Topic A

Decomposition and Fraction Equivalence

4.NF.3b, 4.NF.4a, 4.NF.3a

Focus Standard: 4.NF.3b  Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

4.NF.4a  Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).

Instructional Days: 6
Coherence - Links from: G3–M5 Fractions as Numbers on the Number Line
-Links to: G5–M3 Addition and Subtraction of Fractions

Topic A builds on Grade 3 work with unit fractions. Students explore fraction equivalence through the decomposition of non-unit fractions into unit fractions, as well as the decomposition of unit fractions into smaller unit fractions. They represent these decompositions, and prove equivalence, using visual models.

In Lessons 1 and 2, students decompose fractions as unit fractions, drawing tape diagrams to represent them as sums of fractions with the same denominator in different ways, e.g.,

\[
\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5},
\]

\[
\frac{2}{3} = \frac{1}{3} + \frac{1}{3}.
\]

In Lesson 3, students see that representing a fraction as the repeated addition of a unit fraction is the same as multiplying that unit fraction by a whole number. This is already a familiar
fact in other contexts. For example,

\[3 \text{ bananas} = 1 \text{ banana} + 1 \text{ banana} + 1 \text{ banana} = 3 \times 1 \text{ banana},\]

\[3 \text{ twos} = 2 + 2 + 2 = 3 \times 2\]

\[3 \text{ fourths} = 1 \text{ fourth} + 1 \text{ fourth} + 1 \text{ fourth} = 3 \times 1 \text{ fourth},\]

\[\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}\]

By introducing multiplication as a record of the decomposition of a fraction early in the module, students are accustomed to the notation by the time they work with more complex problems in Topic G.

Students continue with decomposition in Lesson 4, where they represent fractions, e.g., \(\frac{1}{2}, \frac{1}{3}, \text{ and } \frac{2}{3}\), as the sum of smaller unit fractions. They fold a paper strip to see that the number of fractional parts in a whole increases, while the size of the pieces decreases. Students investigate and verify this idea through a paper folding activity and record the results with tape diagrams, e.g., \(\frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \left(\frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{8} + \frac{1}{8}\right) = \frac{4}{8}\)

In Lesson 5, this idea is further investigated as students represent the decomposition of unit fractions in area models. In Lesson 6, students use the area model for a second day, this time to represent fractions with different numerators. They explain why two different fractions represent the same portion of a whole.
### A Teaching Sequence Towards Mastery of Decomposition and Fraction Equivalence

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Lesson 1

Objective: Decompose fractions as a sum of unit fractions using tape diagrams.

Suggested Lesson Structure

- Fluency Practice (9 minutes)
- Application Problem (8 minutes)
- Concept Development (33 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (9 minutes)

- Read Tape Diagrams 3.OA.3 (5 minutes)
- Addition of Fractions in Unit Form 3.NF.1 (4 minutes)

Read Tape Diagrams (5 minutes)

Materials: (S) Personal white boards

Note: This fluency activity prepares students for G4–M5–Lesson 1.

T: (Project a tape diagram partitioned into 2 equal parts. Write 10 at the top.) Say the value of the whole.

S: 10.

T: Write the value of one unit as a division problem.

S: (Write 10 ÷ 2 = 5.)

T: Write the whole as a repeated addition sentence.

S: (Write 5 + 5 = 10.)

Continue the process for 6 ÷ 2, 15 ÷ 3, 6 ÷ 3, 12 ÷ 4, and 24 ÷ 4.

Addition of Fractions in Unit Form (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity prepares students for G4–M5–Lesson 1.

T: (Project a circle partitioned into 2 equal parts with 1 part shaded.) How many circles do you see?
Lesson 1

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S: 1.
T: How many equal parts are in the circle?
S: 2.
T: What fraction of the circle is shaded?
S: \(\frac{1}{2}\).
T: (Write \(1\) half + \(1\) half = \(2\) halves = \(1\) whole.) True or false?
S: True.
T: Explain why it is true to your partner.
S: \(1 + 1\) is \(2\). That’s kindergarten. \(\rightarrow\) Two halves is the same as \(1\). \(\rightarrow\) Half an apple + half an apple is \(1\) apple.
T: (Project a circle partitioned into \(4\) equal parts with \(1\) part shaded.) How many circles do you see?
S: 1.
T: How many equal parts does this circle have?
S: 4.
T: Write the fraction that is represented by the shaded part.
S: (Write \(\frac{1}{4}\).)
T: (Write \(1\) fourth + \(1\) fourth + \(1\) fourth + \(1\) fourth = \(4\) fourths = \(1\) whole.) True or false?
S: True.

Continue the process with the other fraction graphics.

Application Problem (8 minutes)

Materials: (S) 1 index card with diagonals drawn, 1 pair of scissors per pair of students

Use your scissors to cut your rectangle on the diagonal lines. Prove that you have cut the rectangle into \(4\) fourths. Include a drawing in your explanation.

Note: This Application Problem reviews and reinforces the concept that fractional parts have the same area. Many students may say that the diagonal lines do not create fourths because the triangles created by the diagonals do not look alike. Exploration will help students see the areas are in fact equal and prepare them for the work with tape models in this lesson.
Lesson 1

Concept Development (33 minutes)

Materials:  (T) 3 strips of paper, markers  (S) 3 strips of paper, colored markers or colored pencils, personal white board

Problem 1: Fold a strip of paper to create thirds and sixths. Record the decompositions represented by the folded paper with addition.

T: The area of this strip of paper is my whole. What number represents this strip of paper?
S: 1.
T: Fold to decompose the whole into 3 equal parts. (Demonstrate.)
T: Draw lines on the creases you made. (Demonstrate.)
T: Draw a number bond to represent the whole decomposed into 3 units of...?
S: 1 third.
T: (Allow students time to draw.) Tell me an addition number sentence to describe this decomposition starting with “1 equals....” (Record the sentence as students speak.)
S: $1 = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$.
T: Let’s show this decomposition in another way.
T: $1 = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$. Tell me a new addition sentence that matches the new groups starting with “1 equals....”
S: $1 = \frac{2}{3} + \frac{1}{3}$.
T: Decompose 5 sixths into 5 units of 1 sixth with a number bond. (Allow students time to work.)
T: Give me an addition sentence representing this decomposition starting with “5 sixths equals....” (Record the sentence as students speak.)
S: $\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$.
T: Let’s double the number of units in our whole. Fold your strip on the creases. Fold one more time in half. Open up your strip. Into how many parts have we now decomposed the whole?
S: 6!
Lesson 1: Decompose fractions as a sum of unit fractions using tape diagrams.

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Lesson 1

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Problem 1:

T: On the other side that has no lines, draw lines on the creases you made and shade \( \frac{5}{6} \) sixths.

T: Show this decomposition in another way.

T: (Insert parentheses.) \( \frac{5}{6} = \left( \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \right) + \left( \frac{1}{6} + \frac{1}{6} \right) \). Tell me a new addition sentence that matches this new decomposition, starting with “5 sixths equals.” (Record the sentence as students speak.)

S: \( \frac{5}{6} = \frac{3}{6} + \frac{2}{6} \).

T: Draw a number bond and addition sentence to match.

S: (Draw a number bond and addition sentence.)

T: Use your paper strip to show your partner the units that match each part.

S: \( \frac{5}{6} = \frac{3}{6} + \frac{2}{6} \).

Problem 2: Fold two strips of paper into fourths. Shade \( \frac{7}{4} \). Write the number sentence created.

T: Take a new strip of paper. The area of this strip of paper is the whole. Fold this paper to create 4 equal parts. (Demonstrate creating fourths vertically.) Shade all 4 of the parts. Take one more strip of paper, fold it, and shade 3 of the 4 parts. How much is shaded?

S: The first strip of paper represents \( \frac{4}{4} \). On the second strip of paper, we shaded \( \frac{3}{4} \). \( \frac{7}{4} = \frac{4}{4} + \frac{3}{4} \).

T: Draw a number bond to represent the 2 parts and their sum.

S: (Draw.)

T: Can \( \frac{4}{4} \) be renamed?

S: Yes, \( \frac{4}{4} \) is equal to 1.

T: Draw another number bond to replace \( \frac{4}{4} \) with 1 whole.

S: (Draw.)

T: Write a number sentence that represents this number bond.

S: (Write \( 1\frac{3}{4} = 1 + \frac{3}{4} \)).

T: We say this is one and three-fourths. \( 1\frac{3}{4} \) is another way to record the decomposition of \( \frac{7}{4} \) as \( \frac{4}{4} \) and \( \frac{3}{4} \). Compare and contrast \( 1\frac{3}{4} \) to \( \frac{7}{4} \).

S: One has a whole number. The other has just a fraction. → They both represent the same area, so they are equivalent. → So, when a fraction is greater than 1, we can write it using a whole number and a fraction.
Problem 3: Write decompositions of fractions represented by tape diagrams as number sentences.

Display the following tape diagram:

```
      1
    /   
   /     
  /       
```

T: The rectangle represents 1 whole. Name the unit fraction.
S: 1 fifth.
T: (Label \(\frac{1}{5}\) underneath both shaded unit fractions.) Name the shaded fraction.
S: 2 fifths.
T: Decompose \(\frac{2}{5}\) into unit fractions.
S: \(\frac{2}{5} = \frac{1}{5} + \frac{1}{5}\).

Display the tape diagram pictured to the right.

T: What is the unit fraction?
S: 1 fifth.
T: Use the model to write an addition sentence for the tape diagram showing the decomposition of 4 fifths indicated by the braces.
S: \(\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{2}{5}\).

Display the tape diagram pictured to the right.

T: What is the unit fraction?
S: 1 sixth. \(\rightarrow\) 1 fifth.
T: How do you know it is not \(\frac{1}{6}\)?
S: This tape diagram shows 5 equal parts shaded as being 1. Then, there’s another unit after that. \(\rightarrow\) This tape diagram represents a number greater than 1. \(\rightarrow\) This tape diagram is showing a mixed number.
T: Tell your partner the number this tape diagram represents.
S: \(\frac{6}{5} \Rightarrow 1\frac{1}{5}\).
T: On your boards, write the number sentence for the tape diagram showing a sum equal to 6 fifths.
Lesson 1: Decompose fractions as a sum of unit fractions using tape diagrams.

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Problem 4: Draw decompositions of fractions with tape diagrams from number sentences.

Display the number sentence $\frac{6}{6} = \frac{1}{6} + \frac{2}{6} + \frac{3}{6}$.

T: Discuss with your partner how this number sentence can be modeled as a tape diagram.

S: Well, the sum is a whole because $\frac{6}{6}$ is equal to 1. The unit fraction is $\frac{1}{6}$, so we should partition the tape diagram into 6 equally sized pieces. We can use brackets to label the whole and the addends.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the careful sequencing of the Problem Set guide your selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Assign incomplete problems for homework or at another time during the day.

Student Debrief (10 minutes)

Lesson Objective: Decompose fractions as a sum of unit fractions using tape diagrams.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a
partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- How do Problems 1(f), 1(g), and 1(h) differ from Problems 1(a–e)? How do the tape diagrams model a number greater than 1?
- Compare the size of the shaded fractions in Problem 1 (c) and 1(e). Assume the wholes are equal. What can you infer about the two number sentences?
- How do the number bonds connect to the number sentences?
- How did using the paper strips during our lesson help you to visualize the tape diagrams you had to draw in Problem 2?
- What relationship does the unit fraction have with the number of units in a whole?
- How did the Application Problem connect to today’s lesson?

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. Draw a number bond and write the number sentence to match each tape diagram. The first one is done for you.

a. \[1 = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}\]

b. [Tape diagram with 1 whole divided into 3 parts]

c. [Tape diagram with 1 whole divided into 3 parts]

d. [Tape diagram with 1 whole divided into 3 parts]

e. [Tape diagram with 1 whole divided into 3 parts]

f. [Tape diagram with 1 whole divided into 3 parts]

g. [Tape diagram with 1 whole divided into 3 parts]

h. [Tape diagram with 1 whole divided into 3 parts]
2. Draw and label tape diagrams to model each decomposition.

   a. \( 1 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \)
   
   b. \( \frac{4}{5} = \frac{1}{5} + \frac{2}{5} + \frac{1}{5} \)

   c. \( \frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} \)
   
   d. \( \frac{11}{8} = \frac{7}{8} + \frac{1}{8} + \frac{3}{8} \)

   e. \( \frac{12}{10} = \frac{6}{10} + \frac{4}{10} + \frac{2}{10} \)
   
   f. \( \frac{15}{12} = \frac{8}{12} + \frac{3}{12} + \frac{4}{12} \)

   g. \( \frac{2}{3} = 1 + \frac{2}{3} \)
   
   h. \( 1\frac{5}{8} = 1 + \frac{1}{8} + \frac{1}{8} + \frac{3}{8} \)
Lesson 1 Exit Ticket

1. Draw a number bond and write the number sentence to match the tape diagram.
   a.
   \[
   \begin{align*}
   1 & = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}
   \end{align*}
   \]

2. Draw and label tape diagrams to model each number sentence.
   a. \(1 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}\)
   b. \(\frac{5}{6} = \frac{2}{6} + \frac{2}{6} + \frac{1}{6}\)
1. Draw a number bond and write the number sentence to match each tape diagram. The first one is done for you.

a. \[ \frac{2}{3} = \frac{1}{3} + \frac{1}{3} \]
   \[
   \begin{array}{c}
   \frac{1}{3} \\
   \frac{1}{3}
   \end{array}
   \]

b.

c.

d.

e.

f.

g.

h.
2. Draw and label tape diagrams to match each number sentence.
   a. $\frac{5}{8} = \frac{2}{8} + \frac{2}{8} + \frac{1}{8}$
   b. $\frac{12}{8} = \frac{6}{8} + \frac{2}{8} + \frac{4}{8}$
   c. $\frac{11}{10} = \frac{5}{10} + \frac{5}{10} + \frac{1}{10}$
   d. $\frac{13}{12} = \frac{7}{12} + \frac{1}{12} + \frac{5}{12}$
   e. $1 \frac{1}{4} = 1 + \frac{1}{4}$
   f. $1 \frac{2}{7} = 1 + \frac{2}{7}$
Lesson 2

Objective: Decompose fractions as a sum of unit fractions using tape diagrams.

Suggested Lesson Structure

- Fluency Practice (10 minutes)
- Application Problem (6 minutes)
- Concept Development (34 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (10 minutes)

- Read Tape Diagrams 3.OA.3 (4 minutes)
- Break Apart Fractions 4.NF.3 (6 minutes)

Read Tape Diagrams (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity prepares students for today’s lesson.

T: (Project a tape diagram partitioned into 3 equal parts. Write 15 at the top.) Say the value of the whole.

S: 15.

T: Write the value of 1 unit as a division problem.

S: (Write 15 ÷ 3 = 5.)

T: (Write 5 in each unit.) Write the whole as a repeated addition sentence.

S: (Write 5 + 5 + 5 = 15.)

T: (Write 3 fives = 5 + 5 + 5 = 3 × __.) Write the whole as a multiplication equation.

S: (Write 3 × 5 = 15.)

Continue the process for 8 ÷ 2, 20 ÷ 5, 12 ÷ 2, 8 ÷ 4, 21 ÷ 3, and 32 ÷ 4.

Break Apart Fractions (6 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–M5–Lesson 1.
Lesson 2

T: (Project a circle partitioned into 4 equal parts with 3 parts shaded.) How many circles do you see?
S: 1 circle.
T: How many equal parts does the circle have?
S: 4.
T: What fraction of the circle is shaded?
S: 3 fourths.
T: How many fourths are in 3 fourths?
S: 3.
T: (Write $\frac{3}{4} = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$.) On your boards, write $\frac{3}{4}$ as a repeated addition sentence.
S: (Write $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$)
T: (Write $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.) Beneath it, write $\frac{3}{4} = \frac{2}{4} + \underline{\hspace{1cm}}$. Fill in the unknown fraction.
S: (Write $\frac{3}{4} = \frac{2}{4} + \frac{1}{4}$)

Continue the process with the other fraction graphics.

Application Problem (6 minutes)

Mrs. Salcido cut a small birthday cake into 6 equal pieces for 6 children. One child was not hungry, so she gave the birthday boy the extra piece. Draw a tape diagram to show how much cake the five children each got.

Note: This Application Problem is a review of the material presented in G4–M5–Lesson 1 and will prepare students for the more advanced portion of this lesson objective that they will encounter in today’s lesson.

Concept Development (34 minutes)

Materials: (T) 2 strips of paper, markers (S) 2 strips of paper, markers or colored pencils, personal white board
Lesson 2: Decompose fractions as a sum of unit fractions using tape diagrams.

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NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:
If paper strip eighths prove difficult to color and manipulate in Problem 2 of the Concept Development, use concrete manipulatives, such as fraction strips, Cuisenaire rods, or linking cubes. Alternatively, use larger fraction strips or tape diagram drawings.

Problem 1: Use a number bond to show how 1 can be decomposed into fourths and how fourths can be composed to make 1.

T: (Display a number bond to show 1 decomposed into 4 units of 1 fourth.) What does the number bond show?
S: 1 is the whole. The four 1 fourths are the parts. 4 fourths make a whole.
T: Let’s say it as an addition sentence starting with “1 equals....”
S: 1 = \(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\).

T: Fold a strip of paper to represent the same parts that our number bond showed. Work with a partner to see if there are any different number sentences we could create for decomposing 1 into fourths. Draw number bonds, and then write number sentences.
T: What number sentences did you create?
S: 1 = \(\frac{3}{4} + \frac{1}{4}\) → We could write 1 = \(\frac{1}{4} + \frac{1}{4} + \frac{2}{4}\). They both equal 1!

Problem 2: Fold a piece of paper to create eighths. Decompose fractions of the whole in different ways.

T: Turn your strip of paper over. The length of this strip of paper represents 1 whole. Fold this paper to create 8 equal parts. (Demonstrate folding vertically.) Shade 7 of your 8 parts.
T: Give me one number sentence that shows the decomposition of your strip of paper into unit fractions.
S: (Write the sum of 7 units of 1 eighth.)
T: Use parentheses to decompose your sum of unit fractions into two parts.
S: Students write examples such as \(\frac{7}{8} = \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right)\).

T: On your boards, record your decomposition of 7 eighths with 2 parts, and then look for other ways to decompose with 2 or more parts.
S: \(\frac{7}{8} \frac{3}{8} + \frac{4}{8} \rightarrow \frac{7}{8} \frac{7}{8} + 0 \rightarrow \frac{7}{8} \frac{2}{8} + \frac{3}{8} + \frac{0}{8}\)
T: What do all of the number sentences have in common? Discuss with a partner.
S: They all add up to \(\frac{7}{8}\). The parts are eighths in all of them.
Lesson 2: Decompose fractions as a sum of unit fractions using tape diagrams.

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Problem 3: Write number sentences to decompose $\frac{5}{6}$ as a sum of unit fractions.

T: Form groups of three. Work on your personal boards. Each of you should write a number bond and sentence showing a decomposition of 5 sixths. If you have the same decomposition as someone else in your group, one of you must change your work. (Allow time for students to work.)

T: Let’s share. What number bonds did you create? (Record number sentences.)

S: $\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \rightarrow \frac{5}{6} = \frac{3}{6} + \frac{2}{6} \rightarrow \frac{5}{6} = \frac{4}{6} + \frac{1}{6}$

T: Now, on your boards, instead of drawing number bonds, draw tape diagrams to show three different ways of decomposing the fraction $\frac{5}{4}$. Write the number sentence describing each tape diagram you drew next to the tape diagrams. What number sentences did you write?

S: $\frac{5}{4} = \frac{1}{4} + \frac{1}{4} \rightarrow \frac{5}{4} = \frac{2}{4} + \frac{2}{4} \rightarrow \frac{5}{4} = \frac{1}{4} + \frac{1}{4}$

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Lesson 2

Student Debrief (10 minutes)

Lesson Objective: Decompose fractions as a sum of unit fractions using tape diagrams.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Look at your answers for Problems 1(b) and 1(c). Problem 1(c) is a fraction greater than 1, but it has fewer ways to be decomposed. Why is that?
- In Problem 1(a), which was completed for you, the first number sentence was decomposed into the sum of unit fractions. The second number sentence bonded some of these unit fractions. Which ones? (\(\frac{2}{8}\) bonded \(\frac{1}{8}\)). Draw parentheses around the unit fractions in the first number sentence that match the second number sentence. Do the same for Problems 1(b) and 1(c). (Answers will vary.)
- Give examples of when you decomposed numbers in earlier grades.
- How did the Application Problem connect to today’s lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 2 Problem Set 4•5

Name ____________________________________________ Date _____________________

1. Step 1: Draw and shade a tape diagram of the given fraction.
   Step 2: Record the decomposition as a sum of unit fractions.
   Step 3: Record the decomposition of the fraction two more ways.
   (The first one has been done for you.)

   a. \( \frac{5}{8} \)
      
      \[
      \frac{5}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} 
      \]

   b. \( \frac{9}{10} \)

   c. \( \frac{3}{2} \)
2. **Step 1:** Draw and shade a tape diagram of the given fraction.

   **Step 2:** Record the decomposition of the fraction in three different ways using number sentences.

   a. \( \frac{7}{8} \)

   b. \( \frac{5}{3} \)

   c. \( \frac{7}{5} \)

   d. \( 1 \frac{1}{3} \)
Lesson 2 Exit Ticket

Name ____________________________________________________________________ Date ______________

1. Step 1: Draw and shade a tape diagram of the given fraction.
   Step 2: Record the decomposition of the fraction in three different ways using number sentences.

\[
\frac{4}{7}
\]
Lesson 2 Homework

1. Step 1: Draw and shade a tape diagram of the given fraction.
   Step 2: Record the decomposition as a sum of unit fractions.
   Step 3: Record the decomposition of the fraction two more ways.
   (The first one has been done for you.)

   a. \( \frac{5}{6} \)

   \[
   \frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}
   \]

   \[
   \frac{5}{6} = \frac{2}{6} + \frac{2}{6} + \frac{1}{6}
   \]

   \[
   \frac{5}{6} = \frac{1}{6} + \frac{4}{6}
   \]

   b. \( \frac{6}{8} \)

   c. \( \frac{7}{10} \)
2. Step 1: Draw and shade a tape diagram of the given fraction.
   Step 2: Record the decomposition of the fraction in three different ways using number sentences.
   
   a. \( \frac{10}{12} \)

   b. \( \frac{5}{4} \)

   c. \( \frac{6}{5} \)

   d. \( 1 \frac{1}{4} \)
Lesson 3

Objective: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Multiply Mentally 4.OA.4 (4 minutes)
- Repeated Addition as Multiplication 4.OA.4 (4 minutes)
- Add Fractions 4.NF.3 (4 minutes)

Multiply Mentally (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–Module 3 content.

T: (Write 34 × 2 = ____) Say the multiplication sentence.
S: 34 × 2 = 68.
T: (Write 34 × 2 = 68. Below it, write 34 × 20 = ____) Say the multiplication sentence.
S: 34 × 20 = 680.
T: (Write 34 × 20 = 680. Below it, write 34 × 22 = ____) On your boards, solve 34 × 22.
S: (Write 34 × 22 = 748.)

Repeat the process for the following possible sequence: 23 × 3, 23 × 20, 23 × 23, and 12 × 4, 12 × 30, and 12 × 34.

Repeated Addition as Multiplication (4 minutes)

Materials: (S) Personal white boards

NOTES ON MULTIPLE MEANS OF REPRESENTATION:
Scaffold the Multiply Mentally fluency activity for students working below grade level and others. Clarify that (34 × 2) + (34 × 20) is the same as 34 × 22, and so on. Ask, “Why is this true?”
Lesson 3:
Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

Note: This fluency activity reviews G4–Module 3 content.

T: (Write $2 + 2 + 2 = \_\_\_\_\_\_$.) Say the addition sentence.
S: $2 + 2 + 2 = 6$.

T: (Write $2 + 2 + 2 = 6$. Beneath it, write $\_\_\_\_\_\_ \times 2 = 6$.) On your boards, fill in the missing factor.
S: (Write $3 \times 2 = 6$.)

T: (Write $3 \times 2 = 6$. To the right, write $30 + 30 + 30 = \_\_\_\_\_\_$.) Say the addition sentence.
S: $30 + 30 + 30 = 90$.

T: (Write $30 + 30 + 30 = 90$. Beneath it, write $\_\_\_\_\_\_ \times 30 = 90$.) Fill in the missing factor.
S: (Write $3 \times 30 = 90$.)

T: (Write $3 \times 30 = 90$. To the right, write $32 + 32 + 32 = \_\_\_\_\_\_$.) On your boards, write the repeated addition sentence. Then, beneath it, write a multiplication sentence to reflect the addition sentence.
S: (Write $32 + 32 + 32 = 96$. Beneath it, write $3 \times 32 = 96$.)

Continue process for the following possible sequence: $1 + 1 + 1 + 1$, $4 \times 1$, $20 + 20 + 20 + 20$, $4 \times 20$, $21 + 21 + 21 + 21$, $4 \times 21$, $23 + 23 + 23$, and $3 \times 23$.

Add Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–M5–Lesson 2.

T: (Write $\frac{4}{5} = \_\_\_\_\_\_$.) Say the fraction.
S: $\frac{4}{5}$.

T: On your boards, draw a tape diagram representing 4 fifths.
S: (Draw a tape diagram partitioned into 5 equal units. Shade 4 units.)

T: (Write $\frac{4}{5} = \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_$.) Write $\frac{4}{5}$ as the sum of unit fractions.
S: (Write $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$)

T: (Write $\frac{4}{5} = \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_$.) Bracket 2 fifths on your diagram, and complete this number sentence.
S: (Group 2 fifths on diagram. Write $\frac{4}{5} = \frac{2}{5} + \frac{2}{5}$ or $\frac{4}{5} = \frac{1}{5} + \frac{3}{5}$)

T: (Write $\frac{4}{5} = \_\_\_\_\_\_ + \_\_\_\_\_\_$.) Bracket fifths again on your diagram, and write a number sentence to match. There’s more than one correct answer.
S: (Group fifths on diagram. Write $\frac{4}{5} = \frac{2}{5} + \frac{2}{5} + \frac{1}{5}$ or $\frac{4}{5} = \frac{1}{5} + \frac{3}{5}$)
Lesson 3: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

Note: This Application Problem builds from Grade 3 knowledge of interpreting products of whole numbers. This Application Problem bridges to today’s lesson where students will come to understand that a non-unit fraction can be decomposed and represented as a whole number times a unit fraction.

Concept Development (30 minutes)

Materials: (S) Personal white boards

Problem 1: Express a non-unit fraction less than 1 as a whole number times a unit fraction using a tape diagram.

T: Look back at the tape diagram that we drew in the Application Problem. What fraction is represented by the shaded part?

S: \( \frac{3}{4} \).

T: Say \( \frac{3}{4} \) decomposed as the sum of unit fractions.

S: \( \frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \).

T: How many fourths are there in \( \frac{3}{4} \)?

S: 3.
Lesson 3: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

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T: We know this because we count 1 fourth 3 times. Discuss with a partner. How might we express this using multiplication?

S: We have 3 fourths. That’s $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ or three groups of one-fourth. Could we multiply $3 \times \frac{1}{4}$?

T: Yes! If we want to add the same fraction of a certain amount many times, instead of adding, we can multiply. Just like you multiplied 6 copies 4 times, we can multiply 1 fourth 3 times. What is 3 copies of $\frac{1}{4}$?

S: It’s $\frac{3}{4}$. My tape diagram proves it!

Repeat with $\frac{2}{4}$ and $\frac{7}{8}$. Direct students to draw a tape diagram to represent each fraction (as above), to shade the given number of parts, and then to write an addition number sentence and a multiplication number sentence.

Problem 2: Determine the non-unit fraction greater than 1 that is represented by a tape diagram, and then write the fraction as a whole number times a unit fraction.

T: (Project tape diagram of $\frac{10}{8}$ as shown below.) What fractional part does the tape diagram show?

S: It shows tenths! → It shows eighths!

T: We first must identify 1 whole. It’s bracketed here. (Point.) How many parts is our whole partitioned into?

S: 8!

T: The bracketed portion of the tape diagram shows 8 fractional parts. What is the total number of eighths?

S: 10.

T: What is the fraction?

S: 10 eighths.

T: Say this as an addition number sentence. Use your fingers to keep track of the number of units as you say them.

S: $\frac{10}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

T: As a multiplication number sentence?

S: $\frac{10}{8} = 10 \times \frac{1}{8}$

T: What are the advantages of multiplying fractions instead of adding?

S: It’s easier to write. → It’s faster. → It’s more efficient.

Problem 3: Express a non-unit fraction greater than 1 as a whole number times a unit fraction using a tape diagram.

T: Let’s put parentheses around 8 eighths so that we can see 10 eighths can also be written to show 1 whole and 2 more eighths. (Write $\frac{10}{8} = (8 \times \frac{1}{8}) + (2 \times \frac{1}{8})$.)
Lesson 3: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

Date: 1/7/14

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:
Offer an alternative to Problem 2 on the Problem Set for students working above grade level. Challenge students to compose a word problem of their own to match one or more of the tape diagrams they construct for Problem 2. Always offer challenges and extensions to learners as alternatives, rather than additional “busy” work.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.
Lesson 3

Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

You may choose to use any combination of the questions below to lead the discussion.

- In all the problems, why do we need to label the whole, as 1, on our tape diagrams? What would happen if we did not label the whole?
- What is an advantage to representing the fractions using multiplication?
- What is similar in Problems 3(c), 3(d), and 3(e)? Which fractions are greater than 1? Which is less than 1?
- Are you surprised to see multiplication sentences with products less than 1? Why?
- In our lesson when we expressed $\frac{5}{5}$ as $\left(3 \times \frac{1}{5}\right) + \left(2 \times \frac{1}{5}\right)$, what property were we using?
- Consider the work we did in G4–M5–Lessons 1 and 2 where we decomposed a tape diagram multiple ways. Can we now rewrite those number sentences using addition and multiplication? Try it with this tape diagram (as seen below).

```
1

\[
\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = \left(3 \times \frac{1}{5}\right) + \left(2 \times \frac{1}{5}\right)
\]
```

- How is multiplying fractions like multiplying whole numbers?
- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. Decompose each fraction modeled by a tape diagram as a sum of unit fractions. Write the equivalent multiplication sentence. The first one has been done for you.

a. \[
\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \quad \frac{3}{4} = 3 \times \frac{1}{4}
\]

b. 

\[
\text{Diagram}
\]

c. 

\[
\text{Diagram}
\]

d. 

\[
\text{Diagram}
\]

e. 

\[
\text{Diagram}
\]
2. Write the following fractions greater than 1 as the sum of two products.
   
   a. \[ \frac{1}{\phantom{2}} \]
   
   b. \[ \frac{1}{\phantom{2}} \]

3. Draw a tape diagram and record the given fraction’s decomposition into unit fractions as a multiplication sentence.
   
   a. \[ \frac{4}{5} \]
   
   b. \[ \frac{5}{8} \]
   
   c. \[ \frac{7}{9} \]
   
   d. \[ \frac{7}{4} \]
   
   e. \[ \frac{7}{6} \]
Name ________________________________ Date ________________

1. Decompose each fraction modeled by a tape diagram as a sum of unit fractions. Write the equivalent multiplication sentence.
   a. 
   \[
   \begin{array}{c}
   \hline
   1 \\
   \hline
   \end{array}
   \]
   \[
   \begin{array}{cccc}
   & & & \\
   \hline
   & & & \\
   \hline
   \end{array}
   \]
   
   b. 
   \[
   \begin{array}{c}
   1 \\
   \hline
   \end{array}
   \]
   \[
   \begin{array}{cccccccc}
   & & & & & & & \\
   \hline
   & & & & & & & \\
   \hline
   \end{array}
   \]

2. Draw a tape diagram and record the given fraction's decomposition into unit fractions as a multiplication sentence.
   a. \(\frac{6}{9}\)
1. Decompose each fraction modeled by a tape diagram as a sum of unit fractions. Write the equivalent multiplication sentence. The first one has been done for you.

   a. \[ \frac{2}{3} = \frac{1}{3} + \frac{1}{3} \quad \frac{2}{3} = 2 \times \frac{1}{3} \]

   b. 

   c. 

   d. 

Name ____________________________ Date ________________

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2. Write the following fractions greater than 1 as the sum of two products.

a. 

\[
\frac{1}{3} \quad \frac{2}{3}
\]

b. 

\[
\frac{1}{4} \quad \frac{3}{4}
\]

3. Draw a tape diagram and record the given fraction’s decomposition into unit fractions as a multiplication sentence.

a. \(\frac{3}{5}\)

b. \(\frac{3}{8}\)

c. \(\frac{5}{9}\)

d. \(\frac{8}{5}\)

e. \(\frac{12}{4}\)
Lesson 4

Objective: Decompose fractions into sums of smaller unit fractions using tape diagrams.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (4 minutes)
- Concept Development (34 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Break Apart Fractions 4.NF.3 (7 minutes)
- Count by Equivalent Fractions 3.NF.3 (5 minutes)

Break Apart Fractions (7 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–M5–Lesson 3.

T: (Project a tape diagram partitioned into 3 equal units. Write 1 above it. Shade 2 units.) How many equal parts does this 1 whole have?

S: 3 parts.

T: Say the value of 1 unit.

S: 1 third.

T: What fraction of 1 whole is shaded?

S: 2 thirds.

T: On your boards, write the value of the shaded part as a sum of unit fractions.

S: \( \frac{2}{3} = \frac{1}{3} + \frac{1}{3} \)

T: (Write \( \_ \times \frac{1}{3} = \frac{2}{3} \)) On your boards, complete the number sentence.

S: (Write \( 2 \times \frac{1}{3} = \frac{2}{3} \))

Continue the process for the following possible sequence: \( \frac{3}{5}, \frac{5}{8}, \) and \( \frac{5}{4} \).
Lesson 4

Decompose fractions into sums of smaller unit fractions using tape diagrams.

Date: 1/15/14

5.A.38

Count by Equivalent Fractions (5 minutes)

Materials: (S) Personal white boards

Note: This fluency activity prepares students for G4–M5–Lesson 4.

T: Count by ones to 6.
S: 1, 2, 3, 4, 5, 6.

T: Count by sixths to 6 sixths. Start at 0 sixths. (Write as students count.)
S: 0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$

T: 6 sixths is the same as one of what unit?
S: 1 whole.

T: (Beneath $\frac{6}{6}$, write 1.) Count by sixths again. This time, say “1 whole” when you arrive at 6 sixths. Start at zero.
S: 0, $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, 1 whole.

T: Let’s count by thirds to 6 thirds. Start at 0 thirds. (Write as students count.)
S: 0 $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$

T: How many thirds are in 1?
S: 3 thirds.

T: (Beneath $\frac{3}{3}$, write 1.) How many thirds are in 2?
S: 6 thirds.

T: (Beneath $\frac{6}{3}$, write 2.) Let’s count by thirds again. This time, when you arrive at 3 thirds and 6 thirds,
Lesson 4: Decompose fractions into sums of smaller unit fractions using tape diagrams.

Date: 1/15/14

5.A.39

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:
Cuisenaire rods can be used to model 1 whole (brown), 2 halves (pink), 4 fourths (red), and 8 eighths (white). If concrete Cuisenaire rods are unavailable or otherwise challenging, virtual rods can be found at the link below:
http://nrich.maths.org/4348.

Application Problem (4 minutes)
A recipe calls for \(\frac{3}{4}\) cup of milk. Saisha only has a \(\frac{1}{4}\) cup measuring cup. If she doubles the recipe, how many times will she need to fill the \(\frac{1}{4}\) cup with milk? Draw a tape diagram and record as a multiplication sentence.

Note: This Application Problem reviews students’ knowledge of fractions from G4–M5–Lesson 3 and prepares them for today’s objective of decomposing unit fractions into sums of smaller unit fractions.

Concept Development (34 minutes)

Materials: (S) Personal white boards

Problem 1: Use tape diagrams to represent the decomposition of \(\frac{1}{3}\) as the sum of unit fractions.

T: Draw a tape diagram that represents 1 whole, and shade 1 third. Decompose each of the thirds in half. How many parts are there now?
S: 6.
T: What fraction of the whole does each part represent?
S: 1 sixth.
T: How many sixths are shaded?
S: 2 sixths.
T: What can we say about 1 third and 2 sixths?
S: They are the same.
T: How can you tell?
S: They both take up the same amount of space.
T: Let’s write that as a number sentence: \(\frac{1}{3} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}\).
T: Now, decompose each sixth into 2 equal parts on your tape diagram. How many parts are in the whole now?

say the whole number. Start at zero.
S: 0, \(\frac{1}{3}\), \(\frac{2}{3}\), \(\frac{1}{2}\), \(\frac{4}{3}\), \(\frac{5}{3}\), 2.

Repeat the process, counting by halves to 6 halves.
Lesson 4: Decompose fractions into sums of smaller unit fractions using tape diagrams.

MP.2

Problem 1:

T: What fractional part of the whole does each piece represent?
S: 1 twelfth.

T: How many twelfths equal 1\(\frac{1}{6}\)?
S: \(\frac{2}{12}\) equals \(\frac{1}{6}\).

T: Work with your partner to write a number sentence for how many twelfths equal \(\frac{1}{3}\).
S: \(\frac{1}{3} = \frac{1}{6} + \frac{1}{6} = (\frac{1}{12} + \frac{1}{12}) + (\frac{1}{12} + \frac{1}{12})\).

T: We can put parentheses around two groups of 1 twelfth to show that each combines to make \(\frac{1}{6}\).

\[\frac{1}{3} = (\frac{1}{12} + \frac{1}{12}) + (\frac{1}{12} + \frac{1}{12})\]

T: How can we represent this using multiplication?
S: \(\frac{1}{3} = (2 \times \frac{1}{12}) + (2 \times \frac{1}{12}) \Rightarrow \frac{1}{3} = 4 \times \frac{1}{12}\).

Problem 2: Use tape diagrams to represent the decomposition of \(\frac{1}{5}\) and \(\frac{2}{5}\) as the sum of smaller unit fractions.

T: Draw a tape diagram and shade \(\frac{1}{5}\). Decompose each of the fifths into 3 equal parts. How many parts are there now?
S: There are 15 parts.

T: What fraction does each part represent?
S: \(\frac{1}{15}\).

T: Write an addition sentence to show how many fifteenths it takes to make 1 fifth.
S: \(\frac{1}{5} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15}\)

T: What can we say about one-fifth and three-fifteenths?
S: They are equal.

T: With your partner, write a number sentence that represents \(\frac{2}{5}\).
S: \(\frac{2}{5} = \frac{1}{15} + \frac{3}{15} = \frac{6}{15} \Rightarrow \frac{2}{5} = (3 \times \frac{1}{15}) + (3 \times \frac{1}{15}) = \frac{6}{15} \Rightarrow \frac{2}{5} = 2 \times \frac{1}{5} = 2 \times \frac{3}{15} = \frac{6}{15}\).

Problem 3: Draw a tape diagram and use addition to show that \(\frac{2}{6}\) is the sum of 4 twelfths.

T: (Project \(\frac{2}{6} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}\)) Using what you just learned, how can you model to show that \(\frac{2}{6}\) is equal to \(\frac{4}{12}\)?
S: We can draw a tape diagram and shade \(\frac{2}{6}\). Then, we can decompose it into twelfths.
Lesson 4

Decompose fractions into sums of smaller unit fractions using tape diagrams.

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5.A.41

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T: How many twelfths are shaded?
S: 4.
T: We have seen that 1 third is equal to 2 sixths. We have seen 1 sixth is equal to 2 twelfths. So, how many twelfths equal 1 third?
S: 4 twelfths!
T: So, 2 thirds is how many twelfths? Explain to your partner how you know using your diagrams.
S: 1 third is 4 twelfths, so 2 thirds is 8 twelfths. → It's just double. → It's twice the area on the tape diagram. → It's the same as 4 sixths. 1 third is 2 sixths. 2 thirds is 4 sixths. 1 sixth is the same as 2 twelfths, so 4 times 2 is 8. 8 twelfths.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Decompose fractions into sums of smaller unit fractions using tape diagrams.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- For Problems 1(a–d), what were some different ways that you decomposed the unit fraction?
- What is different about Problems 3(c) and 3(d)? Explain how fourths can be decomposed into both eighths and twelfths.
- For Problems 4, 5, and 6, explain the process you used to show equivalent fractions.
- Without using a tape diagram, what strategy would you use for decomposing a unit fraction?
Lesson 4

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Lesson 4: Decompose fractions into sums of smaller unit fractions using tape diagrams.

Date: 1/15/14

How did the Application Problem connect to today’s lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Name ________________________________ Date _______________

1. The total length of each tape diagram represents 1 whole. Decompose the shaded unit fractions as the sum of smaller unit fractions in at least two different ways. The first one has been done for you.

   a. \[ \frac{1}{2} = \frac{1}{4} + \frac{1}{4} \]

   b. \[ \frac{1}{3} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \]

   c. 

   d. 

2. The total length of each tape diagram represents 1 whole. Decompose the shaded fractions as the sum of smaller unit fractions in at least two different ways.

   a. 

   b. 

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Lesson 4: Decompose fractions into sums of smaller unit fractions using tape diagrams.

3. Draw and label tape diagrams to prove the following statements. The first one has been done for you.

   a. \[ \frac{2}{5} = \frac{4}{10} \]

   b. \[ \frac{2}{6} = \frac{4}{12} \]

   c. \[ \frac{3}{4} = \frac{6}{8} \]

   d. \[ \frac{3}{4} = \frac{9}{12} \]

4. Show that \( \frac{1}{2} \) is equivalent to \( \frac{4}{8} \) using a tape diagram and a number sentence.

5. Show that \( \frac{2}{3} \) is equivalent to \( \frac{6}{9} \) using a tape diagram and a number sentence.

6. Show that \( \frac{4}{6} \) is equivalent to \( \frac{8}{12} \) using a tape diagram and a number sentence.
1. The total length of the tape diagram represents 1 whole. Decompose the shaded unit fraction as the sum of smaller unit fractions in at least two different ways.

2. Draw a tape diagram to prove the following statement.

\[
\frac{2}{3} = \frac{4}{6}
\]
1. The total length of each tape diagram represents 1 whole. Decompose the shaded unit fractions as the sum of smaller unit fractions in at least two different ways. The first one has been done for you.

   a. 
   \[
   \frac{1}{2} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}
   \]

   b. 
   \[
   \frac{1}{4} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}
   \]

2. The total length of each tape diagram represents 1 whole. Decompose the shaded fractions as the sum of smaller unit fractions in at least two different ways.

   a. 

   b. 

   c. 

   \[
   \frac{2}{4} = \frac{1}{2} + \frac{1}{2}
   \]
Lesson 4 Homework

3. Draw tape diagrams to prove the following statements. The first one has been done for you.

a. \( \frac{2}{5} = \frac{4}{10} \)

b. \( \frac{3}{6} = \frac{6}{12} \)

c. \( \frac{2}{6} = \frac{6}{18} \)

d. \( \frac{3}{4} = \frac{12}{16} \)

4. Show that \( \frac{1}{2} \) is equivalent to \( \frac{6}{12} \) using a tape diagram and a number sentence.

5. Show that \( \frac{2}{3} \) is equivalent to \( \frac{8}{12} \) using a tape diagram and a number sentence.

6. Show that \( \frac{4}{5} \) is equivalent to \( \frac{12}{15} \) using a tape diagram and a number sentence.
Lesson 5

Objective: Decompose unit fractions using area models to show equivalence.

Suggested Lesson Structure

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluency Practice</td>
<td>(12 min)</td>
</tr>
<tr>
<td>Application Problem</td>
<td>(8 min)</td>
</tr>
<tr>
<td>Concept Development</td>
<td>(30 min)</td>
</tr>
<tr>
<td>Student Debrief</td>
<td>(10 min)</td>
</tr>
<tr>
<td>Total Time</td>
<td>(60 min)</td>
</tr>
</tbody>
</table>

Fluency Practice (12 minutes)

- Count by Equivalent Fractions 3.NF.3 (4 minutes)
- Add Fractions 4.NF.3 (4 minutes)
- Break Apart the Unit Fraction 4.NF.3 (4 minutes)

Count by Equivalent Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–M5–Lesson 4.

T: Count from 0 fourths to 4 fourths by 1 fourths. (Write as students count.)
S: 0, 1, 2, 3, 4
T: 4 fourths is the same as one of what unit?
S: 1 whole.
T: (Beneath $\frac{4}{4}$, write 1.) Count by fourths again. This time, say “1 whole” when you arrive at 4 fourths. Start at zero.
S: 0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, 1 whole.

T: Let’s count by halves to 4 halves. (Write as students count.)
S: 0, $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$
T: How many halves are equal to 1?
Lesson 5: Decompose unit fractions using area models to show equivalence.

Date: 1/7/14

Add Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–M5–Lesson 2.

T: (Write \( \frac{4}{5} = \_ \)) Say the fraction.
S: \( \frac{4}{5} \).

T: On your boards, draw a tape diagram representing \( \frac{4}{5} \) fifths.
S: (Draw a tape diagram partitioned into 5 equal units. Shade 4 units.)

T: (Write \( \frac{4}{5} = \_ + \_ + \_ + \_ \)) On your boards, fill in the unknown fractions.
S: (Write \( \frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \))

T: (Write \( \frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \). Beneath it, write \( \frac{4}{5} = \_ \times \frac{1}{5} \)) Fill in the unknown factor.
S: (Write \( \frac{4}{5} = 4 \times \frac{1}{5} \))

Continue the process with \( \frac{5}{8} \) and \( \frac{3}{7} \).

Break Apart the Unit Fraction (4 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–M5–Lesson 4.

T: (Project a tape diagram partitioned into 2 equal units. Shade 1 unit.) Name the fraction of the diagram that is shaded.
S: 1 half.

T: (Write \( \frac{1}{2} \) above the shaded unit.) Decompose the shaded unit into 3 equal units.

T: What fraction of the tape diagram is each smaller unit?
S: 1 sixth.

T: (Write \( \frac{1}{2} = \_ + \_ + \_ \)) On your boards, complete the number sentence.
Lesson 5

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Drawing an area model representing fifths or other odd numbers may be challenging for some students. Slip grid paper into personal white boards to assist them, if beneficial. Students who find it easier may continue using folded paper strips to model fractions.

S: (Write $\frac{1}{2} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$)

Repeat the process with $\frac{1}{3}$.

T: (Write $\frac{2}{3}$) On your boards, draw and shade a tape diagram to show $\frac{2}{3}$.

T: Decompose each third into 3 equal parts on your model and with an addition sentence. (Pause.) Each third is the same as 3 of what unit?

S: 3 ninths.

T: (Write $\frac{2}{3} = \ldots$) 2 thirds is the same as how many ninths? Write the answer on your boards.

S: (Write $\frac{2}{3} = \frac{6}{9}$)

Continue the process with $\frac{1}{2} = \frac{4}{8}, \frac{3}{8}, \frac{3}{4} = \frac{9}{12}$, and $\frac{5}{6} = \frac{10}{12}$.

Application Problem (8 minutes)

A loaf of bread was cut into 6 equal slices. Each of the 6 slices was cut in half to make thinner slices for sandwiches.

Mr. Beach used 4 slices. His daughter said, “Wow, you used $\frac{2}{6}$ of the loaf!” His son said, “No, he used $\frac{4}{12}$.”

Work with a partner to explain who was correct using a tape diagram.

Note: This Application Problem builds on G4–M5–Lesson 4’s objective of decomposing a fraction as the sum of smaller fractions, which bridges to today’s lesson where students will use the area model as another way to show both the decomposition and equivalence.
Concept Development  (30 minutes)

Materials:  (S) Personal white boards

Problem 1: Draw an area model to illustrate that $\frac{1}{5}$ is equal to $\frac{2}{10}$.

T:  Draw an area model that is partitioned into 5 equal parts. Shade 1 of them. If the entire figure represents the whole, what fractional part is shaded?
S:  1 fifth.

T:  Draw a horizontal dotted line to decompose the whole into two equal rows. (Demonstrate.) What happened? Discuss with your partner.
S:  There were 5 pieces, but now there are 10. → We had fifths, but now we have tenths. → We doubled the number of original units (fifths) to make a new unit (tenths). → We cut each fifth into 2 equal pieces to make tenths. → There are more parts, but they are smaller, so 2 times 1 tenth is the same as 1 fifth.
T:  How many tenths are shaded?
S:  2 tenths.

T:  Even though the parts changed, did the area covered by the shaded region change?
S:  No.

T:  What relationship does this show between $\frac{1}{5}$ and $\frac{2}{10}$? Say your answer as an addition sentence.
S:  $\frac{1}{5} = \left(\frac{1}{10} + \frac{1}{10}\right) = \frac{2}{10}$. 1 fifth equals 2 tenths.

Problem 2: Decompose $\frac{1}{3}$ as $\frac{4}{12}$ represented in an area model and as the sum and product of unit fractions.

T:  Draw an area model that is partitioned into 3 equal parts. Shade 1 of them. If the entire figure represents the whole, what fraction is shaded?
S:  1 third.

T:  Discuss with your partner how to draw horizontal dotted lines to decompose 1 third to demonstrate that $\frac{1}{3} = \frac{4}{12}$.
S:  We can draw a horizontal line. → One line won’t be enough. That will make sixths. Two lines will make ninths. Three lines!
T:  How many parts do we have now?
S:  12.

T:  How many twelfths are shaded?
S:  $\frac{4}{12}$. 

$\frac{1}{3} = \left(\frac{1}{12} + \frac{1}{12} + \frac{1}{12}\right) = 4 \times \frac{1}{12} = \frac{4}{12}$
Lesson 5: Decompose unit fractions using area models to show equivalence.

Date: 1/7/14

T: Represent the decomposition of $\frac{1}{3}$ as the sum of unit fractions.

S: $\frac{1}{3} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12}$

T: Now, like in the last lesson, represent this decomposition of $\frac{1}{3}$ using a multiplication sentence.

S: $\frac{1}{3} = (2 \times \frac{1}{12}) + (2 \times \frac{1}{12}) = \frac{4}{12}$

Problem 3: Model $\frac{1}{2} = \frac{5}{10}$ and represent the decomposition as the sum and product of unit fractions.

T: (Display $\frac{5}{10}$.) Discuss with your partner how to represent this equivalence using an area model.

S: We can partition an area model in half. We can draw lines across so that they make equal parts. $\rightarrow$ We need 10 parts. Since there are 2 halves, that would be 5 on each side.

T: Work with your partner to draw the model, and write a number sentence to represent the decomposition.

S: $\frac{1}{2} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} \rightarrow \frac{1}{2} = 5 \times \frac{1}{10} = \frac{5}{10}$

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Decompose unit fractions using area models to show equivalence.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1, why do you think the directions tell...
Lesson 5:
Decompose unit fractions using area models to show equivalence.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. Draw horizontal lines to decompose each rectangle into the number of rows as indicated. Use the model to give the shaded area as both a sum of unit fractions and as a multiplication sentence.

a. 2 rows

\[
\frac{1}{4} = \frac{2}{8}
\]

\[
\frac{1}{4} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8}
\]

\[
\frac{1}{4} = 2 \times \frac{1}{8} = \frac{2}{8}
\]

b. 2 rows

c. 4 rows
2. Draw area models to show the decompositions represented by the number sentences below. Represent the decomposition as a sum of unit fractions and as a multiplication sentence.

a. \( \frac{1}{2} = \frac{3}{6} \)  
b. \( \frac{1}{2} = \frac{4}{8} \)

c. \( \frac{1}{2} = \frac{5}{10} \)  
d. \( \frac{1}{3} = \frac{2}{6} \)

e. \( \frac{1}{3} = \frac{4}{12} \)  
f. \( \frac{1}{4} = \frac{3}{12} \)

3. Explain why \( \frac{1}{12} + \frac{1}{12} + \frac{1}{12} \) is the same as \( \frac{1}{4} \).
1. Draw horizontal lines to decompose each rectangle into the number of rows as indicated. Use the model to give the shaded area as both a sum of unit fractions and as a multiplication sentence.

   a. 2 rows

   

   b. 3 rows

   

2. Draw an area model to show the decomposition represented by the number sentence below. Represent the decomposition as a sum of unit fractions and as a multiplication sentence.

   \[
   \frac{3}{5} = \frac{6}{10}
   \]
1. Draw horizontal lines to decompose each rectangle into the number of rows as indicated. Use the model to give the shaded area as both a sum of unit fractions and as a multiplication sentence.

   a. 3 rows
   \[
   \frac{1}{2} = \frac{3}{6} \quad \frac{1}{2} = \frac{1}{6} + \frac{1}{6} = \frac{3}{6} \\
   \frac{1}{2} = 3 \times \frac{1}{6} = \frac{3}{6}
   \]

   b. 2 rows

   c. 4 rows
2. Draw area models to show the decompositions represented by the number sentences below. Represent the decomposition as a sum of unit fractions and as a multiplication sentence.

   a. \( \frac{1}{3} = \frac{2}{6} \)  
   b. \( \frac{1}{3} = \frac{3}{9} \)

   c. \( \frac{1}{3} = \frac{4}{12} \)  
   d. \( \frac{1}{3} = \frac{5}{15} \)

   e. \( \frac{1}{5} = \frac{2}{10} \)  
   f. \( \frac{1}{5} = \frac{3}{15} \)

3. Explain why \( \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} \) is the same as \( \frac{1}{3} \).
Lesson 6
Objective: Decompose fractions using area models to show equivalence.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Sprint: Multiply Whole Numbers Times Fractions 4.NF.4 (9 minutes)
- Find Equivalent Fractions 4.NF.1 (3 minutes)

Sprint: Multiply Whole Numbers Times Fractions (9 minutes)

Materials: (S) Multiply Whole Numbers Times Fractions Sprint

Note: This fluency activity reviews G4–M5–Lesson 3.

Find Equivalent Fractions (3 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews G4–M5–Lesson 5.

T: (Write $\frac{1}{3}$) Say the fraction.
S: $\frac{1}{3}$.

T: On your boards, draw a model to show $\frac{1}{3}$.
S: (Draw a model partitioned into 3 equal units. Shade 1 unit.)

T: (Write $\frac{1}{3} = \frac{2}{6}$) Draw a dotted horizontal line to decompose 1 third into an equivalent fraction.
S: (Draw a dotted horizontal line, breaking 3 units into 6 smaller units. Write $\frac{1}{3} = \frac{2}{6}$)

Continue the process for the following possible sequence: $\frac{1}{3} = \frac{3}{9}$, $\frac{1}{2} = \frac{2}{4}$, $\frac{1}{2} = \frac{4}{8}$, $\frac{1}{4} = \frac{2}{8}$, and $\frac{1}{5} = \frac{3}{15}$.
Application Problem (8 minutes)

Use area models to prove that \( \frac{1}{2} = \frac{2}{4} = \frac{4}{8} \), \( \frac{1}{2} = \frac{3}{6} = \frac{6}{12} \), and \( \frac{1}{2} = \frac{5}{10} \). What conclusion can you make about \( \frac{4}{8} = \frac{6}{12} \) and \( \frac{5}{10} \)? Explain.

Note: This Application Problem builds from G4–M5–Lesson 5 where students decomposed unit fractions using area models to show equivalence. Consider leading a discussion with a question such as, “Why can you show \( \frac{1}{2} = \frac{2}{4} = \frac{4}{8} \) on one model, \( \frac{1}{2} = \frac{3}{6} = \frac{6}{12} \) on another, and \( \frac{1}{2} = \frac{5}{10} \) on another?” Or perhaps, “Why can’t you show \( \frac{1}{2} = \frac{2}{4} = \frac{5}{10} \) on the same area model?”

Concept Development (30 minutes)

Materials: (S) Personal white boards

Problem 1: Use an area model to show that \( \frac{3}{4} = \frac{6}{8} \).

T: Draw an area model representing 1 whole, and then shade \( \frac{3}{4} \).

T: Discuss with a partner how you can use this model to show the decomposition of 3 fourths into eighths.

S: We could draw a line so that each of the fourths is split into 2 equal parts. That would give us eighths. \( \Rightarrow \) Drawing a line will make each unit into 2 smaller units, which would be eighths.

T: How many eighths are shaded?

S: 6 eighths.

T: Work with a partner to write an addition and multiplication sentence to describe the decomposition.

S: \( \frac{3}{4} = \left( \frac{1}{8} + \frac{1}{8} \right) + \left( \frac{1}{8} + \frac{1}{8} \right) = \frac{6}{8} \). \( \Rightarrow \) \( \frac{3}{4} = 3 \times \frac{2}{8} = 6 \times \frac{1}{8} = \frac{6}{8} \). \( \Rightarrow \) \( \frac{3}{4} \) is equal to \( \frac{6}{8} \).

T: What do these addition and multiplication sentences tell you?

S: The shaded area didn’t change. It’s still the same amount. The number of pieces increased, but the size of the pieces got smaller. \( \Rightarrow \) Adding together all the smaller units equals the total of the larger units shaded. \( \Rightarrow \) Multiplying also equals the total of the larger units shaded and is easier to write out!
Problem 2: Draw an area model to represent the equivalence of two fractions and express the equivalence as the sum and product of unit fractions.

T: Let’s draw an area model to show that $\frac{2}{3} = \frac{8}{12}$. What fraction will you model first and why? Discuss with a partner.

S: I will represent $\frac{2}{3}$ first, since thirds are the bigger pieces. I can draw 1 whole divided into thirds and then shade 2 of them. Then, it’s easy to split the thirds into parts to make twelfths. → We have to draw the larger units first and then decompose them into smaller ones, don’t we?

T: Draw an area model representing 2 thirds.

T: How can we show that $\frac{2}{3} = \frac{8}{12}$? Discuss.

S: We can split the thirds into parts until we have 12 of them. → Yes, but we need to make sure that they are equal parts. → We might have to erase our lines and then redraw to make them look equal. → We can draw three lines across the thirds. This will make 12 groups. → When I do that, the eight pieces are already shaded!

T: Express the equivalence as a multiplication sentence.

S: $\frac{2}{3} = \left(8 \times \frac{1}{12}\right) = \frac{8}{12}$ → $\frac{2}{3} = \left(4 \times \frac{1}{12}\right) + \left(4 \times \frac{1}{12}\right) = \frac{8}{12}$ → $\frac{2}{3} = \frac{4}{12} + \frac{4}{12}$

Problem 3: Decompose to create equivalent fractions by drawing an area model and then dividing the area model into smaller parts.

T: Let’s use what we know to model equivalent fractions.

Step 1: Draw an area model. The entire figure is 1 whole.

Step 2: Choose a fraction, and partition the whole using vertical lines.

Step 3: Shade your fraction.

Step 4: Switch papers with a partner. Write down the fraction that your partner has represented.

Step 5: Draw one to three horizontal lines. What equivalent fraction have you modeled?

T: How could we model 5 thirds?

S: We can draw an area model and partition it into 5 parts. Each part is 1 third. We have to label 1 whole after 3 units.

T: Draw one horizontal line to model an equivalent fraction. How many units are in the whole?

S: 6.

T: What fraction is represented?

S: $\frac{10}{6}$. 
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Decompose fractions using area models to show equivalence.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Look at Problem 1(c) and Problem 2(b). Compare the two. How can $\frac{3}{4}$ be equivalent to both fractions?
- Why do we use parentheses? What does it help to show?
- In Problem 2 of the Concept Development, could you represent $\frac{8}{12}$ first and then show the equivalence to $\frac{2}{3}$? How would you show it?
- How can two different fractions represent the same portion of a whole?
- How did the Application Problem connect to today’s lesson?
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
**Lesson 6: Decompose fractions using area models to show equivalence.**

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<th>Solve.</th>
<th># Correct _______</th>
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<tbody>
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<tr>
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### Lesson 6 Sprint 4.5

**B** Solve.  

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<td>1</td>
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<td>( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = )</td>
<td>39</td>
</tr>
<tr>
<td>18</td>
<td>( 4 \times \frac{1}{4} = )</td>
<td>40</td>
</tr>
<tr>
<td>19</td>
<td>( \frac{1}{2} + \frac{1}{2} = )</td>
<td>41</td>
</tr>
<tr>
<td>20</td>
<td>( 2 \times \frac{1}{2} = )</td>
<td>42</td>
</tr>
<tr>
<td>21</td>
<td>( \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = )</td>
<td>43</td>
</tr>
<tr>
<td>22</td>
<td>( 4 \times \frac{1}{3} = )</td>
<td>44</td>
</tr>
</tbody>
</table>
1. Each rectangle represents 1 whole. Draw horizontal lines to decompose each rectangle into the number of units as indicated. Use the model to give the shaded area as a sum and as a product of unit fractions. Use parentheses to show the relationship between the number sentences. The first one has been partially done for you.

a. Sixths

\[
\begin{align*}
\frac{2}{3} & = \frac{4}{6} \\
\frac{1}{6} + \frac{1}{6} + \frac{1}{6} & = \left(\frac{1}{6} + \frac{1}{6}\right) + \frac{1}{6} = \frac{4}{6} \\
\left(\frac{1}{6} + \frac{1}{6}\right) + \frac{1}{6} & = \left(2 \times \frac{1}{6}\right) + \frac{1}{6} = \frac{4}{6} \\
\frac{2}{3} & = 4 \times \frac{1}{6} = \frac{4}{6}
\end{align*}
\]

b. Tenths

c. Twelfths
2. Draw area models to show the decompositions represented by the number sentences below. Express each as a sum and product of unit fractions. Use parentheses to show the relationship between the number sentences.

   a. \[ \frac{3}{5} = \frac{6}{10} \]

   b. \[ \frac{3}{4} = \frac{6}{8} \]

3. Step 1: Draw an area model for a fraction with the denominator of 3, 4, or 5.
   Step 2: Shade in more than one fractional unit.
   Step 3: Partition the area model again to find an equivalent fraction.
   Step 4: Write the equivalent fractions as a number sentence. (If you’ve written a number sentence already on this Problem Set, start over.)
Lesson 6 Exit Ticket

Name ____________________________________________ Date ______________________

1. The rectangle below represents 1 whole. Draw horizontal lines to decompose the rectangle into eighths. Use the model to give the shaded area as a sum and as a product of unit fractions. Use parentheses to show the relationship between the number sentences.

2. Draw an area model to show the decomposition represented by the number sentence below.

\[
\frac{4}{5} = \frac{8}{10}
\]
1. Each rectangle represents 1 whole. Draw horizontal lines to decompose each rectangle into the number of units as indicated. Use the model to give the shaded area as a sum and as a product of unit fractions. Use parentheses to show the relationship between the number sentences. The first one has been partially done for you.

   a. Tenths

   \[ \frac{2}{5} = \frac{4}{20} \]

   \[ \frac{\square}{5} + \frac{\square}{5} = \left( \frac{1}{10} + \frac{1}{10} \right) + \left( \frac{1}{10} + \frac{1}{10} \right) = \frac{4}{10} \]

   \[ \left( \frac{1}{10} + \frac{1}{10} \right) + \left( \frac{1}{10} + \frac{1}{10} \right) = \left( 2 \times \frac{1}{10} \right) + \left( 2 \times \frac{1}{10} \right) = \frac{4}{10} \]

   \[ \frac{2}{5} = 4 \times \frac{1}{20} = \frac{4}{20} \]

   b. Eighths

   c. Fifteenths
2. Draw area models to show the decompositions represented by the number sentences below. Express each as a sum and product of unit fractions. Use parentheses to show the relationship between the number sentences.
   a. $\frac{2}{3} = \frac{4}{6}$
   b. $\frac{4}{5} = \frac{8}{10}$

3. Step 1: Draw an area model for a fraction with the denominator of 3, 4, or 5.
   Step 2: Shade in more than one fractional unit.
   Step 3: Partition the area model again to find an equivalent fraction.
   Step 4: Write the equivalent fractions as a number sentence. (If you have written a number sentence like this one already in this homework, start over.)