In Topic C, students transition from the number line model to represent points in the coordinate plane (6.NS.C.6c). Their conceptual understanding of symmetry from Grade 4 and their experience with the first quadrant of the coordinate plane in Grade 5 (4.G.A.3, 5.G.A.1, 5.G.A.2) serve as a significant foundation as they extend the plane to all four quadrants. In Lesson 14, students use ordered pairs of rational numbers to
name points on a grid, and given a point’s location, they identify the first number in the ordered pair as the first coordinate and the second number as the second coordinate. In Lessons 15–17, students construct the plane; identify the axes, quadrants, and origin; and graph points in the plane, using an appropriate scale on the axes. Students recognize the relationship that exists between points whose coordinates differ only by signs (as reflections across one or both axes) and locate such points using the symmetry of the plane (6.NS.C.6b). For instance, they recognize that the points (3, 4) and (3, −4) are both equal distance from the $x$-axis on the same vertical line, and so the points are reflections in the $x$-axis. In Lessons 18 and 19, students graph points in the coordinate plane and use absolute value to find the lengths of vertical and horizontal segments to solve real-world problems (6.NS.C.8).
Lesson 14: Ordered Pairs

Student Outcomes

- Students use ordered pairs to name points in a grid and to locate points on a map.
- Students identify the first number in an ordered pair as the first coordinate and the second number as the second coordinate.

Lesson Notes

Students will understand the use of ordered pairs of numbers as describing the locations of points on a plane in various situations. They will recognize the significance of the order of numbers in ordered pairs by looking at the different interpretations.

Classwork

Opening Exercise (5 minutes)

Before students arrive, arrange their desks into straight rows. Assign a number (1, 2, 3, …) to each row, and also to the seats in each row starting at the front with seat 1. As students enter the room, give them a sticky note containing a pair of numbers corresponding with the seating locations in the room. Instruct students to find the seat described by their sticky note, apply the sticky note to the desk, and be seated.

Most students will be confused as only those with matching numbers will be able to find their seats. Monitor conversations taking place between students as they agree upon a convention (e.g., that the first number will represent the row, and the second number will represent the seat).

- How did you find your seat in the classroom?
- Did the order of the numbers matter? Why or why not?
  - The order mattered since there are two different seats that involve the numbers 2 and 3. For instance, row 2, seat 3 and row 3, seat 2.

Example 1 (5 minutes): The Order in Ordered Pairs

Instruct students to rotate their desks 90 degrees in one direction. This changes the orientation of the rows, so that students can better see the meanings of each of the coordinates. Students understand that the coordinates of their location from the opening exercise (in most cases) are different in Example 1. For example, the student sitting in row 1, seat 3 for the Opening Exercise, is now sitting in row 3, seat 1.

Example 1

The first number of an ordered pair is called the first coordinate.

The second number of an ordered pair is called the second coordinate.
Define the first and second coordinates in this example as \((\text{row } \#, \text{ seat } \#)\). Ask all students in the classroom to stand. Call out an appropriate ordered pair and ask for the student in that location to raise his or her hand, briefly explain why the ordered pair of numbers describes that student’s position in the room, then be seated. Now have that student call out a different ordered pair that corresponds with the location of another student. Continue this process until all students have participated.

**Example 2 (10 minutes): Using Ordered Pairs to Name Locations**

Task: Divide students into small groups and provide each group with one of the ordered pair scenarios below. Students will read their scenario and describe how the ordered pair is being used, indicating what defines the first coordinate and what defines the second coordinate. Allow groups 5 minutes to read and discuss the scenario and prepare a response to report out to the class.

**Example 2: Using Ordered Pairs to Name Locations**

Describe how the ordered pair is being used in your scenario. Indicate what defines the first coordinate and what defines the second coordinate in your scenario.

Ordered pairs are like a set of directions; they indicate where to go in one direction, and then indicate where to go in the second direction.

- Scenario 1: The seats in a college football stadium are arranged into 210 sections, with 144 seats in each section. Your ticket to the game indicates the location of your seat using the ordered pair of numbers \((123, 37)\). Describe the meaning of each number in the ordered pair and how you would use them to find your seat.

- Scenario 2: Airline pilots use measurements of longitude and latitude to determine their location and to find airports around the world. Longitude is measured as \(0\text{–}180^\circ\) east, or \(0\text{–}180^\circ\) west of a line stretching from the North Pole to the South Pole through Greenwich, England called the prime meridian. Latitude is measured as \(0\text{–}90^\circ\) north or \(0\text{–}90^\circ\) south of the Earth’s Equator. A pilot has the ordered pair \((90^\circ\text{ west}, 30^\circ\text{ north})\). What does each number in the ordered pair describe? How would the pilot locate the airport on a map? Would there be any confusion if a pilot were given the ordered pair \((90^\circ, 30^\circ)\)? Explain.

- Scenario 3: Each room in a school building is named by an ordered pair of numbers that indicates the number of the floor on which the room lies, followed by the sequential number of the room on the floor from the main staircase. A new student at the school is trying to get to science class in room \(4\text{–}13\). Describe to the student what each number means and how she should use the number to find her classroom. Suppose there are classrooms below the main floor. How might these rooms be described?

Ask student groups to report their answers to the scenarios aloud to the class.
Exercises 1–2 (12 minutes)

Students use the gridded maps in the student materials to name points that correspond with the given ordered pairs (and vice-versa). The first coordinates represent numbers on the line labeled $x$, and the second coordinates represent numbers on the line labeled $y$.

### Exercises 1–2

For Exercises 1 and 2, the first coordinates of the ordered pairs represent the numbers on the line labeled $x$ and the second coordinates represent the numbers on the line labeled $y$.

1. Name the letter from the grid that corresponds with each ordered pair of numbers below.

   a. (1, 4)
      - Point F
   
   b. (4, 1)
      - Point B
   
   c. (5, -2)
      - Point G
   
   d. (2, -1)
      - Point C
   
   e. (0, 5)
      - Point A
   
   f. (8.5, 8)
      - Point L
   
   g. (5.2)
      - Point H
   
   h. (0, 9)
      - Point E

2. List the ordered pair of numbers that corresponds with each letter from the grid below.

   a. Point M
      - (5, 7)
   
   b. Point N
      - (6, 0)
   
   c. Point P
      - (0, 6)
   
   d. Point Q
      - (2, 3)

---

*Scaffolding:*

If students do not understand the negative numbers on the vertical axis, review with students how the floors below ground level might be described in Scenario 3 from Example 2.
e. Point R
   (0,3)

f. Point S
   (−2, 3)

g. Point T
   (−3, 2)

h. Point U
   (7, 5)

i. Point V
   (−1, 6)

Have students provide the correct answers to the exercises.

Closing (5 minutes)

- Why does order matter when using ordered pairs of numbers?
- Alayna says the order in which the values are given in an ordered pair doesn’t always matter. Give an example of when the order does matter and an example of when the order does not matter.
- Explain how to locate points when pairs of integers are used.

Lesson Summary

- The order of numbers in an ordered pair is important because the ordered pair should describe one location in the coordinate plane.
- The first number (called the first coordinate) describes a location using the horizontal direction.
- The second number (called the second coordinate) describes a location using the vertical direction.

Exit Ticket (8 minutes)
Lesson 14: Ordered Pairs

Exit Ticket

1. On the map below, the fire department and the hospital have one matching coordinate. Determine the proper order of the ordered pairs in the map, and write the correct ordered pairs for the locations of the fire department and hospital. Indicate which of their coordinates are the same.

![Map Diagram]

2. On the map above, locate and label the locations of each description below:
   a. The local bank has the same first coordinate as the Fire Department, but its second coordinate is half of the fire department’s second coordinate. What ordered pair describes the location of the bank? Locate and label the bank on the map using point $B$.

   b. The Village Police Department has the same second coordinate as the bank, but its first coordinate is $-2$. What ordered pair describes the location of the Village Police Department? Locate and label the Village Police Department on the map using point $P$. 

Exit Ticket Sample Solutions

1. On the map below, the fire department and the hospital have one matching coordinate. Determine the proper order of the ordered pairs in the map, and write the correct ordered pairs for the locations of the fire department and hospital. Indicate which of their coordinates are the same.

   *The order of the numbers is (x, y); Fire Department: (6, 7) and Hospital: (10, 7); they have the same second coordinate.*

![Map with coordinates labeled](image)

2. On the map above, locate and label the location of each description below:
   
a. The local bank has the same first coordinate as the Fire Department and its second coordinate is half of the fire department’s second coordinate. What ordered pair describes the location of the bank? Locate and label the bank on the map using point $B$.

   $(6, 3.5)$; *See the map image for the correct location of point $B$.*

   b. The Village Police Department has the same second coordinate as the bank, but its first coordinate is $−2$. What ordered pair describes the location of the Village Police Department? Locate and label the Village Police Department on the map using point $P$.

   $(−2, 3.5)$; *See the map image for the correct location of point $P$.*

Problem Set Sample Solutions

1. Use the set of ordered pairs below to answer each question:
   $\{(4, 20), (8, 4), (2, 3), (15, 3), (6, 15), (6, 30), (1, 5), (6, 18), (0, 3)\}$
   
a. Write the ordered pair(s) whose first and second coordinate have a greatest common factor of 3.

   $(15, 3)$ and $(6, 15)$

   b. Write the ordered pair(s) whose first coordinate is a factor of its second coordinate.

   $(4, 20), (6, 30), (1, 5), (6, 18)$
c. Write the ordered pair(s) whose second coordinate is a prime number.

\((2, 3), (15, 3), (1, 5), \text{ and } (0, 3)\)

2. Write ordered pairs that represent the location of points \(A, B, C, \text{ and } D\), where the first coordinate represents the horizontal direction, and the second coordinate represents the vertical direction.

\(A (4, 1); \ B (1, -3); \ C (6, 0); \ D (1, 4)\)

3. Extension:

Write ordered pairs of integers that satisfy the criteria in each part below. Remember that the origin is the point whose coordinates are \((0, 0)\). When possible, give ordered pairs such that: (i) both coordinates are positive; (ii) both coordinates are negative; and (iii) the coordinates have opposite signs in either order.

a. These points' vertical distance from the origin is twice their horizontal distance.

\(\text{Answers will vary; examples } (5, 10), (-2, -4), (-5, -10), (2, -4)\)

b. These points' horizontal distance from the origin is two units more than the vertical distance.

\(\text{Answers will vary; examples } (3, 1), (-3, 1), (-3, -1), (3, -1)\)

c. These points' horizontal and vertical distances from the origin are equal but only one coordinate is positive.

\(\text{Answers will vary; examples } (3, -3), (-8, 8)\)
Lesson 15: Locating Ordered Pairs on the Coordinate Plane

Student Outcomes

- Students extend their understanding of the coordinate plane to include all four quadrants, and recognize that the axes (identified as the $x$-axis and $y$-axis) of the coordinate plane divide the plane into four regions called quadrants (that are labeled from first to fourth and are denoted by Roman Numerals).
- Students identify the origin, and locate points other than the origin, which lie on an axis.
- Students locate points in the coordinate plane that correspond to given ordered pairs of integers and other rational numbers.

Classwork

Opening Exercise (5 minutes)

Hang posters on the wall, each containing one of the following terms: $x$-axis, $y$-axis, $x$-coordinate, $y$-coordinate, origin, and coordinate pair. Pair students up and have them discuss these vocabulary terms and what they remember about the terms from Grade 5. Student pairs will then write what they discussed on the posters with the appropriate vocabulary term. Some important aspects for students to remember include:

- The $x$-axis is a horizontal number line; the $y$-axis is a vertical number line.
- The axes meet forming a $90^\circ$ angle at the point $(0,0)$ called the origin.

Example 1 (6 minutes): Extending the Axes Beyond Zero

Students recognize that the axes are number lines and using straight edges, extend the axes on the coordinate plane to include negative numbers revealing the second, third, and fourth quadrants.

- Describe the $x$-axis. Considering what we have seen in this module, what types of numbers should it include? Why?
  - The $x$-axis is a horizontal number line that includes positive and negative numbers. The axis extends in both directions (left and right of zero) because signed numbers represent values or quantities that have opposite directions.

Example 1: Extending the Axes Beyond Zero

The point below represents zero on the number line. Draw a number line to the right starting at zero. Then follow directions as provided by the teacher.

Students use straight edges to extend the $x$-axis to the left of zero to represent the real number line horizontally and complete the number line using the same scale as on the right side of zero.
Lesson 15: Locating Ordered Pairs on the Coordinate Plane

Date: 10/15/13

- Describe the y-axis. What types of numbers should it include?
  - The y-axis is a vertical number line that includes numbers on both sides of zero (above and below), and so it includes both positive and negative numbers.

Students use straight edges to extend the y-axis below zero to represent the real number line vertically and complete the number line using the same scale as that shown above zero.

Example 2 (4 minutes): Components of the Coordinate Plane

Students examine how to use the axes and the origin of the coordinate plane to determine other locations in the plane.

Example 2: Components of the Coordinate Plane

All points on the coordinate plane are described with reference to the origin. What is the origin, and what are its coordinates?

The origin is the point where the x- and y-axes intersect. The coordinates of the origin are (0, 0).

The axes of the coordinate plane intersect at their zero coordinates, which is a point called the origin. The origin is the reference point from which all points in the coordinate plane are described.

To describe locations of points in the coordinate plane we use ordered pairs of numbers. Order is important, so on the coordinate plane we use the form (x, y). The first coordinate represents the point’s location from zero on the x-axis, and the second coordinate represents the point’s location from zero on the y-axis.

Exercises 1–3 (8 minutes)

Students locate and label points that lie on the axes of the coordinate plane.

Exercises 1–3

1. Use the coordinate plane below to answer parts (a)–(c):
   a. Graph at least five points on the x-axis and label their coordinates.
      Points will vary.
   b. What do the coordinates of your points have in common?
      Each point has a y-coordinate of 0.
   c. What must be true about any point that lies on the x-axis?
      If a point lies on the x-axis, its y-coordinate must be 0 because the point is located 0 units above or below the x-axis. The x-axis intersects the y-axis at 0.

Scaffolding:
- The term origin means starting point.
- A person’s country of origin is the country from which he or she came.
- When using a global positioning unit (GPS) when travelling, your origin is where your trip began.

MP.3

MP.7
2. Use the coordinate plane to answer parts (a)–(c):
   a. Graph at least five points on the y-axis and label their coordinates. 
   
   Points will vary.

   b. What do the coordinates of your points have in common? 
   Each point has an x-coordinate of 0.

   c. What must be true about any point that lies on the y-axis? Explain. 
   If a point lies on the y-axis, its x-coordinate must be 0 because the point is located 0 units left or right of the y-axis. The y-axis intersects 0 on the x-axis.

3. If the origin is the only point with 0 for both coordinates, what must be true about the origin? 
   The origin is the only point that is on both the x-axis and the y-axis.

Example 3 (6 minutes): Quadrants of the Coordinate Plane

Students examine the four regions of the coordinate plane cut by the intersecting axes.

Example 3: Quadrants of the Coordinate Plane

- The x- and y-axes divide the coordinate plane into regions called Quadrants. Why are the regions called quadrants?
  - The axes cut the plane into four regions. The prefix “quad” means four.

- Which of the four regions did you work with most in Grade 5, and why was it the only region you used?
  - The region on the top right of the coordinate plane. We only used this region because we had not learned about negative numbers yet.

- The four quadrants are numbered one through four using Roman Numerals. The upper right quadrant is Quadrant I and the remaining quadrants are numbered moving counter-clockwise from Quadrant I; Quadrant II, Quadrant III, and Quadrant IV. What was the first axis that we extended in Example 1 and what did it reveal?
  - We extended the x-axis to the left beyond zero and it revealed another region of the coordinate plane.
- This top left region is called Quadrant II. Label Quadrant II in your student materials. These regions only make up half of the coordinate plane. Where does the remaining half of the coordinate plane come from? Explain.
  - We need to extend the \( y \)-axis down below zero to show its negative values. This reveals two other regions on the plane; one to the left of the \( y \)-axis and one to the right of the \( y \)-axis.
- The quadrants of the coordinate plane are in a counter-clockwise direction starting with Quadrant I. Label the remaining quadrants in your student materials.

**Exercise 4–6 (5 minutes)**

Students locate and label points that lie on the coordinate plane and indicate in which of the four quadrants the points lie.

### Exercises 4–6

4. Locate and label each point described by the ordered pairs below. Indicate which of the quadrants the points lie in.
   - a. \((7,2)\)  
     Quadrant I.
   - b. \((3,-4)\)  
     Quadrant IV.
   - c. \((1,-5)\)  
     Quadrant IV.
   - d. \((-3,8)\)  
     Quadrant II.
   - e. \((-2,-1)\)  
     Quadrant III.

5. Write the coordinates of at least one other point in each of the four quadrants.
   - a. Quadrant I
     Answers will vary.
   - b. Quadrant II
     Answers will vary.
   - c. Quadrant III
     Answers will vary.
   - d. Quadrant IV
     Answers will vary.
6. Do you see any similarities in the points within each quadrant? Explain your reasoning.

The ordered pairs describing the points in Quadrant I contain both positive values. The ordered pairs describing the points in Quadrant III contain both negative values. The first coordinate of the ordered pairs describing the points in Quadrant II are negative values but their second coordinates are positive values. The first coordinate of the ordered pairs describing the points in Quadrant IV are positive values, but their second coordinates are negative values.

Closing (4 minutes)

- If a point lies on an axis, what must be true about its coordinates? Specifically, what is true for a point that lies on the x-axis? y-axis?
- What do you know about the location of a point on the coordinate plane if:
  - Both coordinates are positive?
  - Only one coordinate is positive?
  - Both coordinates are negative?
  - One coordinate is zero?
  - Both coordinates are zero?

Lesson Summary

- The x-axis and y-axis of the coordinate plane are number lines that intersect at zero on each number line.
- The axes create four quadrants in the coordinate plane.
- Points in the coordinate plane lie either on an axis or in one of the four quadrants.

Exit Ticket (4 minutes)
Lesson 15: Locating Ordered Pairs on the Coordinate Plane

Exit Ticket

1. Label the second quadrant on the coordinate plane then answer the following questions:
   a. Write the coordinates of one point that lies in the second quadrant of the coordinate plane.
   b. What must be true about the coordinates of any point that lies in the second quadrant?

2. Label the third quadrant on the coordinate plane then answer the following questions:
   a. Write the coordinates of one point that lies in the third quadrant of the coordinate plane.
   b. What must be true about the coordinates of any point that lies in the third quadrant?

3. a. An ordered pair has coordinates that have the same sign. In which quadrant(s) could the point lie? Explain.
   b. Another ordered pair has coordinates that are opposites. In which quadrant(s) could the point lie? Explain.
Exit Ticket Sample Solutions

1. Label the second quadrant on the coordinate plane then answer the following questions:
   a. Write the coordinates of one point that lies in the second quadrant of the coordinate plane.
      Answers will vary.
   b. What must be true about the coordinates of any point that lies in the second quadrant?
      \( x \)-coordinate must be a negative value and \( y \)-coordinate must be a positive value.

2. Label the third quadrant on the coordinate plane then answer the following questions:
   a. Write the coordinates of one point that lies in the third quadrant of the coordinate plane.
      Answers will vary.
   b. What must be true about the coordinates of any point that lies in the third quadrant?
      \( x \)- and \( y \)-coordinates of any point in the third quadrant must both be negative values.

3. a. An ordered pair has coordinates that have the same sign. In which quadrant(s) could the point lie? Explain.
    The point would have to be located either in Quadrant I where both coordinates are positive values, or in Quadrant III where both coordinates are negative values.
   b. Another ordered pair has coordinates that are opposites. In which quadrant(s) could the point lie? Explain.
    The point would have to be located in either Quadrant II or Quadrant IV because those are the two quadrants where the coordinates have opposite signs.

Problem Set Sample Solutions

1. Name the quadrant in which each of the points lies. If the point does not lie in a quadrant, specify which axis the point lies on.
   a. \((-2, 5)\)
      Quadrant II
   b. \((9, 2, 7)\)
      Quadrant I
   c. \((0, -4)\)
      None; the point is not in a quadrant because it lies on the \( y \)-axis.
d. \((8, -4)\)
   *Quadrant IV*

e. \((-1, -8)\)
   *Quadrant III*

2. Jackie claims that points with the same \(x\)- and \(y\)-coordinates must lie in Quadrant I or Quadrant III. Do you agree or disagree? Explain your answer.

   *Disagree; most points with the same \(x\)- and \(y\)-coordinates lie in Quadrant I or Quadrant III, but the origin \((0, 0)\) is on the \(x\)- and \(y\)-axes, not in any quadrant.*

3. Locate and label each set of points on the coordinate plane. Describe similarities of the ordered pairs in each set and describe the points on the plane.

   a. \[((-2, 5), (-2, 2), (-2, 7), (-2, -3), (-2, 0.8))\]
      *The ordered pairs all have \(x\)-coordinates of \(-2\) and the points lie along a vertical line above and below \(-2\) on the \(x\)-axis.*

   b. \[((-9, 9), (-4.4, -2, 2), (1, -1), (3, -3), (0, 0))\]
      *The ordered pairs each have opposite values for their \(x\)- and \(y\)-coordinates. The points in the plane line up diagonally through Quadrant II, the origin, and Quadrant IV.*

   c. \[((-7, -8), (5, -8), (0, -8), (10, -8), (-3, -8))\]
      *The ordered pairs all have \(y\)-coordinates of \(-8\) and the points lie along a horizontal line to the left and right of \(-8\) on the \(y\)-axis.*

4. Locate and label at least five points on the coordinate plane that have an \(x\)-coordinate of 6.

   a. What is true of the \(y\)-coordinates below the \(x\)-axis?
      *The \(y\)-coordinates are all negative values.*

   b. What is true of the \(y\)-coordinates above the \(x\)-axis?
      *The \(y\)-coordinates are all positive values.*

   c. What must be true of the \(y\)-coordinates on the \(x\)-axis?
      *The \(y\)-coordinates on the \(x\)-axis must be 0.*
Lesson 16: Symmetry in the Coordinate Plane

Student Outcomes

- Students understand that two numbers are said to differ only by signs if they are opposite of each other.
- Students recognize that when two ordered pairs differ only by sign of one or both of the coordinates, then the locations of the points are related by reflections across one or both axes.

Classwork

Opening Exercise (3 minutes)

Opening Exercise

Give an example of two opposite numbers and describe where the numbers lie on the number line. How are opposite numbers similar and how are they different?

Example 1 (14 minutes): Extending Opposite Numbers to the Coordinate Plane

Students locate and label points whose ordered pairs differ only by the sign of one or both coordinates. Together, students and their teacher examine the relationships of the points on the coordinate plane, and express these relationships in a graphic organizer.

- Locate and label the points (3,4) and (−3,4).
- Record observations in the first column of the graphic organizer.

The first column of the graphic organizer is teacher-led so that students can pay particular attention to the absolute values of coordinates and the general locations of the corresponding points with regard to each axis. Following this lead, columns 2 and 3 are more student-led.

- Locate and label the point (3,−4).
- Record observations in the second column of the graphic organizer.
- Locate and label the point (−3,−4).
- Record observations in the third column of the graphic organizer.
Extending Opposite Numbers to the Coordinates of Points on the Coordinate Plane

Locate and label your points on the coordinate plane to the right. For each given pair of points in the table below, record your observations and conjectures in the appropriate cell. Pay attention to the absolute values of the coordinates and where the points lie in reference to each axis.

<table>
<thead>
<tr>
<th></th>
<th>(3, 4) and (−3, 4)</th>
<th>(3, 4) and (3, −4)</th>
<th>(3, 4) and (−3, −4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Similarities of Coordinates</strong></td>
<td>Same y-coordinates. The x-coordinates have the same absolute value.</td>
<td>Same x-coordinates. The y-coordinates have the same absolute value.</td>
<td>The x-coordinates have the same absolute value. The y-coordinates have the same absolute value.</td>
</tr>
<tr>
<td><strong>Differences of Coordinates</strong></td>
<td>The x-coordinates are opposite numbers.</td>
<td>The y-coordinates are opposite numbers.</td>
<td>Both the x- and y-coordinates are opposite numbers.</td>
</tr>
<tr>
<td><strong>Similarities in Location</strong></td>
<td>Both points are 4 units above the x-axis; and 3 units away from the y-axis.</td>
<td>Both points are 3 units to the right of the y-axis; and 4 units away from the x-axis.</td>
<td>Both points are 3 units from the y-axis; and 4 units from the x-axis.</td>
</tr>
<tr>
<td><strong>Differences in Location</strong></td>
<td>One point is 3 units to the right of the y-axis; the other is 3 units to the left of the y-axis.</td>
<td>One point is 4 units above the x-axis; the other is 4 units below.</td>
<td>One point is 3 units right of the x-axis; the other is 3 units left. One point is 4 units above the y-axis; the other is 4 units below.</td>
</tr>
<tr>
<td><strong>Relationship between Coordinates and Location on the Plane</strong></td>
<td>The x-coordinates are opposite numbers so the points lie on opposite sides of the y-axis. Because opposites have the same absolute value, both points lie the same distance from the y-axis. The points lie the same distance above the x-axis, so the points are symmetric about the y-axis. A reflection across the y-axis takes one point to the other.</td>
<td>The y-coordinates are opposite numbers so the points lie on opposite sides of the x-axis. Because opposites have the same absolute value, both points lie the same distance from the x-axis. The points lie the same distance right of the y-axis, so the points are symmetric about the x-axis. A reflection across the x-axis takes one point to the other.</td>
<td>The points have opposite numbers for x- and y-coordinates, so the points must lie on opposite sides of each axis. Because the numbers are opposites, and opposites have the same absolute values each point must be the same distance from each axis. A reflection across one axis followed by the other will take one point to the other.</td>
</tr>
</tbody>
</table>
Exercise (5 minutes)

**Exercise**

In each column, write the coordinates of the points that are related to the given point by the criteria listed in the first column of the table. Point \(S(5, 3)\) has been reflected over the \(x\)- and \(y\)-axes for you as a guide and its images are shown on the coordinate plane. Use the coordinate grid to help you locate each point and its corresponding coordinates.

<table>
<thead>
<tr>
<th>Given Point: (S(5, 3))</th>
<th>((-2, 4))</th>
<th>((3, -2))</th>
<th>((-1, -5))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflected across the (x)-axis.</td>
<td>(M(5, -3))</td>
<td>((-2, -4))</td>
<td>((3, 2))</td>
</tr>
<tr>
<td>Reflected across the (y)-axis.</td>
<td>(L(-5, 3))</td>
<td>((2, 4))</td>
<td>((-3, -2))</td>
</tr>
<tr>
<td>Reflected first across the (x)-axis then across the (y)-axis.</td>
<td>(A(-5, -3))</td>
<td>((2, -4))</td>
<td>((-3, 2))</td>
</tr>
<tr>
<td>Reflected first across the (y)-axis then across the (x)-axis.</td>
<td>(A(-5, -3))</td>
<td>((2, -4))</td>
<td>((-3, 2))</td>
</tr>
</tbody>
</table>

- **a.** When the coordinates of two points are \((x, y)\) and \((-x, y)\), what line of symmetry do the points share? Explain.
  
  They share the \(y\)-axis, because the \(y\)-coordinates are the same and the \(x\)-coordinates are opposites, which means the points will be the same distance from the \(y\)-axis, but on opposite sides.

- **b.** When the coordinates of two points are \((x, y)\) and \((x, -y)\), what line of symmetry do the points share? Explain.
  
  They share the \(x\)-axis, because the \(x\)-coordinates are the same and the \(y\)-coordinates are opposites, which means the points will be the same distance from the \(x\)-axis but on opposite sides.

**Example 2 (8 minutes): Navigating the Coordinate Plane using Reflections**

Have students use a pencil eraser or their finger to navigate the coordinate plane given verbal prompts. Then circulate the room during the example to assess students’ understanding and provide assistance as needed.

- **Begin at** \((7, 2)\). Move 3 units down, then reflect over the \(y\)-axis. Where are you?
  - \((-7, -1)\)
- **Begin at** \((4, -5)\). Reflect over the \(x\)-axis, then move 7 units down, then to the right 2 units. Where are you?
  - \((6, -2)\)
Lesson 16
Symmetry in the Coordinate Plane

Begin at \((-3, 0)\). Reflect over the \(x\)-axis then move 6 units to the right. Move up two units, then reflect over the \(x\)-axis again. Where are you?
- \((3, -2)\)

Begin at \((-2, 8)\). Decrease the \(y\)-coordinate by 6. Reflect over the \(y\)-axis, then move down 3 units. Where are you?
- \((2, -1)\)

Begin at \((5, -1)\). Reflect over the \(x\)-axis, then reflect over the \(y\)-axis. Where are you?
- \((-5, 1)\)

Example 3 (7 minutes): Describing How to Navigate the Coordinate Plane

Given a starting point and an ending point, students describe a sequence of directions using at least one reflection about an axis to navigate from the starting point to the ending point. Once students have found a sequence, have them find another sequence while their classmates finish the task.

- Begin at \((9, -3)\) and end at \((-4, -3)\). Use exactly one reflection.  
  Possible Answer: Reflect over the \(y\)-axis then move 5 units to the right.
- Begin at \((0, 0)\) and end at \((5, -1)\). Use exactly one reflection.  
  Possible Answer: Move 5 units right, 1 unit up, then reflect over the \(x\)-axis.
- Begin at \((0, 0)\) and end at \((-1, -6)\). Use exactly two reflections.  
  Possible Answer: Move right 1 unit, reflect over the \(y\)-axis, up 6 units, then reflect over the \(x\)-axis.

Closing (4 minutes)

- When the coordinates of two points differ only by one sign, such as \((-8, 2)\) and \((8, 2)\), what do the similarities and differences in the coordinates tell us about their relative locations on the plane?
- What is the relationship between \((5, 1)\) and \((5, -1)\)? Given one point, how can you locate the other?

Exit Ticket (4 minutes)
Lesson 16: Symmetry in the Coordinate Plane

Exit Ticket

1. How are the ordered pairs (4, 9) and (4, −9) similar, and how are they different? Are the two points related by a reflection over an axis in the coordinate plane? If so, indicate which axis is the line of symmetry between the points. If they are not related by a reflection over an axis in the coordinate plane, explain how you know.

2. Given the point (−5, 2), write the coordinates of a point that is related by a reflection over the x- or y-axis. Specify which axis is the line of symmetry.
Exit Ticket Sample Solutions

1. How are the ordered pairs \((4, 9)\) and \((4, -9)\) similar and how are they different? Are the two points related by a reflection over an axis in the coordinate plane? If so, indicate which axis is the line of symmetry between the points. If they are not related by a reflection over an axis in the coordinate plane, explain how you know?

   The \(x\)-coordinates are the same, but the \(y\)-coordinates are opposites, meaning they are the same distance from zero on the \(x\)-axis, and the same distance but opposite sides of zero on the \(y\)-axis. Reflecting about the \(x\)-axis interchanges these two points.

2. Given the point \((-5, 2)\), write the coordinates of a point that is related by a reflection over the \(x\)- or \(y\)-axis. Specify which axis is the line of symmetry.

   Using the \(x\)-axis as a line of symmetry, \((-5, -2)\); using the \(y\)-axis as a line of symmetry, \((5, 2)\).

Problem Set Sample Solutions

1. Locate a point in Quadrant IV of the coordinate plane. Label the point \(A\) and write its ordered pair next to it.

   Answers will vary; Quadrant IV \((5, -3)\);

   a. Reflect point \(A\) over an axis so that its image is in Quadrant III. Label the image \(B\) and write its ordered pair next to it. Which axis did you reflect over? What is the only difference in the ordered pairs of points \(A\) and \(B\)?

      \(B(-5, -3)\); Reflected over the \(y\)-axis.

      The ordered pairs differ only by the sign of their \(x\)-coordinates: \(A(5, -3)\) and \(B(-5, -3)\).

   b. Reflect point \(B\) over an axis so that its image is in Quadrant II. Label the image \(C\) and write its ordered pair next to it. Which axis did you reflect over? What is the only difference in the ordered pairs of points \(B\) and \(C\)? How does the ordered pair of point \(C\) relate to the ordered pair of point \(A\)?

      \(C(-5, 3)\); Reflected over the \(x\)-axis.

      The ordered pairs differ only by the sign of their \(y\)-coordinates: \(B(-5, -3)\) and \(C(-5, 3)\).

      The ordered pair for point \(C\) differs from the ordered pair for point \(A\) by the signs of both coordinates: \(A(5, -3)\) and \(C(-5, 3)\).

   c. Reflect point \(C\) over an axis so that its image is in Quadrant I. Label the image \(D\) and write its ordered pair next to it. Which axis did you reflect over? How does the ordered pair for point \(D\) compare to the ordered pair for point \(C\)? How does the ordered pair for point \(D\) compare to points \(A\) and \(B\)?

      \(D(5, 3)\); Reflected over the \(y\)-axis again.

      Point \(D\) differs from point \(C\) by only the sign of its \(x\)-coordinate: \(D(5, 3)\) and \(C(-5, 3)\).

      Point \(D\) differs from point \(B\) by the signs of both coordinates: \(D(5, 3)\) and \(B(-5, -3)\).

      Point \(D\) differs from point \(A\) by only the sign of the \(y\)-coordinate: \(D(5, 3)\) and \(A(5, -3)\).
2. Bobbie listened to her teacher’s directions and navigated from the point \((-1, 0)\) to \((5, -3)\). She knows that she has the correct answer but, she forgot part of the teacher’s directions. Her teacher’s directions included the following:

“Move 7 units down, reflect about the \(\_\_\_\_\) -axis, move up 4 units, then move right 4 units.”

Help Bobbie determine the missing axis in the directions, and explain your answer.

The missing line is a reflection over the \(y\)-axis. The first line would move the location to \((-1, -7)\). A reflection over the \(y\)-axis would move the location to \((1, -7)\) in Quadrant IV, which is 4 units left and 4 units down from the end point \((5, -3)\).
Lesson 17: Drawing the Coordinate Plane and Points on the Plane

Student Outcomes

- Students draw a coordinate plane on graph paper in two steps: (1) Draw and order the horizontal and vertical axes; (2) Mark the number scale on each axis.
- Given some points as ordered pairs, students make reasonable choices for scales on both axes, and locate and label the points on graph paper.

Classwork

Opening Exercise (5 minutes)

Instruct students to draw all necessary components of the coordinate plane on the blank 20 × 20 grid provided below, placing the origin at the center of the grid and letting each grid line represent 1 unit. Observe students as they complete the task, using their prior experience with the coordinate plane.
Students and teacher together discuss the need for every coordinate plane to have the following:

- The $x$- and $y$-axes drawn using a straight edge
- The horizontal axis labeled $x$
- The vertical axis labeled $y$
- Each axis labeled using an appropriate scale as dictated by the problem or set of ordered pairs to be graphed.

Students should erase errors and make any necessary changes before proceeding to Example 1.

**Example 1 (8 minutes): Drawing the Coordinate Plane using a 1:1 Scale**

- Is the size of the coordinate grid that we discussed in the opening exercise sufficient to graph the points given in the set in Example 1?
  - Yes. All $x$- and $y$-coordinates are between $-10$ and 10 and both axes on the grid range from $-10$ to 10.

**Example 1: Drawing the Coordinate Plane using a 1:1 Scale**

Locate and label the points $\{(3, 2), (8, 4), (-3, 8), (-2, -9), (0, 6), (-1, -2), (10, -2)\}$ on the grid above.

- Can you name a point that could not be located on this grid? Explain.
  - The point $(18, 5)$ could not be located on this grid because 18 is greater than 10 and therefore to the right of 10 on the $x$-axis. 10 is the greatest number shown on this grid.

- Discuss ways in which the point $(18, 5)$ could be graphed without changing the size of the grid.
  - Changing the number of units that each grid line represents would allow us to fit greater numbers on the axes. Changing the number of units per grid line to 2 units would allow a range of $-20$ to 20 on the $x$-axis.
Example 2 (8 minutes): Drawing the Coordinate Plane Using an Increased Number Scale for One Axis

Students increase the number of units represented by each grid line in the coordinate plane in order to graph a given set of ordered pairs.

Example 2: Draw the Coordinate Plane Using an Increased Number of Scale for One Axis

Draw a coordinate plane on the grid below, then locate and label the following points:

\((-4, 20), (-3, 35), (1, -35), (6, 10), (9, -40)\)

- What is the range of values used as \(x\)-coordinates? How many units should we assign per grid line to show this range of values? Explain.
  - The \(x\)-coordinates range from \(-4\) to \(9\), all within the range of \(-10\) to \(10\), so we will assign each grid line to represent 1 unit.

- What is the range of values used as \(y\)-coordinates? How many units should we assign per grid line to show this range of values? Explain.
  - The \(y\)-coordinates range from \(-40\) to \(35\). If we let each grid line represent 5 units, then the \(x\)-axis will include the range \(-50\) to \(50\).

- Draw and label the coordinate plane then locate and label the set of points.
**Example 3 (8 minutes): Drawing the Coordinate Plane Using a Decreased Number Scale for One Axis**

Students divide units among multiple grid lines in the coordinate plane in order to graph a given set of ordered pairs.

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**Example 3: Drawing the Coordinate Plane Using a Decreased Number Scale for One Axis**

Draw a coordinate plane on the grid below, then locate and label the following points:

\[(0.1, 4), (0.5, 7), (-0.7, -5), (-0.4, 3), (0.8, 1)\]

---

- Will either the \(x\)- or \(y\)-coordinates require a change of scale in the plane? Explain.
  - The \(x\)-coordinates range from \(-0.7\) to \(0.8\) which, if each grid line represented one unit, means the points would all be very close to the \(y\)-axis, and therefore difficult to interpret.

- How could we change the number of units represented per grid line to better show the points in the given set?
  - Divide 1 unit into tenths so that each grid line represents a tenth of a unit, and the \(x\)-axis then ranges from \(-1\) to \(1\).

- Draw and label the coordinate plane then locate and label the set of points.
Example 4 (8 minutes): Drawing the Coordinate Plane Using a Different Number Scale for Both Axes

Students appropriately scale the axes in the coordinate plane in order to graph a given set of ordered pairs. Note that the provided grid is 16 × 16, with fewer grid lines than the previous examples.

Example 4: Drawing a Coordinate Plane Using a Different Number Scale for Both Axes

Draw a coordinate plane on the grid below then locate and label the following points: 
{(−14, 2), (−4, −0.5), (6, −3.5), (14, 2.5), (0, 3.5), (−8, −4)}

Determine a scale for the x-axis that will allow all x-coordinates to be shown on your grid.

*The grid is 16 units wide and the x-coordinates range from −14 to 14. If I let each grid line represent 2 units, then the x-axis will range from −16 to 16.*

Determine a scale for the y-axis that will allow all y-coordinates to be shown on your grid.

*The grid is 16 units high and the y-coordinates range from −4 to 3.5. I could let each grid line represent one unit, but if I let each grid line represent \( \frac{1}{2} \) of a unit, the points will be easier to graph.*

Draw and label the coordinate plane then locate and label the set of points.

- How was this example different than the first three examples in this lesson?
  - The given set of points caused me to change the scales on both axes and the given grid had fewer grid lines.
- Did these differences affect your decision making as you created the coordinate plane? Explain.
  - Shrinking the scale of the x-axis allowed me to show a larger range of numbers, but fewer grid lines limited that range.
Closing (2 minutes)

- Why is it important to label the axes when setting up a coordinate plane?
  - So that the person viewing the graph knows which axis represents which coordinate and also so they know what scale is being used. If a person does not know the scale being used, they will likely misinterpret the graph.

- Why shouldn’t you draw and label the entire coordinate grid before looking at the points to be graphed?
  - Looking at the range of values in a given set of points allows you to decide whether or not a change of scale is necessary (or desired). If you set a scale before observing the given values, you will likely have to change the scale on your axes.

Lesson Summary

- The axes of the coordinate plane must be drawn using a straight edge and labeled x (horizontal axis) and y (vertical axis).
- Before assigning a scale to the axes it is important to assess the range of values found in a set of points, as well as the number of grid lines available. This will allow you to determine if the number of units per grid line should be increased or decreased so that all points can be represented on the coordinate plane that you construct.

Exit Ticket (4 minutes)
Lesson 17: Drawing the Coordinate Plane and Points on the Plane

Exit Ticket

Determine an appropriate scale for the set of points given below. Draw and label the coordinate plane then locate and label the set of points.

\{(10, 0.2)\(-25, 0.8), (0, -0.4), (20, 1), (-5, -0.8)\}
Exit Ticket Sample Solutions

Determine an appropriate scale for the set of points given below. Draw and label the coordinate plane, then locate and label the set of points.

\{(10, 0.2), (−25, 0.8), (0, −0.4), (20, 0.1), (−5, −0.8)\}

The x-coordinates range from −25 to 20. The grid is 10 units wide. If I let each grid line represent 5 units, then the x-axis will range from −25 to 25.

The y-coordinates range from −0.8 to 1. The grid is 10 units high. If I let each grid line represent 2-tenths of a unit, then the y-axis will range from −1 to 1.

Problem Set Sample Solutions

1. Label the coordinate plane then locate and label the set of points below.

\{ (0.3, 0.9), (−0.1, 0.7), (−0.5, −0.1), (0, −0.4) \}
Lesson 17: Drawing the Coordinate Plane and Points on the Plane

Date: 10/14/13

2. Label the coordinate plane then locate and label the set of points below.

\[
\{(90, 9), (-110, -11), (40, 4)\}
\[
\{(60, -6), (-80, -8)\}
\]

Extension:

On the back, describe the pattern you see in the coordinates in question 2 and the pattern you see in the points. Are these patterns consistent for other points too?

The x-coordinate for each of the given points is 10 times its y-coordinate. When I graphed the points, they appear to make a straight line. I checked other ordered pairs with the same pattern, such as \((-100, -10)\), \((20, 2)\) and even \((0, 0)\) and it appears that these points are also in that line.
Lesson 18: Distance on the Coordinate Plane

Student Outcomes

- Students compute the length of horizontal and vertical line segments with integer coordinates for endpoints in the coordinate plane by counting the number of units between end points and using absolute value.

Classwork

Opening Exercise (5 minutes)

Opening Exercise

Four friends are touring on motorcycles. They come to an intersection of two roads; the road they are on continues straight, and the other is perpendicular to it. The sign at the intersection shows the distances to several towns. Draw a map/diagram of the roads and use it and the information on the sign to answer the following questions:

What is the distance between Albertsville and Dewey Falls?

Albertsville is 8 miles to the left and Dewey Falls is 6 miles to the right. Since the towns are in opposite directions from the intersection, their distances must be combined. By addition, $8 + 6 = 14$, so the distance between Albertsville and Dewey Falls is 14 miles.

What is the distance between Blossville and Cheyenne?

Blossville and Cheyenne are both straight ahead from the intersection in the direction that they are going. Since they are on the same side of the intersection, Blossville is on the way to Cheyenne so the distance to Cheyenne includes the 3 miles to Blossville. To find the distance from Blossville to Cheyenne, I have to subtract, $12 - 3 = 9$. So the distance from Blossville to Cheyenne is 9 miles.

On the coordinate plane, what represents the intersection of the two roads?

The intersection is represented by the origin.

Example 1 (6 minutes): The Distance Between Points on an Axis

Students find the distance between points on the $x$-axis by finding the distance between numbers on the number line. They find the absolute values of the $x$-coordinates and add or subtract their absolute values to determine the distance between the points.

Example 1: The Distance Between Points on an Axis

What is the distance between $(−4,0)$ and $(5,0)$?
Lesson 18

Lesson 18: Distance on the Coordinate Plane

What do the ordered pairs have in common and what does that mean about their location in the coordinate plane?

Both of their \( y \)-coordinates are zero so each point lies on the \( x \)-axis, the horizontal number line.

How did we find the distance between two numbers on the number line?

We calculated the absolute values of the numbers, which told us how far the numbers were from zero. If the numbers were located on opposite sides of zero, then we added their absolute values together. If the numbers were located on the same side of zero, then we subtracted their absolute values.

Use the same method to find the distance between \((-4, 0)\) and \((5, 0)\).

\[ |-4| = 4 \text{ and } |5| = 5. \text{ The numbers are on opposite sides of zero, so the absolute values get combined, so } 4 + 5 = 9. \]

The distance between \((-4, 0)\) and \((5, 0)\) is 9 units.

Example 2 (5 minutes): The Length of a Line Segment on an Axis

Students find the length of a line segment that lies on the \( y \)-axis by finding the distance between its endpoints.

Example 2: The Length of a Line Segment on an Axis

What is the length of the line segment with endpoints \((0, -6)\) and \((0, -11)\)?

What do the ordered pairs of the endpoints have in common and what does that mean about the line segment’s location in the coordinate plane?

The \( x \)-coordinates of both endpoints are zero so the points lie on the \( y \)-axis, the vertical number line. If its endpoints lie on a vertical number line, then the line segment itself must also lie on the vertical line.

Find the length of the line segment described by finding the distance between its endpoints \((0, -6)\) and \((0, -11)\).

\[ |-6| = 6 \text{ and } |-11| = 11. \text{ The numbers are on the same side of zero which means the longer distance contains the shorter distance, so the absolute values need to be subtracted. } 11 - 6 = 5. \text{ The distance between } (0, -6) \text{ and } (0, -11) \text{ is 5 units.} \]

Example 3 (10 minutes): Length of a Horizontal or Vertical Line Segment that Does Not Lie on an Axis

Students find the length of a vertical line segment which does not lie on the \( y \)-axis by finding the distance between its endpoints.

Example 3: Length of a Horizontal or Vertical Line Segment that Does Not Lie on an Axis

Find the length of the line segment with endpoints \((-3, 3)\) and \((-3, -5)\).

What do the endpoints, which are represented by the ordered pairs, have in common? What does that tell us about the location of the line segment on the coordinate plane?

Both endpoints have \( x \)-coordinates of \(-3\) so the points lie on the vertical line that intersects the \( x \)-axis at \(-3\). This means that the endpoints of the line segment, and thus the line segment, lie on a vertical line.

Find the length of the line segment by finding the distance between its endpoints.

The endpoints are on the same vertical line, so we only need to find the distance between 3 and \(-5\) on the number line.

\[ |3| = 3 \text{ and } |(-5)| = 5, \text{ and the numbers are on opposite sides of zero so the values must be added; } 3 + 5 = 8. \text{ So the distance between } (-3, 3) \text{ and } (-3, -5) \text{ is 8 units.} \]

Scaffolding:

- Students may need to draw an auxiliary line through the endpoints to help visualize a horizontal or vertical number line.

MP.7
Exercise 1 (10 minutes)

Students calculate the distance between pairs of points using absolute values.

Exercise 1

1. Find the lengths of the line segments whose endpoints are given below. Explain how you determined that the line segments are horizontal or vertical.

   a. $(-3, 4)$ and $(-3, 9)$
      
      Both endpoints have $x$-coordinates of $-3$, so the points lie on a vertical line that passes through $-3$ on the $x$-axis. $|4| = 4$ and $|9| = 9$, and the numbers are on the same side of zero. By subtraction, $9 - 4 = 5$, so the length of the line segment with endpoints $(-3, 4)$ and $(-3, 9)$ is 5 units.

   b. $(2, -2)$ and $(-8, -2)$
      
      Both endpoints have $y$-coordinates of $-2$, so the points lie on a horizontal line that passes through $-2$ on the $y$-axis. $|2| = 2$ and $|-8| = 8$, and the numbers are on opposite sides of zero, so the absolute values must be added. By addition $8 + 2 = 10$, so the length of the line segment with endpoints $(2, -2)$ and $(-8, -2)$ is 10 units.

   c. $(-6, -6)$ and $(-6, 1)$
      
      Both endpoints have $x$-coordinates of $-6$, so the points lie on a vertical line. $|-6| = 6$ and $|1| = 1$, and the numbers are on opposite sides of zero, so the absolute values must be added. By addition $6 + 1 = 7$, so the length of the line segment with endpoints $(-6, -6)$ and $(-6, 1)$ is 7 units.

   d. $(-9, 4)$ and $(-4, 4)$
      
      Both endpoints have $y$-coordinates of 4, so the points lie on a horizontal line. $|-9| = 9$ and $|-4| = 4$, and the numbers are on the same side of zero. By subtraction $9 - 4 = 5$, so the length of the line segment with endpoints $(-9, 4)$ and $(-4, 4)$ is 5 units.

   e. $(0, -11)$ and $(0, 8)$
      
      Both endpoints have $x$-coordinates of 0, so the points lie on the $y$-axis. $|-11| = 11$ and $|8| = 8$, and the numbers are on opposite sides of zero, so their absolute values must be added. By addition $11 + 8 = 19$, so the length of the line segment with endpoints $(0, -11)$ and $(0, 8)$ is 19 units.

Closing (3 minutes)

- Why can we find the length of a horizontal or vertical line segment even if it’s not on the $x$- or $y$-axis?
- Can you think of a real-world situation where this might be useful?

Lesson Summary

To find the distance between points that lie on the same horizontal line or on the same vertical line, we can use the same strategy that we used to find the distance between points on the number line.

Exit Ticket (6 minutes)
Lesson 18: Distance on the Coordinate Plane

Exit Ticket

Determine whether each given pair of endpoints lies on the same horizontal or vertical line. If so, find the length of the line segment that joins the pair of points. If not, explain how you know the points are not on the same horizontal or vertical line.

a.  \((0, -2)\) and \((0, 9)\)

b.  \((11, 4)\) and \((2, 11)\)

c.  \((3, -8)\) and \((3, -1)\)

d.  \((-4, -4)\) and \((5, -4)\)
Exit Ticket Sample Solutions

Determine whether each given pair of endpoints lies on the same horizontal or vertical line. If so, find the length of the line segment that joins the pair of points. If not, explain how you know the points are not on the same horizontal or vertical line.

a. (0, −2) and (0, 9)

The endpoints both have x-coordinates of 0 so they both lie on the y-axis which is a vertical line. They lie on opposite sides of zero so their absolute values have to be combined to get the total distance. |−2| = 2 and |9| = 9, so by addition, 2 + 9 = 11. The length of the line segment with endpoints (0, −2) and (0, 9) is 11 units.

b. (11, 4) and (2, 11)

The points do not lie on the same horizontal or vertical line because they do not share a common x- or y-coordinate.

c. (3, −8) and (3, −1)

The endpoints both have x-coordinates of 3, so the points lie on a vertical line that passes through 3 on the x-axis. The y-coordinates numbers lie on the same side of zero. The distance between the points is determined by subtracting their absolute values, |−8| = 8 and |−1| = 1. So by subtraction, 8 − 1 = 7. The length of the line segment with endpoints (3, −8) and (3, −1) is 7 units.

d. (−4, −4) and (5, −4)

The endpoints have the same y-coordinate of −4, so they lie on a horizontal line that passes through −4 on the y-axis. The numbers lie on opposite sides of zero on the number line, so their absolute values must be added to obtain the total distance, |−4| = 4 and |5| = 5. So by addition, 4 + 5 = 9. The length of the line segment with endpoints (−4, −4) and (5, −4) is 9 units.

Problem Set Sample Solutions

1. Find the length of the line segment with endpoints (7, 2) and (−4, 2), and explain how you arrived at your solution.

11 units. Both points have the same y-coordinate, so I knew they were on the same horizontal line. I found the distance between the x-coordinates by counting the number of units on a horizontal number line from −4 to zero, and then from zero to 7, and 7 + 4 = 11.

2. Sarah and Jamal were learning partners in math class and were working independently. They each started at the point (−2, 5) and moved 3 units vertically in the plane. Each student arrived at a different endpoint. How is this possible? Explain and list the two different endpoints.

It is possible because Sarah could have counted up and Jamal could have counted down, or vice-versa. Moving 3 units in either direction vertically would generate the following possible endpoints: (−2, 8) or (−2, 2).

3. The length of a line segment is 13 units. One endpoint of the line segment is (−3, 7). Find four points that could be the other endpoints of the line segment.

(−3, 20), (−3, −6), (−16, 7) or (10, 7)
Lesson 19: Problem-Solving and the Coordinate Plane

Student Outcomes

- Students solve problems related to the distance between points that lie on the same horizontal or vertical line.
- Students use the coordinate plane to graph points, line segments and geometric shapes in the various quadrants and then use the absolute value to find the related distances.

Lesson Notes

The grid provided in the Opening Exercise is also used for Exercises 1–6 since each exercise is sequential. Students extend their knowledge about finding distances between points on the coordinate plane to the associated lengths of line segments and sides of geometric figures.

Classwork

Opening Exercise (3 minutes)

Opening Exercise

In the coordinate plane, find the distance between the points using absolute value.

The distance between the points is 8 units. The points have the same first coordinates and therefore lie on the same vertical line. \(|-3| = 3\), and \(|5| = 5\), and the numbers lie on opposite sides of 0 so their absolute values are added together; \(3 + 5 = 8\). We can check our answer by just counting the number of units between the two points.
Exercises 1–2 (8 minutes): The Length of a Line Segment is the Distance Between its Endpoints

Students relate the distance between two points lying in different quadrants of the coordinate plane to the length of a line segment with those endpoints. Students then use this relationship to graph a horizontal or vertical line segment using distance to find the coordinates of endpoints.

Exercises

1. Locate and label (4, 5) and (4, –3). Draw the line segment between the endpoints given on the coordinate plane. How long is the line segment that you drew? Explain.

   *The length of the line segment is also 8 units. I found that the distance between (4, –3) and (4, 5) is 8 units, and because these are the endpoints of the line segment, the line segment begins and ends at these points, so the distance from end to end is 8 units.*

2. Draw a horizontal line segment starting at (4, –3) that has a length of 9 units. What are the possible coordinates of the other endpoint of the line segment? (There is more than one answer.)

   \((-5, -3)\) or \((13, -3)\)

Which point do you choose to be the other endpoint of the horizontal line segment? Explain how and why you chose that point. Locate and label the point on the coordinate grid.

   *The other endpoint of the horizontal line segment is \((-5, -3)\); I chose this point because the other option \((13, -3)\) is located off of the given coordinate grid. Note: Students may choose the endpoint \((13, -3)\) but they must change the number scale of the x-axis to do so.*

Exercise 3 (5 minutes): Extending Lengths of Line Segments to Sides of Geometric Figures

The two line segments that you have just drawn could be seen as two sides of a rectangle. Given this, the endpoints of the two line segments would be three of the vertices of this rectangle.

3. Find the coordinates of the fourth vertex of the rectangle. Explain how you find the coordinates of the fourth vertex using absolute value.

   *The fourth vertex is \((-5, 5)\). The opposite sides of a rectangle are the same length, so the length of the vertical side starting at \((-5, -3)\) has to be 8 units long. Also, the side from \((-5, -3)\) to the remaining vertex is a vertical line, so the endpoints must have the same first coordinate. \(|-3| = 3\), and \(8 - 3 = 5\), so the remaining vertex must be five units above the x-axis.*

   *Students can use a similar argument using the length of the horizontal side starting at \((4, 5)\), knowing it has to be 9 units long.*

   How does the fourth vertex that you found relate to each of the consecutive vertices in either direction? Explain.

   *The fourth vertex has the same first coordinate as \((-5, -3)\) because they are the endpoints of a vertical line segment. The fourth vertex has the same second coordinate as \((4, 5)\) since they are the endpoints of a horizontal line segment.*

   Draw the remaining sides of the rectangle.
Exercises 4–6 (6 minutes): Using Lengths of Sides of Geometric Figures to Solve Problems

4. Using the vertices that you have found and the lengths of the line segments between them, find the perimeter of the rectangle.  
\[ 8 + 9 + 8 + 9 = 34; \text{ The perimeter of the rectangle is 34 units.} \]

5. Find the area of the rectangle.  
\[ 9 \times 8 = 72; \text{ The area of the rectangle is 72 units}^2. \]

6. Draw a diagonal line segment through the rectangle with opposite vertices for endpoints. What geometric figures are formed by this line segment? What are the areas of each of these figures? Explain.  
\[ \text{The diagonal line segment cuts the rectangle into two right triangles. The areas of the triangles are 36 units}^2 \text{ each because the triangles each make up half of the rectangle and half of 72 is 36.} \]

EXTENSION [If time allows]: Line the edge of a piece of paper up to the diagonal in the rectangle. Mark the length of the diagonal on the edge of the paper. Align your marks horizontally or vertically on the grid and estimate the length of the diagonal to the nearest integer. Use that estimation to now estimate the perimeter of the triangles.  
\[ \text{The length of the diagonal is approximately 12 units, and the perimeter of each triangle is approximately 29 units.} \]

Exercise 7 (8 minutes)

7. Construct a rectangle on the coordinate plane that satisfies each of the criteria listed below. Identify the coordinate of each of its vertices.
   - Each of the vertices lies in a different quadrant.
   - Its sides are either vertical or horizontal.
   - The perimeter of the rectangle is 28 units.

   Answers will vary. The example to the right shows a rectangle with side lengths 10 and 4 units. The coordinates of the rectangle's vertices are (-6, 3), (4, 3), (4, -1) and (-6, -1).

   Using absolute value, show how the lengths of the sides of your rectangle provide a perimeter of 28 units.  
   \[ |−6| = 6, |4| = 4, \text{ and } 6 + 4 = 10, \text{ so the width of my rectangle is } 10 \text{ units.} \]
   \[ |3| = 3, |−1| = 1, \text{ and } 3 + 1 = 4, \text{ so the height of my rectangle is } 4 \text{ units.} \]
   \[ 10 + 4 + 10 + 4 = 28; \text{ The perimeter of my rectangle is } 28 \text{ units.} \]
Closing (5 minutes)

- How do we determine the length of a horizontal line segment whose endpoints lie in different quadrants of the coordinate plane?
  - If the points are in different quadrants, then the $x$-coordinates lie on opposite sides of zero. The distance between the $x$-coordinates can be found by adding the absolute values of the $x$-coordinates. (The $y$-coordinates are the same and show that the points lie on a horizontal line.)

- If we know one endpoint of a vertical line segment and the length of the line segment, how do we find the other endpoint of the line segment? Is the process the same with a horizontal line segment?
  - If the line segment is vertical, then the other endpoint could be above or below the given endpoint. If we know the length of the line segment then we can count up or down from the given endpoint to find the other endpoint. We can check our answer using the absolute values of the $y$-coordinates.

Lesson Summary

- The length of a line segment on the coordinate plane can be determined by finding the distance between its endpoints.
- You can find the perimeter and area of figures such as rectangles and right triangles by finding the lengths of the line segments that make up their sides, and then using the appropriate formula.

Exit Ticket (10 minutes)
Lesson 19: Problem-Solving and the Coordinate Plane

Exit Ticket

1. The coordinates of one endpoint of a line segment are \((-2, -7)\). The line segment is 12 units long. Give three possible coordinates of the line segment’s other endpoint.

2. Graph a rectangle with area 12 units², such that its vertices lie in at least two of the four quadrants in the coordinate plane. State the lengths of each of the sides, and use absolute value to show how you determined the lengths of the sides.
Exit Ticket Sample Solutions

1. The coordinates of one endpoint of a line segment are \((-2, -7)\). The line segment is 12 units long. Give three possible coordinates of the line segment’s other endpoint.
   \((10, -7); (-14, -7); (-2, 5); (-2, -19)\)

2. Graph a rectangle with area 12 units\(^2\), such that its vertices lie in at least two of the four quadrants in the coordinate plane. List the lengths of the sides, and use absolute value to show how you determined the lengths of the sides.

   *Answers will vary. The rectangle can have side lengths of 6 and 2 or 3 and 4. A sample is provided on the grid on the right. 6 \times 2 = 12*

Problem Set Sample Solutions

Please provide students with three coordinate grids to use in completing the Problem Set.

1. One endpoint of a line segment is \((-3, -6)\). The length of the line segment is 7 units. Find four points that could serve as the other endpoint of the given line segment.
   \((-10, -6); (4, -6); (-3, 1); (-3, -13)\)

2. Two of the vertices of a rectangle are \((1, -6)\) and \((-8, -6)\). If the rectangle has a perimeter of 26 units, what are the coordinates of its other two vertices?
   \((1, -2)\) and \((-8, -2)\) or \((1, -10)\) and \((-8, -10)\)

3. A rectangle has a perimeter of 28 units, an area of 48 square units, and sides that are either horizontal or vertical. If one vertex is the point \((-5, -7)\) and the origin is in the interior of the rectangle, find the vertex of the rectangle that is opposite \((-5, -7)\).
   \((1, 1)\)
1. Mr. Kindle invested some money in the stock market. He tracks his gains and losses using a computer program. Mr. Kindle receives a daily email that updates him on all his transactions from the previous day. This morning, his email read as follows:

   Good morning, Mr. Kindle,

   Yesterday’s investment activity included a loss of $800, a gain of $960, and another gain of $230. Log in now to see your current balance.

   a. Write an integer to represent each gain and loss.

<table>
<thead>
<tr>
<th>Description</th>
<th>Integer Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss of $800</td>
<td></td>
</tr>
<tr>
<td>Gain of $960</td>
<td></td>
</tr>
<tr>
<td>Gain of $230</td>
<td></td>
</tr>
</tbody>
</table>

   b. Mr. Kindle noticed that an error had been made on his account. The “loss of $800” should have been a “gain of $800.” Locate and label both points that represent “a loss of $800” and “a gain of $800” on the number line below. Describe the relationship of these two numbers, when zero represents no change (gain or loss).
c. Mr. Kindle wanted to correct the error, so he entered $-(-800)$ into the program. He made a note that read, “The opposite of the opposite of $800$ is $800.” Is his reasoning correct? Explain.

2. At 6:00 a.m., Buffalo, NY had a temperature of $10^\circ F$. At noon, the temperature was $-10^\circ F$, and at midnight it was $-20^\circ F$.

a. Write a statement comparing $-10^\circ F$ and $-20^\circ F$.

b. Write an inequality statement that shows the relationship between the three recorded temperatures. Which temperature is the warmest?
c. Explain how to use absolute value to find the number of degrees below zero the temperature was at noon.

d. In Peekskill, NY, the temperature at 6:00 a.m. was $-12^\circ F$. At noon, the temperature was the exact opposite of Buffalo’s temperature at 6:00 a.m. At midnight, a meteorologist recorded the temperature as $-6^\circ F$ in Peekskill. He concluded that, “For temperatures below zero, as the temperature increases, the absolute value of the temperature decreases.” Is his conclusion valid? Explain and use a vertical number line to support your answer.

3. Choose an integer between $0$ and $-5$ on a number line, and label the point $P$. Locate and label each of the following points and their values on the number line.

![Number Line]

a. Label point $A$: the opposite of $P$.

b. Label point $B$: a number less than $P$.

c. Label point $C$: a number greater than $P$.

d. Label point $D$: a number half way between $P$ and the integer to the right of $P$. 
4. Julia is learning about elevation in math class. She decided to research some facts about New York State to better understand the concept. Here are some facts that she found.

- **Mount Marcy** is the highest point in New York State. It is 5,343 feet above sea level.
- **Lake Erie** is 210 feet below sea level.
- The elevation of **Niagara Falls, NY** is 614 feet above sea level.
- The lobby of the **Empire State Building** is 50 feet above sea level.
- **New York State borders the Atlantic Coast, which is at sea level.**
- The lowest point of **Cayuga Lake** is 435 feet below sea level.

a. Write an integer that represents each location in relationship to sea level.

<table>
<thead>
<tr>
<th>Location</th>
<th>Integer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mount Marcy</td>
<td>5,343</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>-210</td>
</tr>
<tr>
<td>Niagara Falls, NY</td>
<td>614</td>
</tr>
<tr>
<td>Empire State Building</td>
<td>50</td>
</tr>
<tr>
<td>Atlantic Coast</td>
<td>0</td>
</tr>
<tr>
<td>Cayuga Lake</td>
<td>-435</td>
</tr>
</tbody>
</table>

b. Explain what negative and positive numbers tell Julia about elevation.
c. Order the elevations from least to greatest, and then state their absolute values. Use the chart below to record your work.

<table>
<thead>
<tr>
<th>Elevations</th>
<th>Absolute Values of Elevations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

d. Circle the row in the table that represents sea level. Describe how the order of the elevations below sea level compares to the order of their absolute values. Describe how the order of the elevations above sea level compares to the order of their absolute values.
5. For centuries, a mysterious sea serpent has been rumored to live at the bottom of Seneca Lake, the longest of the Finger Lakes. A team of historians used a computer program to plot the last five positions of the sightings.

a. Locate and label the locations of the last four sightings: $A \left(-9 \frac{1}{2}, 0\right)$, $B \left(-3, -4.75\right)$, $C \left(9, 2\right)$, and $D \left(8, -2.5\right)$.

b. Over time, most of the sightings occurred in Quadrant III. Write the coordinates of a point that lies in Quadrant III.

c. What is the distance between point $A$ and the point $\left(9 \frac{1}{2}, 0\right)$? Show your work to support your answer.

d. What are the coordinates of point $E$ on the coordinate plane?

e. Point $F$ is related to point $E$. Its $x$-coordinate is the same as point $E$’s, but its $y$-coordinate is the opposite of point $E$’s. Locate and label point $F$. What are the coordinates? How far apart are points $E$ and $F$? Explain how you arrived at your answer.
### A Progression Toward Mastery

<table>
<thead>
<tr>
<th>Assessment Task Item</th>
<th>STEP 1</th>
<th>STEP 2</th>
<th>STEP 3</th>
<th>STEP 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong> a 6.NS.C.5 6.NS.C.6a</td>
<td>Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.</td>
<td>Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.</td>
<td>A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.</td>
<td>A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.</td>
</tr>
<tr>
<td>Student is unable to answer the question. None of the descriptions are correctly represented with an integer although student may have made an effort to answer the question.</td>
<td>Student correctly represents only one of the three descriptions with an integer.</td>
<td>Student correctly represents two of the three descriptions with integers.</td>
<td>Student correctly represents all three descriptions with integers: −800, 960, 230.</td>
<td></td>
</tr>
<tr>
<td><strong>b</strong> 6.NS.C.5 6.NS.C.6a 6.NS.C.6c</td>
<td>Student does not attempt to locate and label −800 and 800 and provides little or no evidence of reasoning.</td>
<td>Student attempts to locate and label −800 and 800 but makes an error. For example, both integers are not equidistant from 0. Student may or may not have correctly identified the relationship as opposites.</td>
<td>Student accurately locates but does not label −800 and 800; student correctly identifies the relationship between the integers as opposites. OR Student accurately locates and labels −800 and 800 on the number line but does not identify the relationship between the integers as opposites.</td>
<td>The student accurately locates and labels −800 and 800 on the number line and identifies the relationship between the integers as opposites.</td>
</tr>
<tr>
<td><strong>c</strong> 6.NS.C.5 6.NS.C.6a</td>
<td>Student response is incorrect, and no evidence of reasoning, such as an explanation and/or diagram is provided.</td>
<td>Student response is incorrect, but the student attempts to answer the question with an explanation and/or diagram that demonstrates an understanding of the word opposite although</td>
<td>Student response correctly states that: Yes, Mr. Kindle’s reasoning is correct. But the explanation and/or diagram provided does not completely explain why Mr. Kindle’s statement is correct.</td>
<td>Student response correctly states that: Yes, Mr. Kindle’s reasoning is correct. AND The stance is supported with a valid explanation that demonstrates a solid understanding of the fact that the opposite of</td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>6.NS.C.7b</td>
<td>Student response is missing.</td>
<td>Student response provides an incorrect statement but provides some evidence of understanding the ordering of rational numbers in the written work.</td>
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<tr>
<td>b</td>
<td>6.NS.C.7a 6.NS.C.7b</td>
<td>Student response is missing.</td>
<td>Student attempts to write an inequality statement, but the statement is incorrect and does not include all three numbers. OR The incorrect inequality statement lists all three numbers but does not list 10 as the greatest value.</td>
<td>Student writes an inequality statement that orders the three values with 10 as the greatest number, but the statement contains an error. For example, (-10 &lt; -20 &lt; 10).</td>
</tr>
<tr>
<td>c</td>
<td>6.NS.C.7c</td>
<td>Student response is missing.</td>
<td>Student response explains how to use a number line to find the number of degrees below zero the temperature is at noon, but the use of absolute value is not included in the explanation, or it is referenced incorrectly such as (</td>
<td>-10</td>
</tr>
<tr>
<td>d</td>
<td>6.NS.C.7c</td>
<td>Student response is missing. OR Student response is an incomplete statement supported by little or no evidence of reasoning.</td>
<td>Student response is incorrect but shows some evidence of reasoning. However, the explanation does not show that as negative numbers decrease, their absolute values increase. Student explanation may or may not be supported with an accurate vertical number line model.</td>
<td>Student response includes “Yes” along with a valid explanation that indicates that as negative numbers decrease, their absolute values increase. But a vertical number line mode is missing or contains an error.</td>
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<tr>
<td><strong>Module 3</strong></td>
<td><strong>Rational Numbers</strong></td>
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<tr>
<td><strong>Date:</strong></td>
<td></td>
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</tr>
<tr>
<td><strong>188</strong></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### 6.NS.C.6a

**Point Location and Value of Points on the Number Line**

<table>
<thead>
<tr>
<th><strong>Task</strong></th>
<th><strong>Student Response</strong></th>
<th><strong>Correct Answer</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3 a</strong></td>
<td>Student response is missing. OR There is little or no evidence of understanding in the work shown to determine the correct location and value of point A.</td>
<td>Student incorrectly locates point A (the opposite of point P) on the number line; however, the location of point A indicates some understanding of an integer’s opposite.</td>
</tr>
<tr>
<td><strong>6.NS.C.6c</strong></td>
<td>Student locates the correct point on the number line for the opposite (1, 2, 3, or 4) based on the integer between 0 and (-5) ((-1, -2, -3, \text{ or } -4)). However, the opposite is not labeled on the number line as point A. OR Student correctly locates and labels point A, the opposite of point P, even though point P does not represent an integer between 0 and (-5).</td>
<td></td>
</tr>
<tr>
<td><strong>6.NS.C.7a</strong></td>
<td>A correct answer of the opposite (1, 2, 3, or 4) is given based on correctly choosing an integer between 0 and (-5) ((-1, -2, -3, \text{ or } -4)) as point P. The opposite is correctly located on the number line and labeled as point A.</td>
<td></td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>Student response is missing. OR There is little or no evidence of understanding in the work shown to determine the correct location and value of point B.</td>
<td>Student incorrectly locates point B on the number line; however, the location of point B on the number line indicates that point B is not equal to point P.</td>
</tr>
<tr>
<td><strong>6.NS.C.6c</strong></td>
<td>Student correctly locates a point on the number line to the left of point P; however, the point is not labeled as point B. OR Student correctly locates and labels point B even though point P does not represent an integer between 0 and (-5).</td>
<td></td>
</tr>
<tr>
<td><strong>6.NS.C.7a</strong></td>
<td>Point B is correctly graphed and labeled on the number line. The point is to the left of point P on the number line; for example, if point P is (-3), point B could be (-5).</td>
<td></td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>Student response is missing. OR There is little or no evidence of understanding in the work shown to determine the correct location and value of point C.</td>
<td>Student incorrectly locates point C on the number line; however, the location of point C on the number line indicates that point C is not equal to point P.</td>
</tr>
<tr>
<td><strong>6.NS.C.6c</strong></td>
<td>Student correctly locates a point on the number line to the right of point P; however, the point is not labeled as point C. OR Student correctly locates and labels point C, even though point P does not represent an integer between 0 and (-5).</td>
<td></td>
</tr>
<tr>
<td><strong>6.NS.C.7a</strong></td>
<td>Point C is correctly graphed and labeled on the number line. The point is to the right of point P on the number line; for example, if point P is (-3), point C could be 0.</td>
<td></td>
</tr>
<tr>
<td><strong>d</strong></td>
<td>Student response is missing. OR There is little or no evidence of understanding in the work shown to determine the correct location and value of point D.</td>
<td>Student incorrectly locates point D on the number line; however, the location of point D is to the right of point P although not half way between the integer to the right of point P and point P.</td>
</tr>
<tr>
<td><strong>6.NS.C.6c</strong></td>
<td>Student correctly locates the number that is halfway between point P and the integer to the right of point P; however, the point is not labeled as point D. OR Student correctly locates and labels point D even though point P does not represent an integer</td>
<td></td>
</tr>
<tr>
<td><strong>6.NS.C.7a</strong></td>
<td>Point D is correctly graphed and labeled on the number line. The point is exactly halfway between point P and the integer to the right of point P on the number line; for example, if point P is (-3), point D would be (-2.5).</td>
<td></td>
</tr>
</tbody>
</table>
### End-of-Module Assessment Task

<table>
<thead>
<tr>
<th></th>
<th>6.NS.C.5</th>
<th>6.NS.C.7c</th>
<th>6.NS.C.7d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Student response is missing. OR Student made an effort to answer the question, but none of the responses are correct.</td>
<td>Student response includes 1, 2, 3, or at most 4 locations represented with correct integers.</td>
<td>Student response includes all six locations represented with the correct integers: 5,343, $-210$, 614, 50, 0, $-435$</td>
</tr>
<tr>
<td>b</td>
<td>Student response is missing. OR Student attempts to provide an explanation, and the explanation is supported with some evidence of reasoning, but it is incomplete. For example, “Positive and negative numbers tell Julia about sea level.”</td>
<td>Student response includes an explanation with evidence of solid reasoning, but the explanation lacks details. For example, “Positive and negative numbers tell Julia how far from sea level a location is.”</td>
<td>Student response is correct. An accurate and complete explanation is given, stating that a positive number indicates an elevation above sea level, and a negative number indicates an elevation below sea level.</td>
</tr>
<tr>
<td>c</td>
<td>Student responses are missing and/or student only partially fills in the chart.</td>
<td>Student fills in the chart attempting to order the elevations and find their absolute values, but more than two numerical errors are made. OR Student fills in the chart and correctly finds the absolute value of each number but does not order the elevations from least to greatest or from greatest to least.</td>
<td>Student response is correct and complete. The chart is accurately completed with elevations ordered from least to greatest and their respective absolute values recorded.</td>
</tr>
<tr>
<td>d</td>
<td>Student responses are missing. OR Student circles the row with zeros in the chart to represent sea level but provides no further explanation.</td>
<td>Student circles the row with zeros in the chart to represent sea level and provides an explanation that contains some evidence of reasoning although the explanation may be incomplete or contain inaccurate statements.</td>
<td>Student circles the row with zeros in the chart to represent sea level AND an accurate explanation is given and is supported with substantial evidence that sea levels below zero will have opposite absolute values as their elevations, and sea levels above zero will have the same absolute values as their elevations.</td>
</tr>
<tr>
<td>---</td>
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</tr>
<tr>
<td>5</td>
<td>Student response is missing. OR All 4 points are inaccurately located.</td>
<td>Student accurately locates and labels 1-2 points.</td>
<td>Student response is incorrect AND neither coordinate is stated as a negative number.</td>
</tr>
</tbody>
</table>
1. Mr. Kindle invested some money in the stock market. He tracks his gains and losses using a computer program. Mr. Kindle receives a daily email that updates him on all his transactions from the previous day. This morning, his email read as follows:

   Good morning, Mr. Kindle,

   Yesterday’s investment activity included a loss of $800, a gain of $960, and another gain of $230. Log in now to see your current balance.

   a. Write an integer to represent each gain and loss.

<table>
<thead>
<tr>
<th>Description</th>
<th>Integer Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss of $800</td>
<td>-800</td>
</tr>
<tr>
<td>Gain of $960</td>
<td>960</td>
</tr>
<tr>
<td>Gain of $230</td>
<td>230</td>
</tr>
</tbody>
</table>

   b. Mr. Kindle noticed that an error had been made on his account. The “loss of $800” should have been a “gain of $800.” Locate and label both points that represent “a loss of $800” and “a gain of $800” on the number line below. Describe the relationship of these two numbers, when zero represents no change (gain or loss).

   -800 and 800 are opposites.
c. Mr. Kindle wanted to correct the error, so he entered \(-(-800)\) into the program. He made a note that read, “The opposite of the opposite of \$800\) is \$800.” Is his reasoning correct? Explain.

Yes, he is correct. The opposite of 800 is \(-800\) and the opposite of that is 800.

2. At 6:00 a.m., Buffalo, NY had a temperature of \(10^\circ F\). At noon, the temperature was \(-10^\circ F\), and at midnight it was \(-20^\circ F\).

a. Write a statement comparing \(-10^\circ F\) and \(-20^\circ F\)

\(-10^\circ F\) is warmer than \(-20^\circ F\).

b. Write an inequality statement that shows the relationship between the three recorded temperatures. Which temperature is the warmest?

\(-20 < -10 < 10\)

\(10^\circ F\) is the warmest temperature.
c. Explain how to use absolute value to find the number of degrees below zero the temperature was at noon.

\[ | -10 | = 10 \quad \text{The temperature at noon was } 10^\circ \text{F below zero.} \]

d. In Peekskill, NY, the temperature at 6:00 a.m. was \(-12^\circ\text{F}\). At noon, the temperature was the exact opposite of Buffalo’s temperature at 6:00 a.m. At midnight, a meteorologist recorded the temperature as \(-6^\circ\text{F}\) in Peekskill. He concluded that, “For temperatures below zero, as the temperature increases, the absolute value of the temperature decreases.” Is his conclusion valid? Explain and use a vertical number line to support your answer.

\[ | -12 | = 12 \quad \text{the absolute values are decreasing} \]
\[ | -10 | = 10 \quad \text{decreasing} \]
\[ | -6 | = 6 \quad \text{Yes, his conclusion is valid. Absolute value is a number’s distance from zero. As the temperatures increase from } -12 \text{ to } -10 \text{ to } -6, \text{ they get closer to zero. So their distances from zero is decreasing.} \]

3. Choose an integer between 0 and \(-5\) on a number line, and label the point \(P\). Locate and label each of the following points and their values on the number line.

\[
\text{B} \quad \text{P} \quad \text{C} \quad \text{A}
\]

a. Label point \(A\): the opposite of \(P\).

b. Label point \(B\): a number less than \(P\).

c. Label point \(C\): a number greater than \(P\).

d. Label point \(D\): a number half way between \(P\) and the integer to the right of \(P\).
4. Julia is learning about elevation in math class. She decided to research some facts about New York State to better understand the concept. Here are some facts that she found.

- *Mount Marcy* is the highest point in New York State. It is 5,343 feet above sea level.
- *Lake Erie* is 210 feet below sea level.
- *The elevation of Niagara Falls, NY* is 614 feet above sea level.
- *The lobby of the Empire State Building* is 50 feet above sea level.
- *New York State borders the Atlantic Coast*, which is at sea level.
- *The lowest point of Cayuga Lake* is 435 feet below sea level.

a. Write an integer that represents each location in relationship to sea level.

<table>
<thead>
<tr>
<th>Location</th>
<th>Integer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mount Marcy</td>
<td>5,343</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>-210</td>
</tr>
<tr>
<td>Niagara Falls, NY</td>
<td>614</td>
</tr>
<tr>
<td>Empire State Building</td>
<td>50</td>
</tr>
<tr>
<td>Atlantic Coast</td>
<td>0</td>
</tr>
<tr>
<td>Cayuga Lake</td>
<td>-435</td>
</tr>
</tbody>
</table>

b. Explain what negative and positive numbers tell Julia about elevation.

   A negative number means the elevation is below sea level. A positive number means the elevation is above sea level.
c. Order the elevations from least to greatest, and then state their absolute values. Use the chart below to record your work.

<table>
<thead>
<tr>
<th>Elevations</th>
<th>Absolute Values of Elevations</th>
</tr>
</thead>
<tbody>
<tr>
<td>-435</td>
<td>435</td>
</tr>
<tr>
<td>-210</td>
<td>210</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>614</td>
<td>614</td>
</tr>
<tr>
<td>5343</td>
<td>5343</td>
</tr>
</tbody>
</table>

d. Circle the row in the table that represents sea level. Describe how the order of the elevations below sea level compares to the order of their absolute values. Describe how the order of the elevations above sea level compares to the order of their absolute values.

The elevations below sea level have absolute values that are their opposites, so the order is opposite. \(-435 < -210\) but \(435 > 235\).

The elevations above sea level are the same as their absolute values, so the order is the same. \(50 < 614 < 5343\)
5. For centuries, a mysterious sea serpent has been rumored to live at the bottom of Seneca Lake, the longest of the Finger Lakes. A team of historians used a computer program to plot the last five positions of the sightings.

a. Locate and label the locations of the last four sightings: \( A \left( -9 \frac{1}{2}, 0 \right) \), \( B \left( -3, -4.75 \right) \), \( C \left( 9, 2 \right) \), and \( D \left( 8, -2.5 \right) \).

b. Over time, most of the sightings occurred in Quadrant III. Write the coordinates of a point that lies in Quadrant III.

\[ (-1, -3) \]

c. What is the distance between point \( A \) and the point \( \left( 9 \frac{1}{2}, 0 \right) \)? Show your work to support your answer.

\[ q' \frac{1}{2} + q' \frac{2}{2} = 19 \]

The distance is 19 units.

d. What are the coordinates of point \( E \) on the coordinate plane?

\[ \left( 5, 2 \right) \]

e. Point \( F \) is related to point \( E \). Its \( x \)-coordinate is the same as point \( E \)'s, but its \( y \)-coordinate is the opposite of point \( E \)'s. Locate and label point \( F \). What are the coordinates? How far apart are points \( E \) and \( F \)? Explain how you arrived at your answer.