve for 68 ÷ 3 by using number disks and represent the problem using numbers, or long division, with your partner.
S: I got 22 with a remainder of 2.
T: How can we check if 22 with a remainder of 2 is the correct answer?
S: We can multiply to check because we know that multiplication and division are related. We can multiply 22 × 3 to check and then we need to add 2.
T: 22 × 3 is?
S: 66.
T: Plus 2?
S: 68. Our answer was right!

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.
You may choose to use any combination of the questions below to lead the discussion.

- How did solving Problem 1 prepare you for solving Problem 2?
- Explain to your partner why only 6 ones could be distributed in Problem 3. What happens to the remaining ones?
- Solve 12 divided by 3. Solve 12 divided by 4. As a divisor gets larger, what will happen to the quotient if the whole stays the same?
- Was the remainder ever larger than the divisor? Why not?
- In the Problem Set, we only had remainders of 1 and 2. Give me an example of a problem that might have a larger remainder.
- Explain the connection between using number disks and long division. Why do you think it is called long division?
- What new math vocabulary did we use today to communicate precisely?

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Name ___________________________ Date ______________

Show the division using disks. Relate your work on the place value chart to long division. Check your quotient and remainder by using multiplication and addition.

1. \(7 \div 2\)
   
   \[
   \begin{array}{c|c}
   \text{Tens} & \text{Ones} \\
   \hline
   2 & 7 \\
   \end{array}
   \]
   
   quotient = _________
   remainder = _________

   \[
   \begin{array}{c|c}
   \text{Check Your Work} \\
   \hline
   & 3 \\
   \times 2 & \\
   \end{array}
   \]

2. \(27 \div 2\)
   
   \[
   \begin{array}{c|c}
   \text{Tens} & \text{Ones} \\
   \hline
   2 & 7 \\
   \end{array}
   \]
   
   quotient = _________
   remainder = _________

   \[
   \begin{array}{c|c}
   \text{Check Your Work} \\
   \hline
   & 3 \\
   \times 2 & \\
   \end{array}
   \]

3. \(8 \div 3\)
   
   \[
   \begin{array}{c|c}
   \text{Ones} \\
   \hline
   3 & 8 \\
   \end{array}
   \]
   
   quotient = _________
   remainder = _________

   \[
   \begin{array}{c|c}
   \text{Check Your Work} \\
   \hline
   & 3 \\
   \times 2 & \\
   \end{array}
   \]
Lesson 16 Problem Set

4. \(38 \div 3\)

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\[
3 \longdiv{38} \\
\text{quotient} = \underline{} \\
\text{remainder} = \underline{} \\
\]

5. \(6 \div 4\)

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\[
4 \longdiv{6} \\
\text{quotient} = \underline{} \\
\text{remainder} = \underline{} \\
\]

6. \(86 \div 4\)

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\[
4 \longdiv{86} \\
\text{quotient} = \underline{} \\
\text{remainder} = \underline{} \\
\]

Check Your Work
Lesson 16 Exit Ticket

Name ___________________________ Date _________________

Show the division using disks. Relate your work on the place value chart to long division. Check your quotient and remainder by using multiplication and addition.

1. 5 ÷ 3

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3  

quotient = __________

remainder = __________

2. 65 ÷ 3

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<tr>
<td></td>
<td>65</td>
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</table>

3  

quotient = __________

remainder = __________
Lesson 16 Homework

Name ________________________________ Date ________________

Show the division using disks. Relate your work on the place value chart to long division. Check your quotient and remainder by using multiplication and addition.

1. $7 \div 3$

   \[
   \begin{array}{c|c}
   \text{Tens} & \text{Ones} \\
   \hline
   \end{array}
   \]

   \[
   \begin{array}{c|c}
   \text{Ones} & \text{Tens} \\
   \hline
   7 & \phantom{1} \\
   \hline
   \end{array}
   \]

   \[
   3 \big| 7
   \]

   quotient = __________

   remainder = __________

2. $67 \div 3$

   \[
   \begin{array}{c|c}
   \text{Tens} & \text{Ones} \\
   \hline
   \end{array}
   \]

   \[
   \begin{array}{c|c}
   \text{Ones} & \text{Tens} \\
   \hline
   67 & \phantom{1} \\
   \hline
   \end{array}
   \]

   \[
   3 \big| 67
   \]

   quotient = __________

   remainder = __________

3. $5 \div 2$

   \[
   \begin{array}{c|c}
   \text{Ones} & \phantom{1} \\
   \hline
   \end{array}
   \]

   \[
   2 \big| 5
   \]

   quotient = __________

   remainder = __________
Lesson 16 Homework

Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

4. \( 85 \div 2 \)

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\[ \begin{array}{c}
 2 \overline{85}
\end{array} \]

quotient = __________
remainder = __________

5. \( 5 \div 4 \)

<table>
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<th>Ones</th>
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\[ \begin{array}{c}
 4 \overline{5}
\end{array} \]

quotient = __________
remainder = __________

6. \( 85 \div 4 \)

<table>
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<th>Tens</th>
<th>Ones</th>
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\[ \begin{array}{c}
 4 \overline{85}
\end{array} \]

quotient = __________
remainder = __________
Lesson 17

Objective: Represent and solve division problems requiring decomposing a remainder in the tens.

Suggested Lesson Structure

<table>
<thead>
<tr>
<th>Suggested Activity</th>
<th>Time</th>
</tr>
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<tbody>
<tr>
<td>Fluency Practice</td>
<td>11 minutes</td>
</tr>
<tr>
<td>Application Problem</td>
<td>5 minutes</td>
</tr>
<tr>
<td>Concept Development</td>
<td>34 minutes</td>
</tr>
<tr>
<td>Student Debrief</td>
<td>10 minutes</td>
</tr>
<tr>
<td>Total Time</td>
<td>60 minutes</td>
</tr>
</tbody>
</table>

Fluency Practice (11 minutes)

- Group Count 4.OA.1 (2 minutes)
- Divide Mentally 4.NBT.6 (4 minutes)
- Divide Using the Standard Algorithm 4.NBT.6 (5 minutes)

Group Count (2 minutes)

Note: This prepares students to divide with remainders during the Concept Development.

Direct students to count forward and backward, occasionally changing the direction of the count.

- Twos to 20
- Threes to 30
- Fours to 40
- Fives to 50

Divide Mentally (4 minutes)

Note: This reviews G4–M3–Lesson 16’s content.

T: (Write $40 \div 2$.) Say the completed division equation in unit form.
S: 4 tens divided by 2 equals 2 tens.
T: (To the right, write $8 \div 2$.) Say the completed division equation in unit form.
S: 8 ones divided by 2 equals 4 ones.
T: (Above both equations, write $48 \div 2$. Draw a number bond to connect the two original problems to this new problem.) Say the completed division equation in unit form.
Lesson 17

Represent and solve division problems requiring decomposing a remainder in the tens.

S: 4 tens 8 ones divided by 2 equals 2 tens 4 ones.
T: Say the division equation in regular form.
S: 48 divided by 2 equals 24.
Continue with the following possible sequence: 93 ÷ 3 and 88 ÷ 4.

**Divide Using the Standard Algorithm (5 minutes)**

Materials: (S) Personal white boards

Note: This reviews G4–M3–Lesson 16’s content.

T: (Write 24 ÷ 2.) On your boards, solve the division problem using the vertical method.
Continue with the following possible sequence: 36 ÷ 3, 37 ÷ 3, 55 ÷ 5, 57 ÷ 5, 88 ÷ 4, 87 ÷ 4, 96 ÷ 3, and 95 ÷ 3.

**Application Problem (5 minutes)**

Audrey and her sister found 9 dimes and 8 pennies. If they share the money equally, how much money will each sister get?

Note: This Application Problem reviews division of ones. Sharing 9 dimes connects to Problems 1 and 2 of this Concept Development, asking students to decompose 1 ten for 10 ones.
Concept Development (34 minutes)

Materials: (S) Personal white boards

Problem 1: Divide two-digit numbers by one-digit numbers using number disks, regrouping in the tens.

3 ones ÷ 2
3 tens ÷ 2

Display 3 ÷ 2 on the board.
T: (Have students model on the place value chart.) 3 ones divided by 2 is?
S: One with a remainder of 1.
T: Record 3 ÷ 2 as long division.

Students complete the algorithm. Encourage the students to share the relationship of their model to the algorithm.

Display 30 ÷ 2 on the board.
T: Using mental math, tell your partner the answer to 30 ÷ 2.
S: Thirty divided by 2 is 15.
T: Let’s confirm your quotient. Represent 30 on the place value chart. Tell your partner how many groups below are needed.
S: Two. (Draw.)
T: 3 tens divided by 2 is? Distribute your disks and cross off what’s been distributed. The answer is?
S: 1 ten with a remainder of 1 ten. That’s an interesting answer.
T: Can we rename the left over ten?
S: Yes! Change 1 ten for 10 ones.
T: Let’s rename 1 ten. Now rename and distribute the 10 ones with your partner.
S: Our answer is 1 ten 5 ones, or 15.
T: Why didn’t we stop when we had a remainder of 1 ten?
S: Because 1 ten is just 10 ones, and you can keep dividing.
T: So why did we stop when we got a remainder of 1 one?
S: The ones are the smallest unit on our place value chart, so we stopped there and made a remainder.
T: Let’s solve 30 ÷ 2 using long division.
T: 3 tens divided by 2?
S: 1 ten.
Lesson 17

Represent and solve division problems requiring decomposing a remainder in the tens.

Date: 8/28/13

Problem 2

4 ones ÷ 3
4 tens 2 ones ÷ 3

Display 4 ÷ 3 on the board.

T: Represent 4 ones on the place value chart. With your partner, solve for 4 ÷ 3 using number disks and long division.

S: The quotient is 1 and the remainder is 1.
Lesson 17

Represent and solve division problems requiring decomposing a remainder in the tens.

Date: 8/28/13

Display 42 ÷ 3 on the board.

T: Represent 4 tens 2 ones on the place value chart and get ready to solve using long division.

T: 4 tens divided by 3 is? Distribute your disks and cross off what is used. The answer is?

S: 1 ten with a remainder of 1 ten. Oh! I remember from last time, we need to change 1 ten for 10 ones.

T: (With students, draw an arrow to show 1 ten decomposed as 10 ones in the place value chart and show 12 ones in the algorithm.) How many ones remain?

S: 12.

T: Yes. 10 ones + 2 ones is 12 ones.

T: Show 12 ones divided by 3. Complete the remaining steps. What is the quotient?

S: Our quotient is 1 ten 4 ones, or 14.

Have students share with a partner how the model matches the algorithm, paying particular attention to the decomposition of 1 ten and how it is combined with the ones. Note that this is just the same process the students use in subtraction. We decompose a larger unit into smaller units.

Problem 3

8 tens 4 ones ÷ 3

Display 84 ÷ 3 on the board.

T: Solve for 84 ÷ 3 by using number disks and long division.

S: The quotient is 28.

T: What was different about the place value chart with this problem?

S: There were a lot more disks! We had to decompose 2 tens this time.

T: How many ones did you have after decomposing your 2 tens?

S: 24 ones.

T: Show your partner where to find 24 ones in the numerical representation. (Students point to the 2 tens remaining and the 4 ones that were bundled.)

T: Check your answer using multiplication.

S: 28 times 3 is 84. Our answer is right!

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Students working above grade level and others can be encouraged to solve without place value charts to become more efficient at solving long division problems. Allow them to share and explain their method with others.
Lesson 17

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Represent and solve division problems requiring decomposing a remainder in the tens.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- How did Problem 2 allow you to see only the remaining 1 ten in the ones column?
- Explain why 1 ten remains in Problem 4?
- How is the long division recording different in today’s lesson compared to yesterday’s lesson?
- What different words are we using to describe what we do when we have a remaining ten or tens? (Break apart, unbundle, change, rename, decompose, regroup.) Which of these words are you most comfortable using yourself?
- What other operation involves changing 1 ten for 10 ones at times? (Subtraction.) What operations involve the opposite, changing 10 ones for 1 ten at times?
- What would happen if we divided the ones before the tens?

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

As learners model with number disks to complete the Problem Set, encourage the following to minimize mistakes:

- Whisper-count as you distribute.
- Cross out to track the number distributed.
- Draw dots in arrays. The “hands way” array may be helpful.
- Circle the remainder.
- Try disks, dots, numbers, etc. Use what’s most efficient for you.
Lesson 17: Represent and solve division problems requiring decomposing a remainder in the tens.

Date: 8/28/13

- What connection can you find between the written division and the multiplication you used to check your work?
- Why are we learning long division after addition, subtraction, and multiplication?
- How did the Application Problem connect to today’s lesson?

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 17 Problem Set

Show the division using disks. Relate your model to long division. Check your quotient and remainder by using multiplication and addition.

1. $5 \div 2$

   \[
   \begin{array}{c|c}
   \text{Tens} & \text{Ones} \\
   \hline
   \text{2} & 5 \\
   \end{array}
   \]

   \[
   \begin{array}{c|c}
   \text{2} & \times \text{2} \\
   \end{array}
   \]

   quotient = __________

   remainder = __________

2. $50 \div 2$

   \[
   \begin{array}{c|c}
   \text{Tens} & \text{Ones} \\
   \hline
   \text{2} & 5 \ 0 \\
   \end{array}
   \]

   \[
   \begin{array}{c|c}
   \text{2} & \times \text{2} \\
   \end{array}
   \]

   quotient = __________

   remainder = __________

3. $7 \div 3$

   \[
   \begin{array}{c|c}
   \text{Ones} \\
   \hline
   \text{3} & 7 \\
   \end{array}
   \]

   \[
   \begin{array}{c|c}
   \text{3} & \times \text{2} \\
   \end{array}
   \]

   quotient = __________

   remainder = __________
4. \(75 \div 3\)

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3 \[7\ 5\]

quotient = \_

remainder = \_

Check Your Work

5. \(9 \div 4\)

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4 \[9\]

quotient = \_

remainder = \_

Check Your Work

6. \(92 \div 4\)

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4 \[9\ 2\]

quotient = \_

remainder = \_

Check Your Work
Show the division using disks. Relate your model to long division. Check your quotient by using multiplication and addition.

1. \(5 \div 4\)
   \[
   \begin{array}{c|c}
   \text{Tens} & \text{One} \\
   \hline
   & \\
   \end{array}
   \]
   \[
   \begin{array}{c}
   4 \overline{5} \\
   \hline
   6 \\
   \end{array}
   \]
   Quotient = __________
   Remainder = __________

2. \(56 \div 4\)
   \[
   \begin{array}{c|c}
   \text{Tens} & \text{One} \\
   \hline
   & \\
   \end{array}
   \]
   \[
   \begin{array}{c}
   4 \overline{5 6} \\
   \hline
   \\
   \end{array}
   \]
   Quotient = __________
   Remainder = __________
Name ____________________________ Date ______________

Show the division using disks. Relate your model to long division. Check your quotient and remainder by using multiplication and addition.

1. 7 ÷ 2

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   2 ) 7

   quotient = __________
   remainder = __________

2. 73 ÷ 2

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   2 ) 7 3

   quotient = __________
   remainder = __________

3. 6 ÷ 4

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   4 ) 6

   quotient = __________
   remainder = __________
4. \(62 \div 4\)

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4 \[ 6 \ 2 \] quotient = \_
remainder = \_

5. \(8 \div 3\)

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3 \[ 8 \] quotient = \_
remainder = \_

6. \(84 \div 3\)

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3 \[ 8 \ 4 \] quotient = \_
remainder = \_
Lesson 18

Objective: Find whole number quotients and remainders.

Suggested Lesson Structure

- **Fluency Practice** (12 minutes)
- **Application Problem** (7 minutes)
- **Concept Development** (31 minutes)
- **Student Debrief** (10 minutes)

Total Time (60 minutes)

### Fluency Practice (12 minutes)

- **Group Count 4.OA.1** (4 minutes)
- **Divide Mentally 4.NBT.6** (4 minutes)
- **Divide Using the Standard Algorithm 4.NBT.6** (4 minutes)

### Group Count (4 minutes)

Note: This prepares students to divide with remainders during this lesson’s Concept Development.

Direct students to count forward and backward, occasionally changing the direction of the count.

- Fours to 40
- Sixes to 60

### Divide Mentally (4 minutes)

Note: This reviews content from G4–M3–Lessons 16 and 17.

Repeat the process from G4–M3–Lesson 17 using the following possible sequence: 48 ÷ 2, 55 ÷ 5, 96 ÷ 3, and 84 ÷ 4.

### Divide Using the Standard Algorithm (4 minutes)

Materials: (S) Personal white boards

Note: This drill reviews G4–M3–Lesson 17’s content.

Repeat the process from G4–M3–Lesson 17 using the following possible sequence: 20 ÷ 3, 50 ÷ 2, 43 ÷ 3, and 64 ÷ 5.
Application Problem (7 minutes)

Mallory’s family is going to buy oranges. The Grand Market sells oranges at 3 pounds for 87 cents. How much does 1 pound of oranges cost at Grand Market?

Note: This fluency reviews division with a remainder in the tens from Lesson 17.

Concept Development (31 minutes)

Materials: (S) Personal white boards

Problem 1: Divide a two-digit number by a one-digit divisor with a remainder in the tens place.

5 tens 7 ones ÷ 3

T: (Write $57 \div 3$.) Let’s divide 57 into 3 equal groups. Break 57 into tens and ones.
S: 5 tens 7 ones.
T: Let’s divide 5 tens first. Why?
S: When we divide, we always start with the larger units. We divide the tens first because we may have to change tens for ones.
T: 5 tens divided by 3?
S: 1 ten in each group with 2 tens remaining.
T: We’ve distributed 3 tens. Let’s write 3 in the tens place. We also write that there are 2 tens remaining because 5 minus 3 is 2.
T: How do we divide the remaining 2 tens?
S: We unbundle the 2 tens as 20 ones.
T: Yes. So, how many ones do we have altogether?
S: 27.
T: Yes, 20 ones plus 7 ones is 27 ones.
T: You know your threes facts. Get ready for some mental math. What’s 27 ones divided by 3?
S: 9 ones!
T: 9 ones in each group is recorded above in the ones place. Read the quotient.
S: 19.
T: Say the division sentence.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Scaffold long division with the following options:
- Provide graph paper for easy alignment of tens and ones.
- Label the ‘tens’ and ‘ones’ place.
- Write zeros as place holders.
Lesson 18

Lesson 18: Find whole number quotients and remainders.

Date: 8/28/13

S: 57 divided by 3 is 19.
T: Check with multiplication. What’s 19 times 3?
S: 57!

Problem 2: Divide with a remainder in the tens and ones place.
8 tens 6 ones ÷ 5

T: (Write 86 ÷ 5.) You’ve solved 57 divided by 3 by unbundling tens. Let’s try a more challenging problem. How many groups will we divide 86 into?
S: 5.
T: Tell me, what is the first step?
S: Start with the tens. Divide 8 tens into 5 groups. That’s 1 ten in each group with 3 tens remaining.
T: Show me on your boards how you’ve recorded the distributed tens and the remaining tens.
T: What will you do with the 3 remaining tens?
S: Unbundle 3 tens as 30 ones.
T: How many ones altogether?
S: 36.
T: Next step?
S: Divide 36 ones into 5 groups. That’s 7 ones in each group with 1 one remaining.
T: How did you record what you distributed? What remains? Check your neighbor’s writing. Thumbs up if you agree.
T: I see you’ve written 35 ones distributed under 36 ones you had at first. Did you write R1? Read your quotient. Read your remainder. What is 86 divided by 5?
S: 17 with a remainder of 1.
T: How could you prove your division is correct?
S: Multiply 17 by 5 and then add 1 more.
T: Work with your partner to check with multiplication.

Problem 3: Use mental math to divide and calculate a remainder.
7 tens 4 ones ÷ 8

T: (Write 74 ÷ 8.) You’ve unbundled tens. And you’ve written remainders in the quotient. Now, take a look at this problem. What’s tricky here?
S: Hey! We can’t divide 7 tens into 8 groups! What will we do?
T: We’ll think of our eights facts. I’m thinking of an eights fact whose product is close to 74. Can you guess?
S: 72. 8 times 9 is 72.
T: Nice job! But 72 is only part of 74. What’s the missing part?
S: 2!
T: Say the division equation.
S: 74 divided by 8 is 9 with a remainder of 2.

Continue with 87 \div 9 and 64 \div 7. Gradually omit the number bond and encourage mental math.

**Problem Set (10 minutes)**

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first.

**Student Debrief (10 minutes)**

**Lesson Objective:** Find whole number quotients and remainders.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Compare the remainders to the divisors on the Problem Set. What do you find is true? Which always has a larger value? Why is that?
- How did the zero effect your division in Problem 9?
- What did you notice about the divisor, the whole, and quotients in Problems 9 and 10?
- Can you predict whether or not there will be a remainder? How?
- The whole is the same on Problems 11 and 12. Why is the quotient smaller on Problem 11?

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 18 Problem Set

Name ______________________________ Date __________________

Solve using the standard algorithm. Check your quotient and remainder by using multiplication and addition.

1. \(46 \div 2\)

2. \(96 \div 3\)

3. \(85 \div 5\)

4. \(52 \div 4\)

5. \(53 \div 3\)

6. \(95 \div 4\)
Lesson 18 Problem Set

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Find whole number quotients and remainders.

Date: 8/28/13
Lesson 18 Exit Ticket

Name ____________________________ Date ________________

Solve using the standard algorithm. Check your quotient and remainder by using multiplication and addition.

1. \( 93 \div 7 \)  
2. \( 99 \div 8 \)
Solve using the standard algorithm. Check your quotient and remainder by using multiplication and addition.

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Lesson 19

Objective: Explain remainders by using place value understanding and models.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Development (25 minutes)
- Student Debrief (15 minutes)

Total Time (60 minutes)

Fluency Practice (12 minutes)

- Sprint: Mental Division 4.NBT.6 (8 minutes)
- Divide Using the Standard Algorithm 4.NBT.6 (4 minutes)

Materials: (S) Mental Division Sprint
Note: This Sprint reviews content from previous lessons and reinforces place value used in the division algorithm.

Divide Using the Standard Algorithm (4 minutes)

Materials: (S) Personal white boards
Note: This reviews G4–M3–Lesson 17’s content.
Repeat the process from G4–M3–Lessons 17 and 18 using the following possible sequence: 37 ÷ 2, 45 ÷ 3, 26 ÷ 4, and 58 ÷ 3.

Application Problem (8 minutes)

Two friends start a business writing and selling comic books. After 1 month, they have earned $38. Show how they can fairly share their earnings, using $1, $5, $10, and/or $20 bills.
Note: Students practice decomposing a ten using long division from Lesson 18 and with a money model. Other acceptable answers are 1 ten 9 ones, 19 ones, or 3 fives 4 ones.

Concept Development (25 minutes)

Materials: (S) Personal white boards

Model division with remainders in the tens and ones places using number disks.

Problem 1

41 ÷ 3

T: (Write 41 ÷ 3). What disks will you draw to represent 41?
S: 4 tens 1 one.
T: How many groups will we divide 41 into?
S: 3.
T: Draw 3 groups, and let’s share 4 tens equally. How many tens in each group? Draw number disks as you distribute 4 tens into 3 groups like you’re dealing cards to 3 players.
S: 1 ten in each group with 1 ten remaining.
T: How can we divide the remaining ten?
S: Unbundle 1 ten as 10 ones.
T: Let’s see you draw that. (Allow students time to draw.) What did you do?
S: I drew an arrow from 1 ten disk in the tens place and drew 10 ones in the ones place.
T: How many ones do you have now?
S: 11 ones.
T: Let’s divide those 11 ones into 3 groups. Divide 11 ones into 3 groups by distributing 1 to each group. How many are remaining?
S: 8.
T: Are there enough to distribute again?
S: Yes. We can distribute another one to each group.
T: How many are left now?
S: Five. We can distribute again. We will have 2 remaining.
T: Explain what happened.

Notes on Multiple Means of Action and Expression:

Some learners may need less guidance to model 41 ÷ 3, and after solving quickly and independently, may benefit more from writing a step-by-step script for solving 41 ÷ 3 in preparation for Problem 5 of the Problem Set. This script might be used in a video of the student supporting his peers as they learn long division.
Lesson 19: Explain remainders by using place value understanding and models.

Date: 8/28/13

S: 2 ones are left after distributing the rest equally. We had to keep distributing until we didn’t have enough to distribute evenly again.

T: Now your number disks clearly show the solution for $41 \div 3$. Tell me the quotient. Tell me the remainder.

S: 41 divided by 3 is 13 with a remainder of 2.

T: With your partner write an equation we can use to check your division.

S: $(13 \times 3) + 2 = 41$.

T: With your partner find where 13, 3, 2, and 41 are represented in the place value chart.

S: Thirteen is the 1 ten and 3 ones in each group. Three is the number of groups we made. Two is the remaining 2 ones from the whole. Forty-one is the whole.

Problem 2

Share $64$ as 6 tens and 4 ones equally between 4 friends.

T: Tell your partner what happens when we have an extra ten we can’t distribute.

S: We break the ten apart into 10 ones. Then, we add the 10 ones to the ones that are already there. Then, we can distribute the ones into 3 equal groups.

T: Can you think of a real life situation in which you might change a ten for 10 ones?

S: Yeah! When you’re getting change for 10 dollars! → If the soda machine doesn’t take tens, you need to change out for ones.

T: Let’s say I give 4 students $64$ to share equally—6 ten dollar bills and 4 one dollar bills. Write an equation and draw number disks to show the money.

T: What happens when you try to share 6 ten dollar bills equally with 4 people?

S: Each person gets $10, but then you have 2 ten dollar bills left.

T: What do you do?

S: Make change! Cash in those 2 ten dollar bills for 20 ones. Then we can share the money fairly. → Or, they could change the 2 tens for 4 fives. That would work too.

T: You’re both correct. Either approach would work. Since we’re using a place value chart to show division, let’s pretend they changed the 2 tens for 20 ones, and model that. Since we have so many ones, model with quick dots as you distribute like a fast card dealer. How will you distribute the ones?

S: I will keep distributing them until I can’t distribute them equally anymore. This time I was able to distribute evenly.

T: Why do you have to keep distributing?

S: If I don’t keep distributing, there will be too many remaining. That means that you would be able to distribute again but didn’t.

T: How much money does each student receive?

S: $16!
Lesson 19

Explain remainders by using place value understanding and models.

Problem Set (15 minutes)

Students should do their personal best to complete the Problem Set within the allotted 15 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (15 minutes)

Lesson Objective: Explain remainders by using place value understanding and models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

 Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 2, Cayman’s remainder is larger than the divisor. What rule can you suggest to Cayman so he doesn’t make this mistake again? Was his answer completely wrong? Why not?

- In Problem 4, the friends have to make change for the 1 ten dollar bill. Why can’t they tear the bill in half? How does that relate to the number disks?

- In Problem 5, how did you script describe the remainder in the tens and ones?

- Select a few students to share and compare their scripts for solving 45 ÷ 3.

- Compare using number disks and other methods to divide. Which do you prefer? Why?
- We related a remainder in the tens place to making money change. What other real life situations can you relate it to? Is this similar to mixed metric units, such as having 5 liters of water to share among 4 people?
- With money, sometimes we might use units other than ones and tens, such as fives or twenties. Why do you think we only use ones and tens to model division on the place value chart?

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
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## Lesson 19 Sprint

**NYS COMMON CORE MATHEMATICS CURRICULUM**

**Lesson 19:** Explain remainders by using place value understanding and models.

**Date:** 8/28/13

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This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.
1. When you divide 94 by 3, there is a remainder of 1. Model this problem with number disks. In the number disk model, how did you show the remainder?

2. Cayman says that $94 \div 3$ is 30 with a remainder of 4. He reasons it is correct because $(3 \times 30) + 4 = 94$. What mistake has Cayman made? Explain how he can correct his work.

3. The number disk model is showing $72 \div 3$. Complete the model. Explain what happens to the 1 ten that is remaining in the tens column.
4. Two friends share 56 dollars.
   a. They have 5 ten dollar bills and 6 dollar bills. Draw a picture to show how the bills will be shared. Will they have to make change at any stage?

   b. Explain how they share the money evenly.

5. Imagine you are filming a video explaining the problem $45 \div 3$ to new fourth graders. Create a script to explain how you can keep dividing after getting a remainder of 1 ten in the first step.
Lesson 19 Exit Ticket

1. Molly’s photo album has a total of 97 pictures. Each page of the album holds 6 pictures. How many pages can Molly fill? Will there be any pictures left? If so, how many? Use number disks to solve.

2. Marti’s photo album has a total of 45 pictures. Each page holds 4 pictures. She said she can only fill 10 pages completely. Do you agree? Explain why or why not.
Lesson 19 Homework

Name ___________________________  Date ________________

1. When you divide 86 by 4 there is a remainder of 2. Model this problem with number disks. In the number disk model, how can you see that there is a remainder?

2. Francine says that 86 ÷ 4 is 20 with a remainder of 6. She reasons it is correct because (4 × 20) + 6 = 86. What mistake has Francine made? Explain how she can correct her work.

3. The number disk model is showing 67 ÷ 4. Complete the model. Explain what happens to the 2 tens that are remaining in the tens column.

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4. Two friends share 76 blueberries.
   a. To count the blueberries, they have already put them into small bowls of 10. Draw a picture to show how the blueberries can be shared equally. Will they have to split apart any of the bowls of 10 blueberries when they share them?

   b. Explain how the friends can share the blueberries fairly.

5. Imagine you are drawing a comic strip showing how to solve the problem 72 ÷ 4 to new fourth graders. Create a script to explain how you can keep dividing after getting a remainder of 3 tens in the first step.
Lesson 20

Objective: Solve division problems without remainders using the area model.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Divide Using the Standard Algorithm 4.NBT.6 (4 minutes)
- Find the Unknown Factor 4.OA.4 (5 minutes)
- Mental Multiplication 4.NBT.5 (3 minutes)

Divide Using the Standard Algorithm (4 minutes)

Materials: (S) Personal white boards

Note: This drill reviews G4–M3–Lesson 17’s content.

Repeat the process from G4–M3–Lessons 17–19 using the following possible sequence: 67 ÷ 2, 60 ÷ 4, 29 ÷ 3, and 77 ÷ 4.

Find the Unknown Factor (5 minutes)

Materials: (S) Personal white boards

Note: This prepares students for G4–M3–Lesson 22’s Concept Development

T: (Write 5 × ___ = 15.) Say the unknown factor.
S: 3.
T: (Write 15 ÷ 5.) On your boards, write the division problem.
S: (Write 15 ÷ 5 = 3.)

Continue with the following possible suggestions: 3 × ___ = 12, 4 × ___ = 12, 5 × ___ = 35, 6 × ___ = 36, 7 × ___ = 49, 9 × ___ = 81, 6 × ___ = 48, 7 × ___ = 42, and 9 × ___ = 54.
Lesson 20

Solve division problems without remainders using the area model.

Date: 8/28/13

Mental Multiplication (3 minutes)

Note: This fluency reviews content taught earlier in the module.

T: (Write $3 \times 2 = \_\_\_\_$.) Say the multiplication sentence in unit form.
S: 3 ones $\times$ 2 = 6 ones.

T: (Write $3 \times 2 = 6$. To the right, write $30 \times 2 = \_\_\_\_$.) Say the multiplication sentence in unit form.
S: 3 tens $\times$ 2 = 6 tens.

T: (Write $30 \times 2 = 60$. To the right, write $30 \times 20 = \_\_\_\_\_.") Say the multiplication sentence in unit form.
S: 3 tens $\times$ 2 tens = 6 hundreds.

T: (Write $30 \times 20 = 600$.)

Continue with the following possible suggestions: $4 \times 2$, $40 \times 2$, $40 \times 20$, $5 \times 3$, $50 \times 3$, and $50 \times 30$.

Application Problem (8 minutes)

Write an equation to find the unknown length of each rectangle. Then find the sum of the two unknown lengths.

a. 4 cm 40 square cm

8 square cm

$(40 \div 4) + (8 \div 4)$

$10 + 2 = 12$

12 centimeters

b. 4 cm 80 square cm

16 square cm

$(80 \div 4) + (16 \div 4)$

$20 + 4 = 24$

24 centimeters

Note: This Application Problem serves as an introduction to the Concept Development, in which the students find the total unknown length of a rectangle with an area of 48, corresponding to Part (a) and 96, corresponding to Part (b).
Lesson 20: Solve division problems without remainders using the area model.

Concept Development (30 minutes)

Materials: (S) Personal white boards

Problem 1: Decompose 48 ÷ 4 from whole to part.

T: Draw a rectangle with an area of 48 square units and a width of 4 units.
S: (Draw.)

T: Draw a new rectangle with the same area directly below but partitioned to match the areas of the rectangles in Part (a) of the Application Problem.

T: Let’s draw a number bond to match the whole and parts of the rectangle. (Students and teacher draw bond as pictured below.)

T: Let’s find the unknown side lengths of the smaller rectangles and add them. (Show as the distribution of the quotients shown to the right.) What is 40 ÷ 4?
S: 10.
T: What is 8 ÷ 4?
S: 2.
T: What is 10 and 2?
S: 12.
T: What is 48 divided by 4?
S: 12.
T: What is the length of the unknown side?
S: 12 units.
T: Take a moment to record the number sentences, reviewing with your partner their connection to both the number bond and the area model.

T: Work with your partner to partition the same area of 48 as 2 twenties and 8. Try to find another way to partition the area of 48 so it’s easy to divide.

T: (Allow students to work for about 4 minutes.) Did anyone find another way to partition the area of 48 so it’s easy to divide?
Lesson 20

Lesson 20: Solve division problems without remainders using the area model.

S: Yes! 24 + 24. 24 divided by 4 is 6. 6 + 6 is 12. → 30 and 18 don’t work well because 30 has a remainder when you divide it by 4. → I did it by using 4 rectangles, each with an area of 12 square units. → Oh yeah, 12 + 12 + 12 + 12.

T: Explain to your partner why different ways of partitioning give us the same correct side length.

S: You are starting with the same amount of area but just chopping it up differently. → The sum of the lengths is the same as the whole length. → You can take a total, break it into two parts, and divide each of them separately. → I use the same break apart and distribute strategy to find the answer to 56 ÷ 8. 40 ÷ 8 is 5. 16 ÷ 8 is 2. 5 and 2 makes 7.

Problem 2: Decompose 96 ÷ 4 from whole to part.

Repeat the same process with Part (b) from the Application Problem.

T: How did you partition the area of 96?

S: We chopped 96 into 40 + 40 + 16. → It was just like 48 ÷ 4. We saw that we could partition 96 into 4 twenties and 2 eights. → We made 96 into 2 forty-eights and used our answer from 48 ÷ 4! All we had to do was double it.

T: Discuss with your partner why we do not decompose 96 as 90 and 6.

S: 9 tens ÷ 4 gives a remainder.

T: True!

Problem 3: Compose 96 ÷ 4 from part to whole.

T: (Write 96 ÷ 4.) Thinking about area, let’s try a new way to divide. The expression 96 ÷ 4 can describe a rectangle with an area of 96 square units. We are trying to find out the length of the unknown side.

T: What is the known side length?

S: 4.

T: (Draw a rectangle with a width of 4.) 4 times how many tens gets us as close as possible to an area of 9 tens? (Point to the 9 tens of the dividend.)
Lesson 20

Solve division problems without remainders using the area model.

Date: 8/28/13

S: 2 tens.

T: Let’s give 2 tens to the length. (Label 2 tens above the rectangle.) Let’s record the 2 tens in the tens place.

T: What is 4 times 2 tens?

S: 8 tens. (Record 8 below the 9 tens.)

T: How many square units is that?

S: 80 square units. (Record 80 square units in the rectangle.)

T: How many tens remain?

S: 1 ten.

T: (Record 1 ten below the 8 tens.) Let’s add the remaining ten to the 6 ones. What is 1 ten + 6 ones? (Record the 6 ones to the right of the 1 ten.)

S: 16.

T: We have 16 square units remaining with a width of 4. (Point to the 16 in the algorithm.) 4 times how many ones gets us as close as possible to an area of 16 ones?

S: 4 ones.

T: Let’s give 4 ones to the length.

T: What is 4 times 4?

S: 16. We have 16 square units.

T: We have no more area to divide.

T: Tell me the length of the unknown side.

S: 24!

T: Our quotient tells us that length.

T: How can we express the length of the unknown side using the distributive property?

S: \((80 \div 4) + (16 \div 4)\)

T: With your partner, draw arrows to connect the distributive property and the area model.

T: Review our four drawings and our process with your partner. Try to reconstruct what we did step by step before we try another one.

T: (Allow time for students to review.) We solved 96 divided by 4 in two very different ways using the area model. First, we started with the whole rectangle and partitioned it. The second way was to go one place value at a time and make the whole rectangle from parts.
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Solve division problems without remainders using the area model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 2, did you partition the rectangle the same way as your partner? Why were we able to go from whole to part?
- In Problems 2 and 3, explain the connection between the written method, the number bond, and the area model.
- In the last problem, explain the connection between the algorithm and the area model.
- Each time we divide, what happens to the amount of area we still have left to divide?
- Even though division is messy, I think it is the most interesting operation of all because, imagine this, sometimes that little piece that is left to divide is always there, even though it gets infinitely small!
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 20: Solve division problems without remainders using the area model.

Date: 8/28/13

Name ____________________________ Date _________________

1. Alfonso solved a division problem by drawing an area model.
   a. Look at the area model. What division problem did Alfonso solve?
      ![Area Model]

      b. Show a number bond to represent Alfonso’s area model. Start with the total and then show how the total is split into two parts. Below the two parts, represent the total length using the distributive property and then solve.

         ![Number Bond]

         \[(\_\div\_) + (\_\div\_)\]

         \[= \_ \quad + \quad \_\]

         \[= \_\]

2. Solve 45 ÷ 3 using an area model. Draw a number bond and use the distributive property to solve for the unknown length.
3. Solve $64 \div 4$ using an area model. Draw a number bond to show how you partitioned the area, and represent the division with a written method.

4. Solve $92 \div 4$ using an area model. Explain, using words, pictures, or numbers, the connection of the distributive property to the area model.

5. Solve $72 \div 6$ using an area model and the standard algorithm.
1. Tony drew the following area model to find an unknown length. What division equation did he model?

![Area Model](image)

2. Solve $42 \div 3$ using the area model, a number bond, and a written method.
1. Maria solved the following division problem by drawing an area model.
   a. Look at the area model. What division problem did Maria solve?

   ![Area Model](image)

   b. Show a number bond to represent Maria’s area model. Start with the total and then show how the total is split into two parts. Below the two parts, represent the total length using the distributive property and then solve.

   ![Number Bond](image)

   \[
   (\_\div\_) + (\_\div\_)
   = \_ + \_
   = 
   \]

2. Solve \(42 \div 3\) using an area model. Draw a number bond and use the distributive property to solve for the unknown length.

   ![Area Model](image)
3. Solve $60 \div 4$ using an area model. Draw a number bond to show how you partitioned the area, and represent the division with a written method.

4. Solve $72 \div 4$ using an area model. Explain, using words, pictures, or numbers, the connection of the distributive property to the area model.

5. Solve $96 \div 6$ using an area model and the standard algorithm.
Lesson 21

Objective: Solve division problems with remainders using the area model.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Sprint: Division with Remainders 4.NBT.6 (8 minutes)
- Find the Unknown Factor 4.OA.4 (4 minutes)

Sprint: Division with Remainders (8 minutes)

Materials: (S) Division with Remainders Sprint

Note: This Sprint reviews content from Topic E, including division with basic one- and two-digit facts with and without remainders.

Find the Unknown Factor (4 minutes)

Materials: (S) Personal white boards

Note: This prepares students for G4–M3–Lesson 22’s Concept Development

T: (Write $6 \times ____ = 18.$) Say the unknown factor.
S: 3.

T: (Write $18 \div 6.$) On your boards, complete the division sentence.
S: (Write $18 \div 6 = 3.$)

Continue with the following possible suggestions: $3 \times ____ = 21$, $4 \times ____ = 20$, $5 \times ____ = 25$, $6 \times ____ = 42$, $7 \times ____ = 56$, $9 \times ____ = 72$, $6 \times ____ = 54$, $7 \times ____ = 63$, and $9 \times ____ = 63$. 
Lesson 21

Solve division problems with remainders using the area model.

Date: 8/28/13

Application Problem (8 minutes)

A rectangle has an area of 36 square units and a width of 2 units. What is the unknown side length?

Method 1:

Method 2:

The unknown side length is 18 units.

Note: This Application Problem serves as an introduction to Problem 1 in the Concept Development, in which the students find the total unknown lengths of a rectangle with an area of 37 and a side length of 2. In this Concept Development, students move on to the complexity of using the area model when there is a remainder.

Concept Development (30 minutes)

Materials: (S) Problem Set

Note: Use the Problem Set for Lesson 21 to record work for Problems 1 and 2 of this Concept Development. Use the remaining problems on the Problem Set for class instruction or independent practice.

Problem 1: 37 ÷ 2

T: (Display the Application Problem with an area of 36 square units on graph paper.) This rectangle has a side length of 18. What would be the area of a rectangle with a width of 2 units and a length of 19 units? (Draw on graph paper.)

S: 38 square units.

T: So we cannot represent a rectangle with an area of 37 square units with whole number side lengths. Let’s build a rectangle part to whole as we did yesterday.
T: Draw a rectangle. Label the width as 2 units. 2 times how many tens gets us as close as possible to an area of 3 tens?
S: 1 ten.
T: Label this rectangle with a length of 1 ten. Record 1 ten in the tens place. What is 1 ten times 2?
S: 2 tens.
T: How many square units of area is that?
S: 20 square units.
T: (Record 20 square units in the rectangle.) How many tens remain?
S: 1 ten. (Record 1 ten below 2 tens. Record 7 ones next to the 1 ten.)
T: 17 ones remain. 2 times how many ones gives us an area close to 17 square units?
S: 8 ones.
T: Extend the rectangle and label its length as 8 ones. 8 ones times 2 is?
S: 16 ones. (Record 16 ones.)
T: 16 ones represents the area of this rectangle. (Label as 16 square units.) How many ones remain?
S: 1 one.
T: To make a new length unit, we must have 2 square units. We only have 1. Let’s draw the remaining 1 square unit.
T: Let’s validate our drawing and algorithm using the distributive property. 20 square units divided by 2 is?
S: 10.
T: 10 length units. 16 square units divided by 2 is?
S: 8 length units.
T: 10 length units plus 8 length units is?
S: 18 length units.
T: Let’s solve for the area. 18 length units times 2 length units equals?
S: 36 square units.
T: We see that in our area model. Add 1 square unit, our remainder.
S: 37 square units.
Problem 2: $76 \div 3$

T: (Write $76 \div 3$.) I’m going to represent this with an area model moving from part to whole by place value just as we did with $37 \div 2$. What should the total area be?

S: 76 square units.

T: (Draw a rectangle.) What is the width or the known side length?

S: 3 length units.

T: (Label a width of 3.) 3 times how many tens gets us as close as possible to an area of 7 tens? (Point to the 7 tens of the dividend.)

S: 2 tens.

T: Let’s give 2 tens to the length. (Write the length on the area model.) Let’s record 2 tens in the tens place.

T: What is 2 tens times 3?

S: 6 tens. (Record 6 tens below the 7 tens.)

T: How many square units of area is that?

S: 60 square units. (Record in the rectangle.)

T: How many tens remain?

S: 1 ten. (Record 1 ten below the 6 tens.)

T: Let’s add the remaining ten to the 6 ones. What is 1 ten + 6 ones? (Record the 6 ones to the right of the 1 ten.)

S: 16 ones.

T: We have an area of 16 square units remaining with a width of 3. (Point to the 16 in the algorithm.) 3 times how many ones gets us as close as possible to an area of 16?

S: 5 ones.

T: Let’s give 5 ones to the length. (Label the length.)

T: This rectangle has an area of?

S: 15 square units.

T: How many square units remaining?

S: 1 square unit.

T: What is the unknown length and how many square units remain?

S: The length of the unknown side is 25 units. One square unit was left over.
Lesson 21

Solve division problems with remainders using the area model.

Date: 8/28/13

Notes on Multiple Means of Representation:

You might give students the option of using graph paper, which gives the concreteness of the squares that make up the area, to draw the area models.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Lesson Objective: Solve division problems with remainders using the area model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 3, explain to your partner the connection between the distributive property and the area model.
- Because we often have remainders when we divide, we have to use the area model by building up from part to whole. What did the first rectangle you drew in Problem 1 represent? The next chunk of the rectangle?
- Each time we divide, what happens to the amount of area we still have left to divide?
- Why don’t we have this complication of leftovers or remainders with multiplication?
- In Problem 4, we didn’t know if we were going to have a remainder in the ones place, so instead we built up to the area working with one place value unit at a time. How might the problems with remainders been challenging if you started with the whole area, like in Lesson 20?
- (Optional.) Let’s look back at Problem 2, 76 ÷ 3. What if we cut this remaining square unit into 3 equal parts with vertical lines? What is the length of one of these units? What if we stack them to add more area? What is the total length of the new rectangle including this tiny piece?
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
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<td></td>
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</tr>
<tr>
<td>17</td>
<td>$4 \div 4$</td>
<td></td>
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<tr>
<td>18</td>
<td>$5 \div 4$</td>
<td></td>
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<tr>
<td>19</td>
<td>$6 \div 2$</td>
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<td>20</td>
<td>$7 \div 2$</td>
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<tr>
<td>21</td>
<td>$8 \div 5$</td>
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<tr>
<td>22</td>
<td>$7 \div 5$</td>
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</tbody>
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© Bill Davidson
1. Solve $37 \div 2$ using an area model. Use long division and the distributive property to record your work.

2. Solve $76 \div 3$ using an area model. Use long division and the distributive property to record your work.

3. Carolina solved the following division problem by drawing an area model.

   ![Area Model](image)

   a. What division problem did she solve?
   b. Show how Carolina’s model can be represented using the distributive property.
Solve the following problems using the area model. Support the area model with long division or the distributive property.

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>4.</td>
<td>48 ÷ 3</td>
</tr>
<tr>
<td>5.</td>
<td>49 ÷ 3</td>
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<tr>
<td>6.</td>
<td>56 ÷ 4</td>
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<td>7.</td>
<td>58 ÷ 4</td>
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<td>8.</td>
<td>66 ÷ 5</td>
</tr>
<tr>
<td>9.</td>
<td>79 ÷ 3</td>
</tr>
</tbody>
</table>

10. Seventy-three students are divided into groups of 6 students each. How many groups of 6 students are there? How many students will not be in a group of 6?
1. Kyle drew the following area model to find an unknown length. What division equation did he model?

   ![Area Model Diagram]

2. Solve $93 \div 4$ using the area model, long division, and the distributive property.
Lesson 21 Homework 4·3

Name ____________________________ Date ________________

1. Solve $35 \div 2$ using an area model. Use long division and the distributive property to record your work.

2. Solve $79 \div 3$ using an area model. Use long division and the distributive property to record your work.

3. Paulina solved the following division problem by drawing an area model.

   ![Area Model](image)

   a. What division problem did she solve?
   b. Show how Paulina’s model can be represented using the distributive property.
Solve the following problems using the area model. Support the area model with long division or the distributive property.

4. \(42 \div 3\)  
5. \(43 \div 3\)  
6. \(52 \div 4\)  
7. \(54 \div 4\)  
8. \(61 \div 5\)  
9. \(73 \div 3\)  

10. Ninety-seven lunch trays were placed equally in 4 stacks. How many lunch trays were in each stack? How many lunch trays will be leftover?