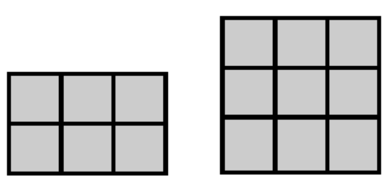
# Topic B:

# Special Notations of Operations

## 6.EE.A.1, 6.EE.A.2c

| | | |
|---|---|---|
| **Focus Standard:** | 6.EE.A.1 | Write and evaluate numeric expressions involving whole-number exponents. |
| | 6.EE.A.2c | Write, read, and evaluate expressions in which letters stand for numbers.<br>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$. |
| **Instructional Days:** | 2 | |
| **Lesson 5:** | Exponents (S)[1] | |
| **Lesson 6:** | The Order of Operations (P) | |

In Topic B, students differentiate between the product of two numbers and whole numbers with exponents. They differentiate between the two through exploration of patterns, specifically noting how squares grow from a $1 \times 1$ measure. They determine that a square with a length and width of three units in measure is constructed with nine square units. This expression is represented as $3^2$ and is evaluated as the product of $3 \times 3 = 9$, not the product of the base and exponent, $6$. They further differentiate between the two by comparing the areas of two models with similar measures, as shown below:



$2a$
When $a = 3$ the area of the rectangle is 6 square units.

$a^2$
When a $= 3$ the area of the rectangle is 9 square units.

[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Once students understand that the base is multiplied by itself the number of times as stated by the exponent, they make a smooth transition into bases that are represented with positive fractions and decimals. They know that for any number, $a$, we define $a^m$ to be the product of $m$ factors of $a$. The number $a$ is the base and $m$ is called the exponent (or the power) of $a$.

In Lesson 6, students build on their previous understanding of the order of operations by including work with exponents. They follow the order of operations to evaluate numerical expressions. They recognize in the absence of parentheses that exponents are evaluated first. Students identify when the order of operations is incorrectly applied and determine the applicable course to correctly evaluate expressions. They understand that the placement of parentheses can alter the final solution when evaluating expressions, as in the following example:

| | |
|---|---|
| $2^4 \cdot (2+8) - 16$ | $2^4 \cdot 2 + 8 - 16$ |
| $2^4 \cdot 10 - 16$ | $16 \cdot 2 + 8 - 16$ |
| $16 \cdot 10 - 16$ | $32 + 8 - 16$ |
| $160 - 16$ | $40 - 16$ |
| $144$ | $24$ |

Students continue to apply the order of operations throughout the module as they evaluate numerical and algebraic expressions.

# Lesson 5: Exponents

## Student Outcomes

- Students discover that $3x = x + x + x$ is not the same thing as $x^3$ which is $x \cdot x \cdot x$.
- Students understand that a base number can be represented with a positive whole number, positive fraction, or positive decimal and that for any number $a$, we define $a^m$ to be the product of $m$ factors of $a$. The number $a$ is the base and $m$ is called the exponent or power of $a$.

## Lesson Notes

In 5th grade, students are introduced to exponents. Explain patterns in the number of zeros of the product when multiplying a number by powers of $10$, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of $10$. Use whole-number exponents to denote powers of $10$ (**5.NBT.A.2**).

In this lesson, students will use new terminology (base, squared, and cubed), and practice moving between exponential notation, expanded notation, and standard notation. The following terms should be displayed, defined, and emphasized throughout Lesson 5: base, exponent, power, squared, and cubed.

### Fluency Exercise (5 minutes)

Multiplication of Decimals White Board Exchange

## Classwork

### Opening Exercise (2 minutes)

**Opening Exercise**

**As you evaluate these expressions, pay attention to how you arrive at your answers.**

$\mathbf{4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4}$

$\mathbf{9 + 9 + 9 + 9 + 9}$

$\mathbf{10 + 10 + 10 + 10 + 10}$

### Socratic Discussion (15 minutes)

- How many of you solved the problems by "counting on"? That is, starting with $4$, you counted on $4$ more each time $(5, 6, 7, \mathbf{8}, 9, 10, 11, \mathbf{12}, 13, 14, 15, \mathbf{16}\ldots)$
- If you did not find the answer that way, could you have done so?
  - *Yes, but it is time-consuming and cumbersome.*
- Addition is a faster way of "counting on."

- How else could you find the sums using addition?
  - *Count by* $4$*,* $9$*, or* $10$*.*
- How else could you solve the problems?
  - *Multiply* $4$ *times* $10$*; multiply* $9$ *times* $5$*; multiply* $10$ *times* $5$*.*
- Multiplication is a faster way to add numbers when the addends are the same.
- When we add five groups of $10$, we use an abbreviation and a different notation, called multiplication. $10 + 10 + 10 + 10 + 10 = 5 \times 10$.
- If multiplication is a more efficient way to represent addition problems involving the repeated addition of the same addend, do you think there might be a more efficient way to represent the repeated multiplication of the same factor, as in $10 \times 10 \times 10 \times 10 \times 10 = ?$

MP.2 & MP.7

Allow students to make suggestions; some will recall this from previous learning.

$$10 \times 10 \times 10 \times 10 \times 10 = 10^5$$

- We see that when we add $5$ groups of $10$, we write $5 \times 10$, but when we multiply $5$ copies of $10$, we write $10^5$. So, multiplication by $5$ in the context of addition corresponds exactly to the exponent $5$ in the context of multiplication.

Make students aware of the correspondence between addition and multiplication because what they know about *repeated addition* will help them learn exponents as *repeated multiplication* as we go forward.

- The little $5$ we write is called an exponent and is written as a superscript. The numeral $5$ is written only half as tall and half as wide as the $10$, and the bottom of the $5$ should be halfway up the number $10$. The top of the $5$ can extend a little higher than the top of the zero in $10$. Why do you think we write exponents so carefully?
  - *It reduces the chance that a reader will confuse* $10^5$ *with* $105$*.*

*Scaffolding:*

When teaching students how to write an exponent as a **super**script, compare and contrast the notation with how to write a **sub**script, as in the molecular formula for water, $H_2O$, or carbon dioxide, $CO_2$. Here the number is again half as tall as the capital letters, and the top of the 2 is halfway down it. The bottom of the subscript can extend a little lower than the bottom of the letter. Ignore the meaning of a chemical subscript.

### Examples 1–5 (5 minutes)

Work through Examples 1–5 as a group, supplement with additional examples if needed.

**Examples 1–5**

**Example 1**

$5 \times 5 \times 5 \times 5 \times 5 = 5^5$

**Example 2**

$2 \times 2 \times 2 \times 2 = 2^4$

**Example 3**

$8^3 = 8 \times 8 \times 8$

**Example 4**

$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

**Example 5**

$g^3 = g \times g \times g$

- The repeated factor is called the **base** and the exponent is also called the **power**. Say the numbers in examples 1–5 to a partner.

Check to make sure students read the examples correctly:

  - *Five to the sixth power, two to the fourth power, eight to the third power, ten to the sixth power, and g to the third power.*

**Go back to Examples 1–4 and use a calculator to evaluate the expressions.**

**Example 1**

$5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 3,125$

**Example 2**

$2 \times 2 \times 2 \times 2 = 2^4 = 16$

**Example 3**

$8^3 = 8 \times 8 \times 8 = 512$

**Example 4**

$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$

**What is the difference between $3g$ and $g^3$?**

$3g = g + g + g$ *or 3 times* $g$ ; $g^3 = g \times g \times g$

Take time to clarify this important distinction.

- The base number can also be written in decimal or fraction form. Try Examples 6, 7, and 8. Use a calculator to evaluate the expressions.

## Example 6–8 (4 minutes)

**Example 6**

$3.8^4 = 3.8 \times 3.8 \times 3.8 \times 3.8 = 208.5136$

**Example 7**

$2.1 \times 2.1 = 2.1^2 = 4.41$

**Example 8**

$0.75 \times 0.75 \times 0.75 = (0.75)^3 = 0.421875$

**The base number can also be a fraction. Convert the decimals to fractions in Examples 7 and 8 and evaluate. Leave your answer as a fraction. Remember how to multiply fractions!**

*Note to teacher:*

If students need additional help multiplying fractions, refer to the first four modules of Grade 5.

## Example 9–10 (1 minute)

| Example 9 | Example 10 |
|---|---|
| $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\left(\frac{1}{2}\right)^3=\frac{1}{8}$ | $\left(\frac{2}{3}\right)^2=\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$ |

- There is a special name for numbers raised to the second power. When a number is raised to the second power, it is called *squared*. Remember that in geometry, squares have the same two dimensions: length and width. For $b > 0$, $b^2$ is the area of a square with side length $b$.
- What is the value of 5 squared?
  - 25
- What is the value of 7 squared?
  - 49
- What is the value of 8 squared?
  - 64
- What is the value of 1 squared?
  - 1

> *Note to Teacher:*
>
> If students ask about exponent values of $n$ that are not positive integers, let them know that positive and negative fractional exponents will be introduced in Algebra II and that negative *integer* exponents will be discussed in eighth grade.

A multiplication chart is included at the end of this lesson. Post or project it as needed.

- Where are square numbers found on the multiplication table?
  - *On the diagonal.*
- There is also a special name for numbers raised to the third power. When a number is raised to the third power, it is called *cubed*. Remember that in geometry, cubes have the same three dimensions: length, width, and height. For $b > 0$, $b^3$ is the volume of a cube with edge length $b$.
- What is the value of 1 cubed?
  - $1 \times 1 \times 1 = 1$
- What is the value of 2 cubed?
  - $2 \times 2 \times 2 = 8$
- What is the value of 3 cubed?
  - $3 \times 3 \times 3 = 27$

MP.6

- In general, for any number $x$, $x^1 = x$, and for any positive integer $n > 1$, $x^n$ is by definition,

  $x^n = \underbrace{(x \cdot x \cdots x)}_{n\ times}$.
- What does the $x$ represent in this expression?
  - *The $x$ represents the factor that will be repeatedly multiplied by itself.*
- What does the $n$ represent in this expression?
  - *$n$ represents the number of times $x$ will be multiplied.*
- Let's look at this with some numbers. How would we represent $4^n$?
  - $4^n = \underbrace{(4 \cdot 4 \cdots 4)}_{n\ times}$

MP.6

- What does the $4$ represent in this expression?
  - The $4$ represents the factor that will be repeatedly multiplied by itself.
- What does the $n$ represent in this expression?
  - *$n$ represents the number of times $4$ will be multiplied.*
- What if we were simply multiplying? How would we represent $4n$?
  - *Because multiplication is repeated addition,* $4n = \underbrace{(4 + 4 \cdots 4)}_{n\ times}$
- What does the $4$ represent in this expression?
  - The $4$ represents the addend that will be repeatedly added to itself.
- What does the $n$ represent in this expression?
  - *$n$ represents the number of times $4$ will be added.*

### Exercise 1–4 (8 minutes)

Ask students to fill in the chart, supplying the missing expressions.

**Exercises 1–4**

**1. Fill in the missing expressions for each row. For whole number and decimal bases, use a calculator to find the standard form of the number. For fraction bases, leave your answer as a fraction.**

***Sample solutions are in italics.***

| Exponential Form | Written as a multiplication expression having repeated factors | Standard Form |
|---|---|---|
| $3^2$ | $3 \times 3$ | $9$ |
| $2^6$ | $2 \times 2 \times 2 \times 2 \times 2 \times 2$ | $64$ |
| $4^5$ | $4 \times 4 \times 4 \times 4 \times 4$ | $1,024$ |
| $\left(\frac{3}{4}\right)^2$ | $\frac{3}{4} \times \frac{3}{4}$ | $\frac{9}{16}$ |
| $1.5^2$ | $1.5 \times 1.5$ | $2.25$ |

**2. Write "five cubed" in all three forms: exponential form, written as a series of products, standard form.**

$5^3$ ; $5 \times 5 \times 5$ ; $125$

**3. Write "fourteen and seven tenths squared" in all three forms.**

$14.7^2$ ; $14.7 \times 14.7$ ; $216.09$

**4. One student thought two to the third power was equal to six. What mistake do you think they made and how would you help them fix their mistake?**

*The student multiplied the base (2) by the exponent (3). This is wrong because the exponent never multiplies bases; the exponent tells how many copies of the base are to be used as factors.*

### Closing (2 minutes)

- We use multiplication as a quicker way to do repeated addition if the addends are the same. We use exponents as a quicker way to multiply if the factors are the same.
- Carefully write exponents as superscripts to avoid confusion.

> Lesson Summary
>
> **Exponential Notation for Whole Number Exponents: Let $m$ be a non-zero whole number. For any number $a$, the expression $a^m$ is the product of $m$ factors of $a$, i.e.,**
>
> $$a^m = \underbrace{a \cdot a \cdot \cdots \cdot a}_{m\ times}.$$
>
> **The number $a$ is called the *base,* and $m$ is called the *exponent* or *power* of $a$.**
>
> **When $m$ is $1$, "the product of one factor of $a$" just means $a$, i.e., $a^1 = a$. Raising any non-zero number $a$ to the power of $0$ is defined to be $1$, i.e., $a^0 = 1$ for all $a \neq 0$.**

### Exit Ticket (3 minutes)

Name ________________________________________________ Date____________________

# Lesson 5: Exponents

## Exit Ticket

1. What is the difference between $6z$ and $z^6$?

2. Write $10^3$ as a multiplication expression having repeated factors.

3. Write $8 \times 8 \times 8 \times 8$ using exponents.

## Exit Ticket Sample Solutions

1. What is the difference between $6z$ and $z^6$?

   $6z = z + z + z + z + z + z$ *or* 6 *times* $z$ ; $z^6 = z \times z \times z \times z \times z \times z$

2. Write $10^3$ as a series of products.

   $10 \times 10 \times 10$

3. Write $8 \times 8 \times 8 \times 8$ using an exponent.

   $8^4$

## Problem Set Sample Solutions

1. Complete the table by filling in the blank cells. Use a calculator when needed.

| Exponential Form | Written as a series of products | Standard Form |
|---|---|---|
| $3^5$ | $3 \times 3 \times 3 \times 3 \times 3$ | $243$ |
| $4^3$ | $4 \times 4 \times 4$ | $64$ |
| $1.9^2$ | $1.9 \times 1.9$ | $3.61$ |
| $\left(\frac{1}{2}\right)^5$ | $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ | $\frac{1}{32}$ |

2. Why do whole numbers raised to an exponent get greater while fractions raised to an exponent get smaller?

   *As whole numbers are multiplied by themselves, products are larger because there are more groups. As fractions of fractions are taken, the product is smaller. A part of a part is less than how much we started with.*

3. The powers of $2$ that are in the range $2$ through $1,000$ are $2, 4, 8, 16, 32, 64, 128, 256$, and $512$. Find all the powers of $3$ that are in the range $3$ through $1,000$.

   $3, 9, 27, 81, 243, 729$

4. Find all the powers of $4$ in the range $4$ through $1,000$.

   $4, 16, 64, 256$

5. Write an equivalent expression for $n \times a$ using only addition.

   $\underbrace{(a + a + \cdots a)}_{n \text{ times}}$

6. Write an equivalent expression for $w^b$ using only multiplication.

   $w^b = \underbrace{(w \cdot w \cdots w)}_{b \text{ times}}$

a. **Explain what $w$ is in this new expression.**

*$w$ is the factor that will be repeatedly multiplied by itself.*

b. **Explain what $b$ is in this new expression.**

*$b$ is the number of times $w$ will be multiplied.*

7. **What are the advantages to using exponential notation?**

*It is a shorthand way of writing a multiplication expression if the factors are all the same.*

8. **What is the difference between $4x$ and $x^4$? Evaluate both of these expressions when $x = 2$.**

*$4x$ means four times $x$, this is the same as $x + x + x + x$. On the other hand, $x^4$ means $x$ to the fourth power, or $x \times x \times x \times x$.*

*When $x = 2$, $4x = 4 \times 2 = 8$*

*When $x = 2$, $x^4 = 2 \times 2 \times 2 \times 2 = 16$*

## White Board Exchange: Multiplication of Decimals

| Progression of Exercises: | Answers: |
|---|---|
| 1. $0.5 \times 0.5$ | $0.25$ |
| 2. $0.6 \times 0.6$ | $0.36$ |
| 3. $0.7 \times 0.7$ | $0.49$ |
| 4. $0.5 \times 0.6$ | $0.3$ |
| 5. $1.5 \times 1.5$ | $2.25$ |
| 6. $2.5 \times 2.5$ | $6.25$ |
| 7. $0.25 \times 0.25$ | $0.0625$ |
| 8. $0.1 \times 0.1$ | $0.01$ |
| 9. $0.1 \times 123.4$ | $12.34$ |
| 10. $0.01 \times 123.4$ | $1.234$ |

Fluency work such as this exercise should take 5–12 minutes of class.

**How to Conduct a White Board Exchange:**

All students will need a personal white board, white board marker, and a means of erasing their work. An economical recommendation is to place card stock inside sheet protectors to use as the personal white boards and to cut sheets of felt into small squares to use as erasers.

It is best to prepare the problems in a way that allows you to reveal them to the class one at a time. For example, use a flip chart or PowerPoint presentation; write the problems on the board and cover with paper beforehand, allowing you to reveal one at a time; or, write only one problem on the board at a time. If the number of digits in the problem is very low (e.g., 12 divided by 3), it may also be appropriate to verbally call out the problem to the students.

The teacher reveals or says the first problem in the list and announces, "Go." Students work the problem on their personal white boards, holding their answers up for the teacher to see as soon as they have them ready. The teacher gives immediate feedback to each student, pointing and/or making eye contact with the student and responding with an affirmation for correct work such as, "Good job!", "Yes!", or "Correct!" For incorrect work, respond with guidance such as "Look again!", "Try again!", or "Check your work!"

If many students have struggled to get the answer correct, go through the solution of that problem as a class before moving on to the next problem in the sequence. Fluency in the skill has been established when the class is able to go through each problem in quick succession without pausing to go through the solution of each problem individually. If only one or two students have not been able to get a given problem correct when the rest of the students are finished, it is appropriate to move the class forward to the next problem without further delay; in this case, find a time to provide remediation to that student before the next fluency exercise on this skill is given.

# Lesson 6: Order of Operations

## Student Outcomes

- Students evaluate numerical expressions. They recognize that in the absence of parentheses, exponents are evaluated first.

## Classwork

### Opening (5 minutes)

Take a few minutes to review the problem set from the previous lesson. Clarify any misconceptions about the use and evaluation of exponents.

### Opening Exercise (5 minutes)

Post the following expression on the board and ask students to evaluate it:

$$3 + 4 \times 2$$

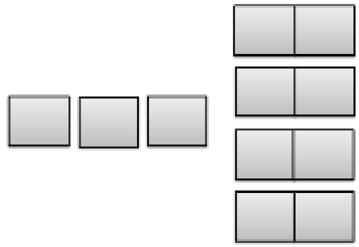
Ask students to record their answer and report it using individual whiteboards, cards, or electronic vote devices, etc. Students who arrive at an answer other than $11$ or $14$ should recheck their work.

### Discussion (5 minutes)

- How could you evaluate the expression $3 + 4 \times 2$ ?
  - $3 + 4$ *could be added first for a sum of* $7$*; then,* $7 \times 2 = 14$.
  - $4 \times 2$ *could be multiplied first for a product of* $8$*; then,* $8 + 3 = 11$.
- Only one of these can be correct. When we evaluate expressions, we must agree to use one set of rules so that everyone arrives at the same correct answer.
- During the last lesson, we said that addition was a shortcut to "counting on."

  How could you think about subtraction?
  - *Subtraction is a shortcut to "counting back."*
- These were the first operations that you learned because they are the least complicated. Next, you learned about multiplication and division.
- Multiplication can be thought of as repeated addition. Thinking back on Lesson 4, how could you think about division?
  - *Division is repeated subtraction.*
- Multiplication and division are more powerful than addition and subtraction, which led mathematicians to develop the order of operations in this way. When we evaluate expressions that have any of these four operations, we always calculate multiplication and division before doing any addition or subtraction. Since multiplication and division are equally powerful, we simply evaluate these two operations as they are written in the expression, from left to right.

- Addition and subtraction are at the same level in the order of operations and are evaluated from left to right in an expression. Now that these rules of Order of Operations are clear, can you go back and evaluate the expression $3 + 4 \times 2$ as $11$?



- The diagram correctly models the expression $3 + 4 \times 2$.
- With addition we are finding the sum of two addends. In this example the first addend is the number $3$. The second addend happens to be the number that is the value of the expression $4 \times 2$, so before we can add we must determine the value of the second addend.

## Example 1 (5 minutes): Expressions with Only Addition, Subtraction, Multiplication, and Division

**Example 1: Expressions with Only Addition, Subtraction, Multiplication, and Division**

**What operations are evaluated first?**

***Multiplication and division, from left to right.***

**What operations are always evaluated last?**

***Addition and subtraction, from left to right.***

Ask students to evaluate the expressions.

### Exercises 1–3

**Exercises**

1. $4 + 2 \times 7$

   $4 + 14$

   $18$

2. $36 \div 3 \times 4$

   $12 \times 4$

   $48$

3. $20 - 5 \times 2$

   $20 - 10$

   $10$

- In the last lesson, you learned about exponents, which are a way of writing repeated multiplication. So, exponents are more powerful than multiplication or division. If exponents are present in an expression, they are evaluated before any multiplication or division.
- When we evaluate expressions, we must agree to use one set of rules so that everyone arrives at the same correct answer. These rules are based on doing the most powerful operations first (exponents), then the less powerful ones (multiplication and division, going from left to right), and finally the least powerful ones last (addition and subtraction, going from left to right).
- Evaluate the expression $4 + 6 \times 6 \div 8$.
  - $4 + (6 \times 6) \div 8$
  - $4 + (36 \div 8)$
  - $4 + 4.5$
  - $8.5$
- Now evaluate the expression $4 + 6^2 \div 8$.
  - $4 + 36 \div 8$
  - $4 + (36 \div 8)$
  - $4 + 4.5$
  - $8.5$
- Why was your first step to find the value of $6^2$?
  - *Because exponents are evaluated first.*

## Example 2 (5 minutes): Expressions with Four Operations and Exponents

Display the following expression.

> *Scaffolding:*
> Some students may benefit from rewriting the expression on successive lines, evaluating only one or two operations on each line.

**Example 2: Expressions with Four Operations and Exponents**

$$4 + 9^2 \div 3 \times 2 - 2$$

**What operation is evaluated first?**

*Exponents ( $9^2 = 9 \times 9 = 81$ )*

**What operations are evaluated next?**

*Multiplication and division, from left to right ( $81 \div 3 = 27$ ; $27 \times 2 = 54$ )*

**What operations are always evaluated last?**

*Addition and subtraction, from left to right ( $4 + 54 = 58$; $58 - 2 = 56$ )*

**What is the final answer?**

$56$

- Evaluate the next two Exercises.

While the answers are provided, it is extremely important to circulate to ensure that students are using the correct order of operations to achieve the answer. For example, in Exercise 5, they should show $4^3$ first, followed by $2 \times 8$.

**Exercises 4–5**

**Exercises 4–5**

4. $90 - 5^2 \times 3$

   $15$

5. $4^3 + 2 \times 8$

   $80$

## Example 3 (5 minutes): Expressions with Parentheses

- The last important rule in the order of operations involves grouping symbols (usually parentheses). These tell us that in certain circumstances or scenarios, we need to do things out of the usual order. Operations inside grouping are always evaluated first, before exponents.

**Example 3: Expressions with Parentheses**

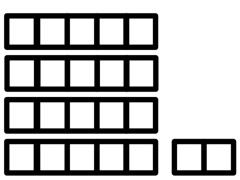
**Consider a family of $4$ that goes to a soccer game. Tickets are $\$5.00$ each. The mom also buys a soft drink for $\$2.00$. How would you write this expression?**

$4 \times 5 + 2$

**How much will this outing cost?**

$\$22$

- Here is a model of the scenario:



**Consider a different scenario: The family goes to the game like before, but each of the family members wants a drink. How would you write this expression?**

$4 \times (5 + 2)$

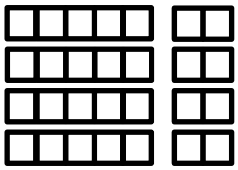
**Why would you add the $5$ and $2$ first?**

***Because we need to determine how much each of them are spending. Each one spends $\$7$, then we multiply by $4$ people to figure out the total cost.***

**How much will this outing cost?**

$\$28$

- Here is a model of the second scenario:



> **How many groups are there?**
>
> $4$
>
> **What is each group comprised of?**
>
> $\$5 + \$2$ *or* $\$7$

- The last complication that can arise is that if two or more sets of parentheses are ever needed, evaluate the innermost parentheses first, and work outward.
- Try Exercises 6 and 7.

### Exercises 6–7

> **Exercises 6–7**
>
> **6.** $2 + (9^2 - 4)$
>
> $79$
>
> **7.** $2 \cdot \big(13 + 5 - 14 \div (3 + 4)\big)$
>
> $32$

If students are confused trying to divide $14$ by $3$, reiterate the rule about nested parentheses.

## Example 4 (5 minutes): Expressions with Parentheses and Exponents

- Let's take a look at how parentheses and exponents work together. Sometimes a problem will have parentheses and the values inside the parentheses have an exponent. Let's evaluate this expression:

Place the expression on the board:

- We will evaluate the parentheses first.

> **Example 4: Expressions with Parentheses and Exponents**
>
> $$2 \times (3 + 4^2)$$

**Which value will we evaluate first within the parentheses?**

*First, evaluate $4^2$ which is sixteen, then add three. The value of the parentheses is nineteen.*

$$2 \times (3 + 4^2)$$
$$2 \times (3 + 16)$$
$$2 \times 19$$

**Evaluate the rest of the expression.**

$$2 \times 19 = 38$$

Place the expression on the board:

**What do you think will happen when the exponent in this expression is outside of the parentheses?**

$$2 \times (3 + 4)^2$$

**Will the answer be the same?**

*Answers will vary*

**Which should we evaluate first? Evaluate.**

*Parentheses*

$$2 \times (3 + 4)^2$$
$$2 \times (7)^2$$

**What happened differently here than in our last example?**

*The four was not raised to the second power because it did not have an exponent. We simply added the values inside the parentheses.*

**What should our next step be?**

*We need to evaluate the exponent next.*

$$7^2 = 7 \times 7 = 49$$

**Evaluate to find the final answer.**

$$2 \times 49$$
$$98$$

**What do you notice about the two answers?**

*The final answers were not the same.*

**What was different between the two expressions?**

*Answers may vary. In the first problem, a value inside the parentheses had an exponent and that value was evaluated first because it was inside of the parentheses. In the second problem, the exponent was outside of the parentheses and we evaluated what was in the parentheses first, and then raised that value to the power of the exponent.*

**What conclusions can you draw about evaluating expressions with parentheses and exponents?**

***Answers may vary. Regardless of where the exponent is in the expression, evaluate the parentheses first. Sometimes there will be values with exponents inside the parentheses. If the exponent is outside the parentheses, evaluate the parentheses first, and then evaluate to the power of the exponent.***

- Try Exercises 8 and 9.

## Exercises 8–9

**Exercises 8–9**

**8.** $7 + (12 - 3^2)$

$10$

**9.** $7 + (12 - 3)^2$

$88$

## Closing (5 minutes)

- When we evaluate expressions, we use one set of rules so that everyone arrives at the same correct answer. Grouping symbols like parentheses tell us to evaluate whatever is inside them before moving on. These rules are based on doing the most powerful operations first (exponents), then the less powerful ones (multiplication and division, going from left to right), and finally the least powerful ones last (addition and subtraction, going from left to right).

### Lesson Summary

**Numerical Expression: A *numerical expression* is a number, or it is any combination of sums, differences, products or divisions of numbers that evaluates to a number.**

**Statements like, "$3 +$" or "$3 \div 0$," are not numerical expressions because neither represents a point on the number line. Note: Raising numbers to whole number powers are considered numerical expressions as well, since the operation is just an abbreviated form of multiplication, e.g., $2^3 = 2 \cdot 2 \cdot 2$.**

**Value of a Numerical Expression: The *value* of a numerical expression is the number found by evaluating the expression.**

**For example: $\frac{1}{3} \cdot (2 + 4) + 7$ is a numerical expression and its value is $9$.**

Note: Please do not stress words over meaning here. It is okay to talk about the "number computed," "computation," "calculation," etc. to refer to the value as well.

## Exit Ticket (5 minutes)

Name ______________________________ Date ______________

# Lesson 6: Order of Operations

## Exit Ticket

1. Evaluate this expression: $39 \div (2 + 1) - 2 \times (4 + 1)$

2. Evaluate this expression: $12 \times (3 + 2^2) \div 2 - 10$

3. Evaluate this expression: $12 \times (3 + 2)^2 \div 2 - 10$

## Exit Ticket Sample Solutions

1. Evaluate this expression: $39 \div (2+1) - 2 \times (4+1)$

   $3$

2. Evaluate this expression: $12 \times (3+2^2) \div 2 - 10$

   $32$

3. Evaluate this expression: $12 \times (3+2)^2 \div 2 - 10$

   $140$

## Problem Set Sample Solutions

Evaluate each expression:

1. $3 \times 5 + 2 \times 8 + 2$

   $33$

2. $(\$1.75 + 2 \times \$0.25 + 5 \times \$0.05) \times 24$

   $\$60.00$

3. $(2 \times 6) + (8 \times 4) + 1$

   $45$

4. $\big((8 \times 1.95) + (3 \times 2.95) + 10.95\big) \times 1.06$

   $37.524$

5. $\big((12 \div 3)^2 - (18 \div 3^2)\big) \times (4 \div 2)$

   $28$