Topic D:
Ratios of Scale Drawings

7.RP.2b, 7.G.1

Focus Standard:

7.RP.2 Recognize and represent proportional relationships between quantities.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Days: 7

Lesson 16: Relating Scale Drawings to Ratios and Rates (E)
Lesson 17: The Unit Rate as the Scale Factor (P)
Lesson 18: Computing Actual Lengths from a Scale Drawing (P)
Lesson 19: Computing Actual Areas from a Scale Drawing (P)
Lesson 20: An Exercise in Creating a Scale Drawing (E)
Lessons 21–22: An Exercise in Changing Scales (E)

In the first lesson of Topic D, students are introduced to scale drawings; they determine if the drawing is a reduction or enlargement of a two-dimensional picture. Pairs of figures are presented for students to match corresponding points. In Lesson 17, students learn the term scale factor and recognize it as the constant of proportionality. With a given scale factor, students make scale drawings of pictures or diagrams. In Lessons 18 and 19, students compute the actual dimensions of objects shown in pictures given the scale factor. They recognize that areas scale by the square of the scale factor that relates lengths. In the final lessons, students engage in their own scale factor projects—first, to produce a scale drawing of the top-view of a furnished room or building, and second, given one scale drawing, to produce new scale drawing using a different scale factor.

¹ Lesson Structure Key: P-Problem Set Less, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson
Lesson 16: Relating Scale Drawings to Ratios and Rates

Student Outcomes

- Students understand that a scale drawing is either the reduction or the enlargement of a two-dimensional picture.
- Students compare the scale drawing picture with the original picture and determine if the scale drawing is a reduction or an enlargement.
- Students match points and figures in one picture with points and figures in the other picture.

Classwork

Intro Activity (3 minutes): Can you guess the image?

Use the attached Intro Activity pages to show students a series of images to see if they can guess what is pictured and then identify whether it is a reduction or enlargement of the original image. The purpose of this activity is for students to get an understanding of the terms reduction and enlargement. The scale drawings produced in grade 7 will focus on creating a scale drawing from a two-dimensional picture. Teachers could also post alternate images of choice on a projector or interactive whiteboard where only one portion is revealed. Students need to guess the object and whether it is a reduction or enlargement of the actual object.

Give students 2 minutes to try to guess each image in the student pages and share responses (see responses to the right). Then show the full size images and have them decide whether the images in the student pages are reductions or enlargements compared to what is now being shown.

Responses for attached images and points for discussion follow.

- It is picture of a subway map. Was the cropped photo that was just seen a reduction or an enlargement of the original picture below? How do you know?
  - Reduction, since it was a scaled down picture of a map of a subway. If you compare the length from one end of a track to the other end, you can see that the cropped photo has a shorter length as compared to the original photo. Any corresponding length could be compared.

- It is a fingerprint. Was the cropped photo that was just seen a reduction or an enlargement of the original picture below? How do you know?
  - Enlargement, since it was from a picture of a fingerprint. If you compare the length of one of the swirls to the actual fingerprint picture, you can see that the cropped photo has a longer length compared to the original fingerprint picture.
This is a reduction of a subway map.

This is an enlarged picture of a fingerprint.

Example 1 (3 minutes)

For each scale drawing, have students identify if it is a reduction or an enlargement to the actual object in real life or to the given original picture. Discuss:

- What are possible uses for enlarged drawings/pictures?
  - Enlarged drawings are good to observe details such as textures and parts that are hard to see to the naked eye. In art, enlargements are used in murals or portraits.

- What are the possible purposes of reduced drawings/pictures?
  - Reductions are purposeful to get a general idea of a picture/object. These scale drawings can fit in folders, books, wallets, etc.

Read over the “Key Idea” with the class and introduce the term “scale drawing”. Emphasize the importance of scale drawings being reductions or enlargements of two-dimensional drawings, not of actual objects.

For the following problems, A is the actual picture and B is the scale drawing. Is the scale drawing an enlargement or a reduction of the actual picture?

1. a. Enlargement  b. Reduction
Example 2 (7 minutes)

Complete this activity together as a class. Discuss:

- Why doesn’t point V correspond with point R?
  - Although both points are on the bottom right hand corner, they are positioned on two different ends of the path. Point V only corresponds to Point W.

- What must we consider before identifying correspond points?
  - We have to make sure we are looking at the maps in the same direction. Then we can see that this is a one-to-one correspondence because they are scale drawings of each other and each point corresponds to one specific point on the other map.

Derek's family took a day trip to a modern public garden. Derek looked at his map of the park that was a reduction of the map located at the garden entrance. The dots represent the placement of rare plants. The diagram below is the top-view as Derek held his map while looking at the posted map.

What are the corresponding points of the scale drawings of the maps?
- Point A to: **Point R**
- Point V to: **Point W**
- Point H to: **Point P**
- Point Y to: **Point N**

Exercise 2 (10 minutes)

In this Exercise, the size of the units on the grid are enlarged, then reduced, to produce two different scale drawings with lengths that are proportional to one another. Guide students to notice that the number of units of length is staying the same but the size of each unit changes from one drawing to another, due to the shrinking and enlarging of the grid. This allows for students to create a scale drawing without having to complete the computation required in finding equivalent ratios (which will be done later in Topic D).

- How will we make the enlarged robot? Will we need to adjust the number of units?
  - No, since the grid is enlarged (thus changing the size of each unit), we can draw the new robot using the same number of units for each given length.

- What is the importance of matching corresponding points and figures from the actual picture to the scale drawing (the mural piece)?
  - The scale drawing will not be proportional and the picture will be distorted.

- How can you check the accuracy of the proportions?
  - You can count the squares and check that the points match.
Create scale drawings of your own modern nesting robots using the grids provided.

Example 3 (7 minutes)

Work on the problem as a class and fill in the table together. Discuss as students record important points in the “Notes” section:

- Is the second image a reduction or enlargement of the first image? How do you know?
  - It is a reduction because the second image is smaller than the first, original image.

- What do you notice about the information on the table?
  - The pairs of corresponding lengths are all proportional.

- Does a constant of proportionality exist? How do you know?
  - Yes, because there is a constant value to get from each length to its corresponding length.

- What is the constant of proportionality and why is it important in scale drawings?
  - The constant of proportionality is 3 and it needs to exist for images to be considered scale drawings. If not, then there would be a lack of proportionality and the images would not be a scaled up or down version of the original image.

<table>
<thead>
<tr>
<th>Lengths of the original drawing</th>
<th>18 units</th>
<th>6 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lengths of the second drawing</td>
<td>9 units</td>
<td>3 units</td>
</tr>
</tbody>
</table>
Exercise 3 (7 minutes)

Have students work in pairs to fill out the table and answer questions. Direct students to the vertical and horizontal lengths of the legs. Reconvene as a class to discuss answers to the given questions and the following:

- Why is it difficult to determine if the second image is a reduction or an enlargement of the first image?
  - The second image is not a scale drawing of the first image so even though it is bigger it is not a true reduction or enlargement of the first image.

- What must one check before determining if one image is a scale drawing of another?
  - The corresponding lengths must all be proportional to each other. If only one pair is proportional and another is not, then the images cannot be identified as scale drawings of one another.

| Lengths of the original image | 5 units | 3 units |
| Lengths of the second image  | 15 units | 10 units |

a. Does a constant of proportionality exist? If so, what is it? If not, explain.
   No, because the ratios of corresponding side lengths are not equal, or proportional to each other.

b. Is Luca’s enlarged mosaic a scale drawing of the first image? Why or why not?
   No, the enlarged mosaic image is not a scale drawing of the first image. We know this because the images do not have all side lengths proportional to each other; there is no constant of proportionality.

Closing (3 minutes)

- What is a scale drawing?
  - It is a drawing that is a reduction or enlargement of an actual picture.

- What is an enlargement? A reduction?
  - A drawing that is larger in scale than its original picture. A drawing that is smaller in scale than its original picture.

- What’s the importance of matching points and figures from one picture/drawing to the next?
  - The corresponding lines, points and figures need to match because otherwise the scale drawing will be distorted and not proportional throughout.

- How do scale drawings related to rates and ratios?
  - The corresponding lengths between scale drawings and original images are equivalent ratios.
Lesson Summary:

**Scale Drawing:** A drawing in which all lengths between points or figures in the drawing are reduced or enlarged proportional to the lengths in the actual picture. A constant of proportionality exists between corresponding lengths of the two images.

**Reduction:** The lengths in the scale drawing are smaller than those in the actual object or picture.

**Enlargement/Magnification:** The lengths in the scale drawing are larger than those in the actual object or picture.

**One-to-one Correspondence:** Each point in one figure corresponds to one and only one point in the second figure.

Exit Ticket (5 minutes)
Lesson 16: Relating Scale Drawings to Ratios and Rates

Exit Ticket

Use the following figure on the graph for problems 1 and 2.

1.  
   a. If the original lengths are multiplied by 2, what are the new coordinates?

   b. Use the table to organize lengths.

<table>
<thead>
<tr>
<th>Actual Picture Lengths (in units)</th>
<th>New Picture Lengths (in units)</th>
</tr>
</thead>
</table>

   c. Is the new picture a reduction or an enlargement?

   d. What is the constant of proportionality?
2. If the original lengths are multiplied by \( \frac{1}{3} \), what are the new coordinates?

b. Use the table to organize the lengths.

<table>
<thead>
<tr>
<th>Actual Picture Lengths (in units)</th>
<th>New Picture Lengths (in units)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. Is the new picture a reduction or an enlargement?

d. What is the constant of proportionality?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. If the original lengths are multiplied by 2, what are the new coordinates?

   \((0,0), (12,18), (12,0)\)

   a. Use the table to organize lengths (the vertical and horizontal legs).

<table>
<thead>
<tr>
<th>Actual Picture</th>
<th>New Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lengths (in units)</td>
<td>6 units</td>
</tr>
<tr>
<td>Lengths (in units)</td>
<td>12 units</td>
</tr>
</tbody>
</table>

   c. Is the new drawing a reduction or an enlargement?

   *Enlargement*

   d. What is the constant of proportionality?

   2

2. If the original lengths are multiplied by \(\frac{1}{3}\), what are the new coordinates?

   \((0,0), (2,3), (2,0)\)

   a. Use the table to organize lengths (the vertical and horizontal legs).

<table>
<thead>
<tr>
<th>Actual Picture</th>
<th>New Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lengths (in units)</td>
<td>6 units</td>
</tr>
<tr>
<td>Lengths (in units)</td>
<td>2 units</td>
</tr>
</tbody>
</table>

   c. Is the new drawing a reduction or an enlargement?

   *Reduction*

   d. What is the constant of proportionality?

   \(\frac{1}{3}\)
Problem Set Sample Solutions

For Problems 1–3, identify if the scale drawing is a reduction or enlargement of the actual picture.

1. **Enlargement**
   a. Actual Picture  
   b. Scale Drawing

![Diamond Actual Picture](image1)

![Diamond Scale Drawing](image2)

2. **Reduction**
   a. Actual Picture
   b. Scale Drawing

![Trailer Actual Picture](image3)

![Trailer Scale Drawing](image4)
3. **Enlargement**
   a. Actual Picture
   b. Scale Drawing

4. Using the grid and the abstract picture of a face, answer the following questions:

   a. On the grid, where is the eye?
      *Intersection BG*
   b. What is located in DH?
      *Tip of the nose*
   c. In what part of the square BI is the chin located?
      *Bottom right corner*
5. Use the graph provided to decide if the rectangular cakes are scale drawings of each other.

Cake 1: (5,3), (5,5), (11,3), (11, 5)
Cake 2: (1,6), (1, 12), (13,12), (13, 6)

How do you know?

These images are not scale drawings of each other because the short length of cake 2 is three times longer than cake 1 but the longer length of cake 2 is only twice as long as cake 1. Both should either be twice as long or three times as long to have one-to-one correspondence and to be scale drawings of each other.
Lesson 16: Intro Activity

Can you guess the image? In each problem, the first image is from the student materials and the second image is the actual picture.

1. 

![Image 1]  

![Image 2]
2.
Lesson 17: The Unit Rate as the Scale Factor

Student Outcomes

- Students recognize that the enlarged or reduced distances in a scale drawing are proportional to the corresponding distance in the original picture.
- Students recognize the scale factor to be the constant of proportionality.
- Given a picture or description of geometric figures, students make a scale drawing with a given scale factor.

Classwork

Example 1 (7 minutes): Devon’s Icon

After reading the prompt with the class, discuss the following questions:

- What type of scale drawing is the sticker?
  - It is an enlargement or a magnification of the original sketch.
- What is the importance of proportionality for Rubin?
  - If the image is not proportional, it looks less professional. The image on the sticker will be distorted.
- How could we go about checking for proportionality of these two images?
  - Measure corresponding lengths and check to see if they all have the same constant of proportionality. (Have students record steps onto student pages)

As a class, label points on the original sketch and then on the sticker sketch correspondingly. Use inches to measure the distance between the points and record on a table.

Rubin created a simple game on his computer and shared it with his friends to play. They were instantly hooked and the popularity of his game spread so quickly that Rubin wanted to create a distinctive icon so players could easily identify his game. He drew a simple sketch. From the sketch, he created stickers to promote his game but Rubin wasn’t quite sure if the stickers were proportional to his original sketch.

<table>
<thead>
<tr>
<th>Original</th>
<th>Sticker</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in.</td>
<td>2 in.</td>
</tr>
<tr>
<td>$\frac{3}{4}$ in.</td>
<td>$\frac{1}{2}$ in.</td>
</tr>
<tr>
<td>1 in.</td>
<td>2 in.</td>
</tr>
<tr>
<td>$\frac{7}{8}$ in.</td>
<td>$\frac{3}{4}$ in.</td>
</tr>
</tbody>
</table>
Discuss:

- What relationship do you see between the measurements?
  - The corresponding lengths are proportional.
- Is the sticker proportional to the original sketch?
  - Yes, the sticker lengths are twice as long as the original sketch.
- How do you know?
  - The unit rate is the same for the corresponding measurements, 2.
- What is this called?
  - Constant of proportionality.
- Introduce the term “scale factor” and review the key idea box with students.
  - Is the new figure bigger or smaller than the original?
  - Bigger.
- What is the scale factor for the sticker as compared to the original sketch? How do you know?
  - Two because the scale factor is the constant of proportionality.
- Each of the corresponding lengths are how many times larger?
  - Two times.
- What can you predict about an image that has a scale factor of 3?
  - The lengths will be three times as long as the original.

**Scaffolding:**
For students with special needs, give the closed sentence: “The _____ of any two _____ lengths in two _____ figures. The scale factor corresponds to the _______ and the _______.”

**Key Idea:**
The scale factor can be calculated from the ratio of any length in the scale drawing to its corresponding length in the actual picture. The scale factor corresponds to the unit rate and the constant of proportionality.

Scaling by factors greater than 1 enlarge the segment and scaling by factors less than 1 reduce the segment.

**Steps to check for proportionality for scale drawing and original object/picture:**
1. Measure lengths of scale drawing. Record on table.
2. Measure corresponding lengths on actual picture/drawing. Record on table.
3. Check for constant of proportionality.
Exercise 1 (5 minutes)

Give students time to measure lengths (in inches) on the app icon that corresponds to the lengths measured in Example 1 and record on tables with partners. Discuss:

- What was the relationship between the sticker and the original sketch?
  - The sticker is bigger than the original.
- What was the constant of proportionality, or scale factor, for this relationship?
  - 2
- What is the relationship between the icon and the original sketch?
  - The icon is smaller than the original sketch.
- What was the constant of proportionality, or scale factor, for this relationship?
  - ½
- How do we determine the scale factor?
  - Measure lengths on the app icon and corresponding lengths on the original sketch and record. Find the constant of proportionality.
- What does the scale factor indicate?
  - A scale factor less than 1 indicates a reduction from the original picture and a scale factor greater than 1 indicates a magnification from the original picture.

Example 2 (7 minutes)

Begin this example by giving the scale factor 3. Demonstrate how to make a scale drawing with the scale factor. Use a table or equation to show how you computed your actual lengths. Discuss:

- Is this a reduction or an enlargement?
  - An enlargement.
- How could you determine even before the drawing?
  - A scale factor greater than one represents an enlargement.
- Can you predict what the scale lengths of the scale drawing will be?
  - Yes, they will be 3 times as big as the actual picture.
- What steps were used to create this scale drawing?
  - Measure lengths of the original drawing and record onto a table. Multiply by 3 to compute the scale drawing lengths. Record and draw.
PENDING FINAL EDITORIAL REVIEW

- How can you double check your work?
  - Divide the scale lengths by 3 to see if they match actual lengths.

Use a scale factor of 3 to create a scale drawing of the picture below.

Picture of the Flag of Columbia:

- A. \( \frac{1}{2} \times 3 = 4 \frac{1}{2} \text{ in.} \)
- B. \( 1 \times 3 = 3 \text{ in.} \)
- C. \( \frac{1}{2} \times 3 = 1 \frac{1}{2} \text{ in.} \)
- D. \( \frac{1}{4} \times 3 = \frac{3}{4} \text{ in.} \)

Exercise 2 (7 minutes)

Have students work with partners to create a scale drawing of the original picture of the flag from Example 2 but now applying a scale factor of 1/2.

- Is this a reduction or an enlargement?
  - A reduction because the scale factor is less than one.
- What steps were used to create this scale drawing?
  - Compute the scale drawing lengths by multiplying by 1/2 or dividing by 2. Record. Measure new segments with a ruler and draw.
Example 3 (5 minutes)

Describe the following: Your family recently took a family portrait. By request, your aunt wanted you to take a picture of the portrait from your phone and send it to her. If the original portrait is 3 feet by 3 feet and the scale factor is 1/18, draw the scale drawing that would be the size of the portrait on your phone. Discuss the questions:

- What is the shape of the portrait?
  - Square.

- Will the resulting picture be a reduction or a magnification?
  - Reduction because the phone picture is smaller than the original portrait. Also, the scale factor is less than one so this indicates a reduction.

- What needs to be done before multiplying the scale lengths by the scale factor?
  - The original portrait’s length must be converted to inches because the scale factor is notated assuming the units are the same.

- What will the scale drawing look like?
  - The scale drawing should be a square measuring 2 inches by 2 inches.

Your family recently had a family portrait taken. Your aunt asked you to take a picture of the portrait using your cell phone and send it to her. If the original portrait is 3 feet by 3 feet and the scale factor is 1/18, draw the scale drawing that would be the size of the portrait on your phone.

Sketch and notes:

\[
3 \times 12 = 36 \\
36 \times \frac{1}{18} = 2 \text{ in.}
\]
Exercise 3 (5 minutes)

Read the problem aloud and ask students to solve the problem with another student.

John is building his daughter a doll house that is a miniature model of their house. The front of their house has a circular window with a diameter of 5 feet. If the scale factor for the model house is \( \frac{1}{30} \), make a sketch of the circular doll house window.

- What is the diameter of the window in the sketch of the model house?
  - 2 inches.

\[
5 \times 12 = 60 \\
60 \times \frac{1}{30} = 2 \text{ in.}
\]

Closing Questions (5 minutes)

- Where is the constant of proportionality represented in scale drawings?
  - Scale Factor.

- What step(s) are used to calculate scale factors?
  - Measure the actual picture lengths and the scale drawing lengths. Write the values as a ratio and determine the constant of proportionality.

- What operation(s) is (are) used to create scale drawings?
  - After the lengths of the actual picture are measured and recorded, multiply each length by the scale factor to find corresponding scale drawing lengths. Measure and draw.

Exit Ticket (5 minutes)
Lesson 17: The Scale Factor for a Scale Drawing

Exit Ticket

A rectangular pool in your friend’s yard is 150 ft. x 400 ft. Create a scale drawing with a scale factor of 1/600. Use a table or equation to show how you computed your scale drawing lengths.
Exit Ticket Sample Solutions

A rectangular pool in your friend’s yard is 150 ft. x 400 ft. Create a scale drawing with a scale factor of 1/600. Use a table or equation to show how you computed your scale drawing lengths.

<table>
<thead>
<tr>
<th>Actual Length</th>
<th>Scale Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 ft</td>
<td>150 ft multiplied by 1/600 = 1/4 ft, or 3 inches</td>
</tr>
<tr>
<td>400 ft</td>
<td>400 ft multiplied by 1/600 = 2/3 ft, or 8 inches</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 in.</td>
</tr>
</tbody>
</table>

Problem Set Sample Solutions

1. Giovanni went to Los Angeles, California, for the summer to visit his cousins. He used a map of bus routes to get from the airport to the nearest bus station from his cousin’s house. The distance from the airport to the bus station is 56 km. On his map, the distance was 4 cm. What is the scale factor?

   The scale factor is 1/140,000. Note: you must change km to cm or cm to km or both to meters to determine the scale factor.

2. Nicole is running for school president and her best friend designed her campaign poster which measured 3 feet by 2 feet. Nicole liked the poster so much she reproduced the artwork on rectangular buttons measuring 2 inches by 1 ½ inches. What is the scale factor?

   The scale factor is 1/18

3. Use a ruler to measure and find the scale factor.

   Scale Factor: 5/3

<table>
<thead>
<tr>
<th>Actual</th>
<th>Scale Drawing</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA-batteri</td>
<td>AA-batteri</td>
</tr>
</tbody>
</table>
4. Find the scale factor using the given scale drawings and measurements below

   Scale Factor: \(\frac{1}{2}\)

   Actual Picture

   Scale Drawing

   24 cm

   6 cm

5. Create a scale drawing from the actual pictures in centimeters:

   a. Scale factor: 3

   b. Scale factor: \(\frac{3}{5}\)
6. Hayden likes building radio-controlled sailboats with her dad. One of the sails, shaped like a right triangle, has side lengths measuring 6 inches, 8 inches and 10 inches. To log her activity, Hayden creates and collects drawings of all the boats her and her dad built together. Using the scale factor of 1/4, draw a scale drawing of sail.

Scaffolding:

- Extension: Students can enlarge an image they want to draw or paint by drawing a grid using a ruler over their reference picture and drawing a grid of equal ratio on their work surface. Direct students to focus on one square at a time until the image is complete. Have students compute the scale factor for the drawing.
Lesson 18: Computing Actual Lengths from a Scale Drawing

Student Outcomes

- Given a scale drawing, students compute the lengths in the actual picture using the scale. Students identify the scale factor in order to make intuitive comparisons of size then devise a strategy for efficiently finding actual lengths using the scale.

Classwork

Example 1 (14 minutes): Basketball at Recess?

The first example has students build upon the previous lesson by applying scale factor to find missing dimensions. This leads into a discussion of whether this method is the most efficient and whether they could find another approach that would be simpler, as demonstrated in Example 2. Guide students to record responses and additional work in their student materials.

Based upon the picture, what are the actual dimensions that the half-court will be? Will the lot be big enough if its width is 25 feet and its length is 75 feet? Explain.

- How can we use the scale factor to determine the actual measurements?
  - Divide each drawing length by the scale factor to find the actual measurement. See table below.

- How can we use the scale factor to write an equation relating the scale drawing lengths to the actual lengths?
  - The scale factor is the constant of proportionality, or the $k$ in the equation $y = kx$ or $x = \frac{y}{k}$ or even $k = \frac{y}{x}$. It is the ratio of drawing length to actual length.

Vincent proposes an idea to the Student Government to install a basketball hoop along with a court marked with all the shooting lines and boundary lines at his school for students to use at recess. He presents a plan to install a half-court design as shown below. After checking with school administration, he is told it will be approved if it will fit on the empty lot that measures 25 feet by 75 feet on the school property. Will the lot be big enough for the court he planned? Explain.

Scaffolding:
A reduction has a scale factor less than 1 and an enlargement has a scale factor greater than 1.
Scale Drawing Lengths | 1 in | 2 in | 1 2/3 in  
--- | --- | --- | ---  
Actual Court Lengths | 15 ft. | 30 ft. | 25 ft.  

Scale Factor: 1 inch corresponds to (15•12) inches, or 180 inches, so the scale factor is 1/180. Let \( k = \frac{1}{180}, x = \text{actual length}, \ y = \text{scale drawing length} \)

To find Actual Length:  
\[
y = \frac{1}{180} x
\]
\[
2 = \frac{1}{180} x
\]
\[
x = 360 \text{ inches, or } 30 \text{ feet}
\]

To find Actual Width:  
\[
y = \frac{1}{180} x
\]
\[
1 \frac{2}{3} = \frac{1}{180} x
\]
\[
\frac{180}{1} \cdot \frac{5}{3} = x
\]
\[
x = 300 \text{ inches, or } 25 \text{ feet}
\]

The actual court measures 25 feet by 30 feet. Yes, the lot will be big enough for the court Vincent planned. The court will take up the entire width of the lot.

Example 2 (5 minutes)

Teacher can guide whole class through completion of exercises below while encouraging student participation through questioning. Student should record information in their materials.

Hold discussion with students regarding the use of the word scale.

- Where have you seen this term used?
  - Bottom of a map, blueprint, etc.
- It refers to a type of ratio. 1 cm represents 20 m is an example of a ratio relationship, and the ratio 1:20 is sometimes called a scale ratio or a scale. Why can’t we call this the scale factor?
  - The scale factor in a scaled drawing is always a scalar between distances measured in the same units.
- Do we always need to use find the scale factor in order to find actual measurements from a scale drawing or could we just use the given scale ratio (or scale)?
  - Take a few minutes to try to find the actual length of the garden. Give your answer in meters. When you are finished, I will ask you how you found your answer.

Allow for students to share approaches with the class. Students could calculate the scale factor and follow the steps from Example 1, or they may realize that it is not necessary to find scale factor. They may apply the scale ratio and work the problem using the ratio 1:20, perhaps setting up a proportion relationship \( y = \frac{1}{20} x \) where \( x \) represents the actual measurement and \( y \) represents the drawing length.

- So then, what two quantities does the constant of proportionality, \( k \), relate?
  - The drawing length to the actual length, when converted to the same units if scale factor is being used. If just the scale ratio is used, then the quantities do not need to be converted to the same units.
PENDING FINAL EDITORIAL REVIEW

- What method was more efficient? Why?
  - Allow for students to respond. If we apply the scale ratio, it requires fewer steps.
- Why would we bother talking about the scale factor then?
  - The scale factor gives us a sense of the comparison. In this example the scale factor is 1/2000 so the scale drawing lengths are 1/2000th of the actual lengths. It is not always easy to see that comparison when you are basing your calculations on the scale. The scale factor helps us reason through the problem and make sense of our results.
- Now, go back and find the actual width of the garden using the scale ratio.

Elicit responses from students, including an explanation of how they arrived at their answer. Record results on board for students to see and be sure students have recorded correct responses in their materials.

The diagram below represents a garden. The scale is 1 cm for every 20 meters of actual length. Each square in the drawing measures 1 cm by 1 cm.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm</td>
<td>8 cm</td>
<td>4 cm</td>
</tr>
<tr>
<td>20 m (or 2000 cm)</td>
<td>160 m (or 16000 cm)</td>
<td>80 m (or 8000 cm)</td>
</tr>
</tbody>
</table>

Method 1:
Using the given scale: 1 cm of scale drawing length corresponds to 20 m of actual length

\[ k = \frac{1}{20} \quad \text{drawing length to actual length} \]

To find actual length:

\[ y = \frac{1}{20} x \quad \text{where } x = \text{actual measurement in m and } y = \text{scale drawing measurement in cm} \]

8 = \frac{1}{20} x \quad \text{substitute scale drawing length in place of } y

x = 160

The actual length is 160 meters.

To find actual width:

Divide the actual length by 2 since its drawing width is half the length.

The actual width is 80 meters.
Method 2:
Use the scale factor: 1 cm of scale drawing length corresponds to 2000 cm of actual length

\[ k = \frac{1}{2000} \]  
\[ \text{drawing length to actual length (in same units)} \]

To find Actual Length: 
\[ y = \frac{1}{2000} x \]  
where \( x \) = actual measurement in cm and \( y \) = drawing measurement in cm

\[ 8 = \frac{1}{2000} x \]  
\[ y = 16,000 \]
\[ \]  

The actual length is 16,000 cm or 160 m.

To find Actual Width: 
\[ y = \frac{1}{2000} x \]  
\[ 4 = \frac{1}{2000} x \]  
\[ \text{substitute the scale drawing width in place of } y \]
\[ y = 8000 \]
\[ \]  

The actual width is 8,000 cm or 80 m.

Example 3 (10 minutes)

A graphic designer is creating an advertisement for a tablet. She needs to enlarge the picture given here so that 0.25 inches on the scale picture will correspond to 1 inch on the actual advertisement. What will be the length and width of the tablet on the advertisement?

Using a Table:

<table>
<thead>
<tr>
<th>Picture, ( y )</th>
<th>Scale</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 in</td>
<td>1 ½</td>
<td>1 1/8 in</td>
<td></td>
</tr>
<tr>
<td>Actual Advertisement, ( x )</td>
<td>1 in</td>
<td>6 in</td>
<td>4 ½ in</td>
</tr>
</tbody>
</table>

Using an Equation:

Find the constant of proportionality, \( k \): 
\[ k = \frac{0.25}{1} \]
\[ k = \frac{1}{4} \]  
\[ \text{(scale factor since units of measure are the same; it is a reduction)} \]

To find Actual Length: 
\[ y = \frac{1}{4} x \]  
where \( x \) = actual measurement and \( y \) = picture measurement

\[ 1 \frac{1}{2} = \frac{1}{4} x \]  
\[ \text{Substitute the picture length in place of } x. \]
\[ x = 6 \]  
\[ \text{Multiply both sides by the reciprocal.} \]

To find Actual Width: 
\[ y = \frac{1}{4} x \]  
\[ 1 \frac{1}{8} = \frac{1}{4} x \]  
\[ \text{Substitute the picture width in place of } y. \]
\[ x = 4 \frac{1}{2} \]  
\[ \text{Multiply both sides by the reciprocal.} \]

The tablet will be 6 inches by 4 ½ inches on the actual advertisement.
PENDING FINAL EDITORIAL REVIEW

- Is it always necessary to write and solve an equation $y = kx$ to find actual measurements?
  - Guide students to conclude that the actual measurement can be found by applying any of the three relationships: $y = kx$, $x = y/k$, or even $k = y/x$. Encourage students to try any of these approaches in the next exercise.

Exercises (10 minutes)

Hold a brief discussion of the problem as a class and identify how to find the answer. Guide students to identify the following big ideas to address as they solve the problem:

- We need to find the relationship between the lengths in the scale drawing and the corresponding actual lengths.
- Use this relationship to calculate the width of the actual mall entrance.
- Compare this with the width of the panels.

Allow time for students to measure and complete problem (see measurement on diagram below). Encourage students to check with elbow partner to ensure that their measurements match each other.

Sample responses shown below include work for two different approaches. Students do not need to apply both and shall receive credit for using either method.

1. Students from the high school are going to perform one of the acts from their upcoming musical at the atrium in the mall. The students want to bring some of the set with them so that the audience can get a better feel of the whole production. The backdrop that they want to bring has panels that measure 10 feet by 10 feet. The students are not sure if they will be able to fit these panels through the entrance of the mall since the panels need to be transported flat (horizontal). They obtain a copy of the mall floor plan, shown below, from the city planning office. Use this diagram to decide if the panels will fit through the entrance. Use a ruler to measure.

Scaffolding:

- Map distance of mall entrance could be noted so that students would not need to measure.
- When determining what unit to use when measuring, look at the given scale.
Answer the following questions.

a. Find the actual distance of the mall entrance and determine whether the set panels will fit.

   **Step 1: Relationship between lengths in drawing and lengths in actual**

   Scale: \( \frac{1}{8} \text{in.} \) \( \frac{4\frac{1}{2}}{ \text{feet}} \) or an equivalent ratio of \( \frac{1}{36} \) inches to feet

   **Scale factor calculations:**

   \[ \left( \frac{1}{8} \right) \times \left( \frac{54}{1} \right) = \frac{1}{432} \text{ a reduction} \]

   **Step 2: Find the actual distance of entrance**

   Use the given scale: \( \frac{3}{8} \times \frac{36}{1} \)

   \[ = 13 \frac{1}{2} \text{ feet wide} \]

   -or-

   Using Scale factor:

   \[ \frac{3}{8} \times \frac{432}{1} = 162 \text{ inches, or } 13 \frac{1}{2} \text{ feet wide} \]

   Yes, the set panels will fit (lying flat) through the mall entrance.

b. What is the scale factor? What does it tell us?

   The scale factor is 1/432. Each length on the scale drawing is 1/432 of the actual length. The actual lengths are 432 times larger than the scale drawing.

Closing (1 minute)

- What does the scale factor tell us about the relationship between the actual picture and the scale drawing?
  - *It gives us an understanding of how much bigger or smaller the scale drawing is compared to the actual picture.*

- How does a scale drawing differ from other drawings?
  - *In a scale drawing, there exists a constant ratio of scale drawing length to actual length, whereas other drawings may not have a constant scale ratio between all corresponding lengths of the drawing and the actual.*

Exit Ticket (5 minutes)
Lesson 18: Computing Actual Lengths from a Scale Drawing

Exit Ticket

A drawing of a surfboard in a catalog shows its length as $8\frac{4}{9}$ inches. Find the actual length of the surfboard if $\frac{1}{2}$ inch length on the drawing corresponds to $\frac{3}{8}$ foot of actual length.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

A drawing of a surfboard in a catalog shows its length as $8\frac{4}{9}$ inches. Find the actual length of the surfboard if $\frac{1}{2}$ inch length on the drawing corresponds to $\frac{3}{8}$ foot of actual length.

<table>
<thead>
<tr>
<th>Scale Drawing Length</th>
<th>equivalent scale ratio</th>
<th>surfboard</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$ inch</td>
<td>1 inch</td>
<td>$\frac{4}{9}$ inches</td>
</tr>
<tr>
<td>$\frac{3}{8}$ foot</td>
<td>$\frac{6}{8} \text{ft. or } \frac{3}{4} \text{ft.}$</td>
<td></td>
</tr>
</tbody>
</table>

$y = \frac{3}{4}x$
$= \frac{8}{9} \div 4$
$= \frac{76}{9} \div 4$
$= \frac{19}{3} \div 1$

The actual surfboard measures $6\frac{1}{3}$ feet long.

Note: Students could also use an equation where $y$ represents the scale drawing and $x$ represents the actual measurement.

$y = \frac{3}{4} \cdot \frac{8}{9}$

Problem Set Sample Solutions

1. A toy company is redesigning their packaging for model cars. The graphic design team needs to take the old image shown below and re-size it so that $\frac{1}{2}$ inch on the old packaging represents $\frac{1}{3}$ inch on the new package. Find the length of the image on the new package.

   Car image length on old packaging measures 2 inches

   $\frac{4}{3}$ inches; Note: It might be interesting to see how the students arrived at this answer. The scale is $\frac{1}{2}$ to $\frac{1}{3}$ and the length of the original figure is 2, which is 4 halves so in the scale drawing the length will be 4 thirds. Another approach would be to set up a proportion and solve:

   $\frac{1}{2} = \frac{2}{x}$
   $\frac{1}{3} \cdot \frac{2}{3} = \frac{x}{3}$
   $x = \frac{4}{3}$
2. The city of St. Louis is creating a welcome sign on a billboard for visitors to see as they enter the city. The following picture needs to be enlarged so that $\frac{1}{2}$ inch represents 7 feet on the actual billboard. Will it fit on a billboard that measures 14 feet in height?

Yes, the drawing measures 1 inch in height, which corresponds to 14 feet on the actual billboard.

3. Your mom is repainting your younger brother’s room. She is going to project the image shown below onto his wall so that she can paint an enlarged version as a mural. How long will the mural be if the projector uses a scale where 1 inch of the image represents $\frac{5}{2}$ feet on the wall?

The scale drawing measures 2 inches, so the image will measure $2 \times 2.5$, or 5 feet on the wall.

4. A model of a skyscraper is made so that 1 inch represents 75 feet. What is the height of the actual building if the height of the model is $18 \frac{3}{5}$ inches?

$1,395$ feet

5. The portrait company that takes little league baseball team photos is offering an option where a portrait of your baseball pose can be enlarged to be used as a wall decal (sticker). Your height in the portrait measures $3 \frac{1}{2}$ inches. If the company uses a scale where 1 inch on the portrait represents 20 inches on the wall decal, find the height on the wall decal. Your actual height is 55 inches. If you stand next to the wall decal, will it be larger or smaller than you?

Your height on the wall decal is 70 inches. The wall decal will be larger than your actual height (when you stand next to it).

6. The sponsor of a 5k run/walk for charity wishes to create a stamp of its billboard to commemorate the event. If the sponsor uses a scale where 1 inch represents 4 feet and the billboard is a rectangle with a width of 14 feet and a length of 48 feet, what will be the shape and size of the stamp?

The stamp will be a rectangle measuring $3 \frac{1}{2}$ inches by 12 inches.

7. Danielle is creating a scale drawing of her room. The rectangular room measures $20 \frac{1}{2}$ feet by 25 feet. If her drawing uses the scale 1 inch represents 2 feet of the actual room, will her drawing fit on an 8 1/2 in by 11 in piece of paper?

No, the drawing would be $10 \frac{7}{8}$ inches by $12 \frac{1}{2}$ inches which is bigger than the piece of paper.
8. A model of an apartment is shown below where 1/4 inch represents 4 feet in the actual apartment. Use a ruler to find the actual length and width of the bedroom.

Ruler measurements: 1 \(\frac{1}{8}\) inches by \(\frac{7}{16}\) inches. The actual length would be 18 feet and the actual width would be 7 ft.
Lesson 19: Computing Actual Areas from a Scale Drawing

Student Outcomes

- Students identify the scale factor.
- Given a scale drawing, students compute the area in the actual picture.

Classwork

Examples 1–3 (13 minutes): Exploring Area Relationships

In this series of examples, students will identify the scale factor. Students can find the areas of the two figures and calculate the ratio of the areas. As students complete a couple more examples, they can be guided to the understanding that the ratio of areas is the square of the scale factor.

Use the diagrams below to find the scale factor. Then find the area of each figure.

Example 1:

Scale factor: \( \frac{2}{1} \)

Actual Picture

| 6 units |
| 4 units |
| 8 units |

Scale Drawing

| 3 units |
| 6 units |

Area = \( 12 \) square units

Scale Drawing Area = \( 48 \) square units

Ratio of Scale Drawing Area to Actual Area: \( \frac{48}{12} = 4 \)

Example 2:

Scale factor: \( \frac{1}{3} \)

Actual Picture

| 9 units |
| 3 units |
| 6 units |
| 2 units |

Scale Drawing

| 3 units |
| 6 units |

Actual Area = \( 54 \) square units

Scale Drawing Area = \( 6 \) square units

Ratio of Scale Drawing Area to Actual Area: \( \frac{6}{54} = \frac{1}{9} \)
Example 3:
Scale factor: \( \frac{4}{3} \)

Actual Area = 27 square units

Scale Drawing Area = 48 square units

Ratio of Scale Drawing Area to Actual Area: \( \frac{48}{27} = \frac{16}{9} \)

Guide students through completing the results statements on the student materials.

Results: What do you notice about the ratio of the areas in Examples 1-3? Complete the statements below.

When the scale factor of the sides was 2, then the ratio of area was 4.

When the scale factor of the sides was 1/3, then the ratio of area was \( \frac{1}{9} \).

When the scale factor of the sides was 4/3, then the ratio of area was \( \frac{16}{9} \).

Based on these observations, what conclusion can you draw about scale factor and area?

The ratio of area is the scale factor multiplied by itself, or squared.

If the scale factor is \( r \), then the ratio of area is \( r^2 \) to 1.

Why do you think this is? Why do you think it is squared (opposed to cubed or something else)?

When you are comparing areas, you are dealing with two dimensions instead of comparing one linear measurement to another.

How might you use this information in working with scale drawings?

In working with scale drawings, you could take the scale factor \( r \), calculate \( r^2 \) to determine the relationship between area of the scale drawing and the actual picture. Given a blueprint for a room, the scale drawing dimensions could be used to find scale drawing area, then apply this new relationship to determine the actual area (the actual dimensions would not be needed).

Suppose a rectangle has an area of 12 square m. If it is enlarged by a scale factor of three, what area would you predict the enlarged rectangle to have based on these three examples. Look and think carefully!

If the scale factor is 3, then the ratio of scale drawing area to actual area is 32 to 12 or 9 to 1. So, if it’s area is 12 square meters before it is enlarged to scale, then the enlarged rectangle will have an area of 12 • \( \left( \frac{9}{1} \right) \), or 12•9, resulting in an area of 108 square meters.
Example 4 (10 minutes): They said yes!

Complete problem as a class asking the guiding questions below. Have students use the space in their student materials to record calculations and work.

Give students time to answer the question, possibly choosing to apply what was discovered in Examples 1–3. Allow for discussion of approaches described below, allowing for students to decide what method they prefer.

The Student Government liked your half-court basketball plan. They have asked you to calculate the actual area of the court so that they can estimate the cost of the project.

Based on the drawing below, what is the area of the planned half-court going to be?

Scale Drawing: 1 inch on drawing corresponds to 15 feet of actual length

Method 1: Use the measurements we found in yesterday’s lesson to calculate the area of the half-court

Actual area = 25 feet \times 30 feet = 750 square feet

Method 2: Apply newly discovered Ratio of Area relationship

This can be applied to the given scale- no unit conversions (shown on left), or to the scale factor (shown on right). Both options are included here as possible student work and would provide for a rich discussion of why they both work and what method is preferred. See questioning below

Using Scale:

The ratio of Area: \( \frac{1}{15} \times \frac{2}{15} = \frac{1}{225} \)

Scale Drawing Area = 2 in. \times \frac{1}{2} \text{ in.} = \frac{10}{3} \text{ square inches}

Let \( x \) = actual area and let \( y \) = scale drawing area

\[ y = kx \]

\[ \frac{10}{3} = \frac{1}{225} \cdot x \]

\[ 225 \cdot \frac{10}{3} = x \]

\[ x = 750 \text{ square feet} \]

Using Scale Factor:

The ratio of Area: \( \left( \frac{1}{180} \right)^2 = \frac{1}{32,400} \)

Scale Drawing Area = 2 in. \times \frac{2}{3} \text{ inches} = \frac{10}{3} \text{ square inches}

Let \( x \) = actual area and let \( y \) = scale drawing area

\[ y = kx \]

\[ \frac{10}{3} = \frac{1}{32,400} \cdot x \]

\[ 324,000 \cdot \frac{10}{3} = x \]

The actual area is \( \frac{324,000}{3} \) square inches, or \( \frac{324,000}{3} \) feet = 750 square feet.
PENDING FINAL EDITORIAL REVIEW

Ask for students to share how they found their answer. Use guiding questions to find all three options as noted above.

- What method do you prefer?
- Is there a time you would choose one method over the other?
  - If we don’t already know the actual dimensions, it might be faster to use Method 1 (ratio of areas). If we are re-carpeting a room based upon a scale drawing, we could just take the dimensions from the scale drawing, calculate area, then apply the ratio of areas to find the actual amount of carpet we need to buy.

Guide students to complete the follow-up question in their student materials.

Does the actual area you found reflect the results we found from Examples 1–3? Explain how you know.

Yes, the scale of 1 inch to 15 feet has a scale factor of \( \frac{1}{180} \), so the ratio of area should be \( \left( \frac{1}{180} \right)^2 \) or \( \frac{1}{32400} \).

The drawing area is \((2)(1 \frac{2}{3})\), or \(\frac{10}{3}\) square inches.

The actual area is 25 feet by 30 feet, or 750 square feet, or 108,000 square inches.

The ratio of area is \(\frac{10}{108,000}\), or \(\frac{10}{324,000}\), or \(\frac{1}{32,400}\).

Although, it would be more efficient to apply this understanding to the scale, eliminating the need to convert units.

If we use the scale of 1/15, then the ratio of area is 1/225.

The drawing area is \((2)(1 2/3)\), or 10/3 square inches.

The actual area is 25 feet by 30 feet, or 750 square feet.

The ratio of area is \(\frac{10}{750}\), or \(\frac{10}{2250}\), or \(\frac{1}{225}\).

Exercises (15 minutes)

Allow time for students to answer independently then share results.

1. The triangle depicted by the drawing has an actual area of 36 square units. What is the scale of the drawing? (Note: each square on grid has a length of 1 unit)

   ![Diagram of triangle]

   \[
   \text{Scale Drawing Area} = \frac{1}{2} \cdot 6 \cdot 3 = 9 \text{ units}^2
   \]

   \[
   \text{Ratio of Scale Drawing Area to Actual Area:} \quad \frac{9}{36} = r^2
   \]

   \[
   \text{Therefore scale factor,} \quad r = \frac{3}{6} \quad \text{since} \quad \frac{3}{6} \cdot \frac{3}{6} = \frac{9}{36}
   \]

   The scale factor is \(\frac{1}{2}\).

   The scale is: 1 unit of drawing length represents 2 units of actual length.
For Exercise 2, allow students the time to measure the apartment using a ruler, then compare measurements with their elbow partner. Students can then continue to complete parts a-f with their elbow partner. Allow students time to share responses comparing to what is given below. Sample answers to questions are given below.

2. Use the scale drawings of two different apartments to answer the questions. Use a ruler to measure.

Suburban Apartment

City Apartment

Scale: 1 inch on scale drawing corresponds to 12 feet in the actual apartment

Scale: 1 inch on scale drawing corresponds to 16 feet in the actual city apartment

a. Find the scale drawing area for both apartments, and then use it to find the actual area of both apartments.

<table>
<thead>
<tr>
<th></th>
<th>Suburban</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Drawing Area</td>
<td>$\frac{3}{2} \times 2$</td>
<td>$2 \times 1 \frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>= 5 square inches</td>
<td>= 3 square inches</td>
</tr>
<tr>
<td>Actual Area</td>
<td>$5(144) = 720$ square feet</td>
<td>$3(256) = 768$ square feet</td>
</tr>
</tbody>
</table>

b. Which apartment has the closet floor with more square footage? Justify your thinking.

<table>
<thead>
<tr>
<th></th>
<th>Suburban</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Drawing Area</td>
<td>$\frac{1}{4} + \frac{1}{2} + \frac{1}{4}$</td>
<td>$\frac{3}{16} + \frac{1}{2} + \frac{1}{4}$</td>
</tr>
<tr>
<td></td>
<td>= $\frac{5}{8}$ square inches</td>
<td>= $\frac{5}{16}$ square inches</td>
</tr>
<tr>
<td>Actual Area</td>
<td>$\frac{5}{8}(144) = 90$ square feet</td>
<td>$\frac{5}{16}(256) = 80$ square feet</td>
</tr>
</tbody>
</table>

The Suburban apartment has greater square footage in the closet floors.
c. Which apartment has the largest bathroom? Justify your thinking.

<table>
<thead>
<tr>
<th></th>
<th>Suburban</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Drawing Area</td>
<td>$\frac{1}{2} \cdot \frac{1}{2}$</td>
<td>$\frac{3}{8} \cdot \frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{4}$ square inches</td>
<td>$= \frac{3}{16}$ square inches</td>
</tr>
<tr>
<td>Actual Area</td>
<td>$(\frac{1}{2}) \cdot (144)$</td>
<td>$(\frac{3}{8}) \cdot (256)$</td>
</tr>
<tr>
<td></td>
<td>$= 72$ square feet</td>
<td>$= 96$ square feet</td>
</tr>
</tbody>
</table>

The city apartment has the largest bathroom.

d. A one-year lease for the suburban apartment costs $750 per month. A one-year lease for the city apartment costs $925. Which apartment offers the greater value in terms of the cost per square foot?

The suburban cost per square foot is $\frac{750}{720}$, or approximately $1.04 per square foot. The city cost per square foot is $\frac{925}{768}$, or approximately $1.20 per square foot. The suburban apartment offers greater value (cheaper cost per square foot): 1.04 versus 1.20

Closing (2 minutes)

- When given a scale drawing, how do we go about finding the area of the actual object?
  - Method 1: Compute each actual length based upon given scale, then use the actual dimensions to compute the actual area.
  - Method 2: Compute the area based upon the given scale drawing dimensions, then use the square of the scale to find actual area.
- Describe a situation where you might need to know the area of an object given a scale drawing or scale model?
  - Something where you might need to purchase materials that are priced per area, something that has a limited amount of space to fill or take up; when comparing two different plans.

Lesson Summary:

Given the scale factor $r$ representing the relationship between scale drawing length and actual length, the square of this scale factor, $r^2$, represents the relationship between scale drawing area and actual area.

For example, if 1 inch on the scale drawing represents 4 inches of actual length, then the scale factor, $r$, is $1/4$. On this same drawing, 1 square inch of scale drawing area would represent 16 square inches of actual area since $r^2$ is $1/16$.

Scaffolding:

Extension to Exit Ticket: Ask students to show multiple methods for finding area of the dining room.

Exit Ticket (5 minutes)
Lesson 19: Computing Actual Areas from a Scale Drawing

Exit Ticket

1. A 1-inch length in the scale drawing below corresponds to a length of 12 feet in the actual room.

   1 inch

   Kitchen

   Dining Room

   1 ½ inches

   ¾ in

   1 ½ inches

   1 ½ inches

   a. Describe how the scale or the scale factor can be used to determine the area of the actual dining room.

   b. Find the actual area of the dining room.

   c. Can a rectangular table that is 7 ft long and 4 ft wide fit into the narrower section of the dining room? Explain your answer.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. A 1-inch length in the scale drawing below corresponds to a length of 12 feet in the actual room.

   ![Scale Drawing Diagram]

   a. Describe how the scale or the scale factor can be used to determine the area of the actual dining room.

   Calculate the area of the scale drawing, then divide it by the square of the scale (or scale factor) to determine the actual area.

   Note: When the scale represents a reduction such as \(\frac{1}{144}\), it may be more logical to take the area and multiply it by the denominator, especially in this case where the numerator is 1.

   b. Find the actual area of the dining room.

   Scale drawing area of dining room: \(\frac{1}{2} \times \frac{3}{4}\) or \(\frac{3}{8}\) square inches

   Actual area of the dining room: \(\frac{12}{8} \div \frac{1}{144}\), or \((12/8)(144)\), or 216 square feet

   Or similar work completing conversions and using scale factor.

   c. Can a rectangular table that is 7 ft long and 4 ft wide fit into the narrower section of the dining room? Explain your answer.

   Narrower section of dining room measures \(\frac{3}{4}\) by \(\frac{3}{4}\) in the drawing, or 9 feet by 6 feet in the actual room. Yes, the table will fit, however it will only allow for 1 additional foot around all sides of the table for movement or chairs.
1. The shaded rectangle shown below is a scale drawing of a rectangle whose area is 288 square feet. What is the scale factor of the drawing? (Note: each square on grid has a length of 1 unit)

The scale factor is \( \frac{1}{3} \).

2. A floor plan for a home is shown below where \( \frac{1}{2} \) inch corresponds to 6 feet of the actual home. Bedroom 2 belongs to 13-year old Kassie, and bedroom 3 belongs to 9-year old Alexis. Kassie claims that her younger sister, Alexis, got the bigger bedroom, is she right? Explain.

Bedroom 2 (Kassie) has an area of 135 sq ft and Bedroom 3 (Alexis) has an area of 144 sq.ft. Therefore, the older sister is correct. Alexis got the bigger bedroom by a difference of 9 square feet.
3. On the mall floor plan, \( \frac{1}{4} \) inch represents 3 feet in the actual store.
   a. Find the actual area of Store 1 and Store 2.

   Store 1 has an area of \( 375 \frac{3}{16} \) square feet and Store 2 has an area of \( 309 \frac{15}{16} \) square feet.
   b. In the center of the atrium, there is a large circular water feature that has an area of \( \frac{9}{64}\pi \) square inches on the drawing. Find the actual area in square feet.

   The water feature has an area of \( \frac{9}{64}\pi \times 144 \), or \( \frac{81}{4}\pi \) square feet, approximately 63.6 square feet.

4. The greenhouse club is purchasing seed for the lawn in the school courtyard. They need to determine how much to buy. Unfortunately, the club meets after school and students are unable to find a custodian to unlock the door. Anthony suggests they just use his school map to calculate the amount of area that will need to be covered in seed. He measures the rectangular area on the map and finds the length to be 10 inches and the width to be 6 inches. The map notes the scale of 1 inch representing 7 feet in the actual courtyard. What is the actual area in square feet?

   \[ 60 \times 49 = 2940 \text{ sq ft} \]

5. The company installing the new in-ground pool in your back yard has provided you with the scale drawing shown below. If the drawing uses a scale of 1 inch to 1 \( \frac{3}{7} \) feet, calculate the total amount of two-dimensional space needed for the pool and its surrounding patio.

   \[ \text{Area} = 780 \text{ square feet} \]
Lesson 20: An Exercise in Creating a Scale Drawing

Student Outcomes

- Students create their own scale drawing of the top-view of a furnished room or building.

Classwork

Preparation (Before Instructional Time): Prepare sheets of grid paper (8.5 x 11 inches), rulers and furniture catalogs for student use. Measure the perimeter of the room to give to students beforehand.

Introduction (3 minutes)

Inform students they will be working in pairs to create their dream classroom. The principal is looking for ideas to create spaces conducive to enjoyable and increased learning. Be as creative as you can be! Didn’t you always think there should be nap time? Now you can create an area for it!

Instruction: Allow students to work at his/her own pace. Guidelines are provided in the Student Pages.

Take measurements: All students will work with the perimeter of the classroom as well as the doors and windows. Give students the dimensions of the room. Have students use the table provided to record measurements.

Create your dream classroom and use the furniture catalog to pick out your furniture: Students will discuss what their ideal classroom will look like with their partners and pick out furniture from the catalog. Students should record the actual measurements on the given table.

Determine scale and calculate scale drawing lengths and widths: Each pair of students will determine their own scale. The calculation of the scale drawing lengths, widths, and areas is to be included.

Scale Drawing: Using a ruler and referring back to the calculated scale length, students will draw the scale drawing including the doors, windows and furniture.

Scaffolding:

Have some students measure the perimeter of the classroom for the class beforehand.

For struggling students: Model the measuring and recording of the perimeter of the classroom.

Extension: Have students choose flooring and record the costs. Including the furniture, students can calculate the cost of the designed room.
## PENDING FINAL EDITORIAL REVIEW

### Measurements:

<table>
<thead>
<tr>
<th></th>
<th>Classroom Perimeter</th>
<th>Windows</th>
<th>Door</th>
<th>Additional Furniture Chairs</th>
<th>Rug</th>
<th>Storage</th>
<th>Bean Bags</th>
<th>Independent Work Tables (x 4)</th>
<th>Board</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Actual:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Length</strong></td>
<td>40 ft.</td>
<td>5 ft.</td>
<td>3 ft.</td>
<td>1 ft.</td>
<td>13 ft.</td>
<td>15 ft.</td>
<td>2 ft.</td>
<td>10 ft.</td>
<td>6 ft.</td>
</tr>
<tr>
<td><strong>Width</strong></td>
<td>30 ft.</td>
<td>/</td>
<td>/</td>
<td>1 ft.</td>
<td>10 ft.</td>
<td>2.5 ft.</td>
<td>2 ft.</td>
<td>3 ft.</td>
<td>/</td>
</tr>
<tr>
<td><strong>Scale Drawing:</strong></td>
<td>4 in.</td>
<td>60 120</td>
<td>36 120</td>
<td>12 120</td>
<td>160 120</td>
<td>180 120</td>
<td>24 120</td>
<td>72 120</td>
<td></td>
</tr>
<tr>
<td><strong>Length</strong></td>
<td>/ 120</td>
<td>/ 3</td>
<td>/ 3</td>
<td>/ 3</td>
<td>/ 3</td>
<td>/ 3</td>
<td>/ 3</td>
<td>/ 3</td>
<td></td>
</tr>
<tr>
<td><strong>Width</strong></td>
<td>3 in.</td>
<td>/</td>
<td>/</td>
<td>1 10</td>
<td>120 120</td>
<td>30 120</td>
<td>36 120</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td><strong>Scale:</strong></td>
<td>1 120</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Scale:** $\frac{1}{120}$

**Initial Sketch:** Use this space to sketch the classroom perimeter, draw out your ideas and play with the placement of the furniture.
### Lesson 20
An Exercise in Creating a Scale Drawing

**Date:** 7/9/13

**Area**

<table>
<thead>
<tr>
<th>Classroom</th>
<th>Chairs</th>
<th>Rug</th>
<th>Storage</th>
<th>Bean Bags</th>
<th>Independent Work Tables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Area</td>
<td>$40 \times 30 = 1200 \text{ ft}^2$</td>
<td>$1 \times 1 = 1 \text{ ft}^2$</td>
<td>$13 \times 10 = 130 \text{ ft}^2$</td>
<td>$15 \times 2.5 = 37.5 \text{ ft}^2$</td>
<td>$2 \times 2 = 4 \text{ ft}^2$</td>
</tr>
<tr>
<td>Scale Drawing Area</td>
<td>$4 \times 3 = 12 \text{ in.}^2$</td>
<td>$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100} \text{ in.}^2$</td>
<td>$1 \times \frac{1}{3} = \frac{1}{3} \text{ in.}^2$</td>
<td>$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8} \text{ in.}^2$</td>
<td>$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15} \text{ in.}^2$</td>
</tr>
</tbody>
</table>
Closing (3 minutes)

- Why are scale drawings used in construction and design projects?
  - Scale drawings can be used to rearrange furniture, find appropriate sizes for new items, and reconfigure room size and building size without having to refer back to the actual room or building being worked on.

- How can we double check our area calculations?
  - We can check to see if our calculations for area are equal to the number of boxes for each object on the graph paper.

- What were the biggest challenges you faced when creating your floor plan? How did you overcome these challenges?
  - Arranging the furniture and realizing the pieces we chose were too big for the space. Choose another item based on the measurements.

Lesson Summary:

Scale Drawing Process:
1. Measure lengths and widths carefully with a ruler or tape measure. Record in an organized table.
2. Calculate the scale drawing lengths, widths and areas using what was learned in previous lessons.
3. Calculate the actual areas.
4. Begin by drawing the perimeter, windows and doorways.
5. Continue to draw the pieces of furniture making note of placement of objects (distance from nearest wall).
6. Check for reasonableness of measurements and calculations.

Exit Ticket (5 minutes)
Lesson 20: An Exercise in Creating a Scale Drawing

Exit Ticket

1. Your sister has just moved into a loft style apartment in Manhattan and has asked you to be her designer. Indicate the placement of the following objects on the floorplan using the appropriate scale: Queen size bed (60 in. by 80 in.), sofa (36 in. by 64 in.) and dining table (48 in. by 48 in.) In the following scale drawing, 1 cm represents 2 ft. Each square on the grid is 1 cm².

   ![Floorplan Drawing]

2. Choose one object and explain the procedure to find the scale lengths.
Exit Ticket Sample Solutions
The following solutions indicate an understanding of the objectives of this lesson:

1. Your sister has just moved into a loft style apartment in Manhattan and has asked you to be her designer. Indicate the placement of the following objects on the floorplan using the appropriate scale: Queen size bed (60 in. by 80 in.), sofa (36 in. by 64 in.) and dining table (48 in. by 48 in.) In the following scale drawing, 1 cm represents 2 ft. Each square on the grid is 1 cm².

   - **Queen Bed:**
     \[ 60 \div 12 = 5, \quad 5 \div 2 = 2\frac{1}{2} \text{ cm} \]
     \[ 80 \div 12 = 6\frac{2}{3}, \quad 6\frac{2}{3} \div 2 = 3\frac{1}{3} \text{ cm} \]

   - **Sofa:**
     \[ 36 \div 12 = 3, \quad 3 \div 2 = 1\frac{1}{2} \text{ cm} \]
     \[ 64 \div 12 = 5\frac{1}{3}, \quad 5\frac{1}{3} \div 2 = 2\frac{2}{3} \text{ cm} \]

   - **Dining Table:**
     \[ 48 \div 12 = 4, \quad 4 \div 2 = 2 \text{ cm} \]

2. Choose one object and explain the procedure to find the scale lengths.
   
   *Take the actual measurements in inches and divide by 12 inches to express the value in feet. Then divide the actual length in feet by two since two feet represent 1 cm. The resulting quotient is the scale length.*

Problem Set Sample Solutions

Interior Designer:
You won a spot on a famous interior designing TV show! The designers will work with you and your existing furniture to re-design a room of your choice. Your job is to create a top-view scale drawing of your room and the furniture within.

- With the scale factor being \( \frac{1}{24} \), create a scale drawing of your room or other favorite room in your home on a sheet of 8.5 x 11 inch graph paper.
- Include the perimeter of the room, windows, doorways, and three or more furniture pieces (such as tables, desks, dressers, chairs, bed, sofa, ottoman, etc.).
- Use the table to record lengths and include calculations of areas.
- Make your furniture “moveable” by duplicating your scale drawing and cutting out the furniture.
- Create a “before” and “after” to help you decide how to rearrange your furniture. Take a photo of your “before”.
- What changed in your furniture plans?
- Why do you like the “after” better than the “before”?

*Answers will vary.*
### Lesson 20: An Exercise in Creating a Scale Drawing

**Date:** 7/9/13

#### Actual Room Dimensions

<table>
<thead>
<tr>
<th>Entire Room</th>
<th>Windows</th>
<th>Doors</th>
<th>Desk/Tables</th>
<th>Seating</th>
<th>Storage</th>
<th>Bed</th>
<th>Shelf</th>
<th>Side Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual : Length</td>
<td>10 ft.</td>
<td>5 ft.</td>
<td>3 ft.</td>
<td>5 ft.</td>
<td>1 ft.</td>
<td>3 ft.</td>
<td>6 ft.</td>
<td>5 1/4 ft.</td>
</tr>
<tr>
<td>Width</td>
<td>13 ft.</td>
<td>/</td>
<td>2 1/2 ft.</td>
<td>2 5/12 ft.</td>
<td>1 ft.</td>
<td>2 ft.</td>
<td>2 1/4 ft.</td>
<td>1 ft.</td>
</tr>
<tr>
<td>Scale Drawing : Length</td>
<td>5 in.</td>
<td>2 1/2 in.</td>
<td>1 1/2 in.</td>
<td>2 1/2 in.</td>
<td>1/2 in.</td>
<td>1 1/2 in.</td>
<td>3 in.</td>
<td>2 3/8 in.</td>
</tr>
<tr>
<td>Width</td>
<td>6 1/2 in.</td>
<td>/</td>
<td>1 1/4 in.</td>
<td>2 1/4 in.</td>
<td>1 1/2 in.</td>
<td>1 1/2 in.</td>
<td>1 1/2 in.</td>
<td>3 3/4 in.</td>
</tr>
</tbody>
</table>

#### Actual Area and Scale Drawing Area

<table>
<thead>
<tr>
<th>Entire Room Length</th>
<th>Desk/Tables</th>
<th>Seating</th>
<th>Storage</th>
<th>Bed</th>
<th>Shelf</th>
<th>Side Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Area</td>
<td>10 \times 13 = 130 \text{ft.}^2</td>
<td>5 \times 2 \frac{5}{12} = \frac{100}{12} = \frac{25}{3} \text{ft.}^2</td>
<td>5 \times \frac{1}{2} = \frac{5}{2} \text{ft.}^2</td>
<td>1 \times 1 = 1 \text{ft.}^2</td>
<td>3 \times 2 = 6 \text{ft.}^2</td>
<td>6 \times 2 \frac{1}{4} = \frac{25}{2} = 12.5 \text{ft.}^2</td>
</tr>
<tr>
<td>Scale Drawing Area</td>
<td>5 \times \frac{1}{2} = \frac{5}{2} = 2 \frac{1}{2} \text{in.}^2</td>
<td>2 \times \frac{1}{2} = \frac{1}{2} \text{in.}^2</td>
<td>1 \times \frac{1}{2} = \frac{1}{2} \text{in.}^2</td>
<td>1 \times \frac{1}{2} = \frac{1}{2} \text{in.}^2</td>
<td>3 \times 1 \frac{1}{2} = 3 \times \frac{3}{2} = \frac{9}{2} = 4.5 \text{in.}^2</td>
<td>2 \times \frac{1}{2} = \frac{1}{2} \text{in.}^2</td>
</tr>
</tbody>
</table>

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Lesson 21: An Exercise in Changing Scales

Student Outcomes

- Given a scale drawing, students produce a scale drawing of a different scale.
- Students recognize that the scale drawing of a different scale is a scale drawing of the original scale drawing.
- For the scale drawing of a different scale, students compute the scale factor for the original scale drawing.

Classwork

Example (20 minutes)

The school plans to publish your work on the dream classroom in the next newsletter. Unfortunately, in order to fit our drawing on the page, it must be ¼ its current length to be published in the magazine. Create a new drawing (SD2) in which all of the lengths are 1/4 those in the original scale drawing (SD1) from lesson 20.

Example is included for students unable to create SD1 from lesson 20 at the end. Pose the following questions:

- Would the new scale create a larger or smaller scale drawing as compared to the original drawing?
  - It would be smaller because ¼ is smaller than one.

- How would you use the scale factor between SD1 to SD2 to calculate the new scale drawing lengths without having to get the actual measurement first?
  - Take the original scale drawing lengths and multiply by this by 1/4 to find the new scale lengths.

Once the students have finished creating SD2, ask students to prove to the architect that SD2 is actually a scale drawing of the original room.

- How can we go about proving that the new scale drawing (SD2) is actually a scale drawing of the original room?
  - The scale lengths of SD2 have to be proportional to the actual lengths. We need to find the constant of proportionality, the scale factor.
PENDING FINAL EDITORIAL REVIEW

- How do we find the new scale factor?
  - Divide one of the new scale lengths by its corresponding actual length.

- If the actual measurement wasn’t known, how could we find it?
  - Calculate the actual length by using the scale factor on the original drawing. Multiply the scale length of the original drawing by the original scale factor.

Exercise (20 minutes)

Write different scale factors on cards, which the students will choose: \( \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, 3, 4 \). They will then create a new scale drawing and calculate the scale factor between their drawing and the original trapezoid in the student material.

After completing a – c independently, have all of the students who were working with enlargements move to the right side of the room and those with reductions to the left. Have students first discuss in smaller groups on their side of the room and then come together as a class to discuss:

- Compare your answers to part a. What can you conclude?
  - All of the enlargements had a scale factor that was greater than 1. The reductions have a scale factor between zero and 1.

- What methods did you use to answer part c?
  - The scale factor between SD2 (student drawn trapezoid) and the original figure can be determined by multiplying the scale factor of SD1 (scale drawing given in the materials) to the Original figure by the scale factor of SD2 to SD1.

The picture shows an enlargement or reduction of a scale drawing of a trapezoid.

Using the scale factor written on the card you chose; draw your new scale drawing with correct calculated measurements:
a. What is the scale factor between the original scale drawing and the one you drew?
   \[
   \frac{1}{3}
   \]

b. The longest base length of the actual trapezoid is 10 cm. What is the scale factor between the original scale drawing and the actual trapezoid?
   \[
   \frac{7}{10}
   \]

c. What is the scale factor between the new scale drawing you drew and the actual trapezoid?
   \[
   \frac{2}{3} = \frac{7}{10} = \frac{7}{3} \times \frac{1}{10} = \frac{7}{30}
   \]

Closing (5 minutes)

- Why might you want to produce a scale drawing of a different scale?
  - To produce multiple formats of a drawing (e.g. different sized papers for a blueprint).

- How do you produce another scale drawing given the original scale drawing and a different scale?
  - Take the lengths of the original scale drawing and multiply by the different scale. Measure and draw out the new scale drawing.

- How can you tell if a new scale drawing is a scale drawing of the original figure?
  - If the new scale drawing (SD2) is a scale drawing of SD1, then it is a scale drawing of the original figure with a different scale.

- How can the scale factor of the new drawing to the original figure be determined?
  - Take the scale length of the new scale drawing and divide it by the actual length of the original figure.

Changing Scale Factors:

- To produce a scale drawing at a different scale, you must determine the new scale factor. The new scale factor is found by dividing the different (new drawing) scale factor by the original scale factor.

- To find each new length, you can multiply each length in the original scale drawing by this new scale factor.

Steps:

- Find each scale factor.
- Divide new scale factor by original scale factor.
- Divide the given length by the new scale factor (the quotient from the prior step)
Problem Set Sample Solutions

1. Jake reads the following problem: If the original scale factor for a scale drawing of a square swimming pool was 1/90 and length of the original drawing measured to be 8 inches, what is the length on the new scale drawing if the scale factor of the new scale drawing length to actual length is 1/144?

He works out the problem like so:

\[
8 \div \frac{1}{90} = 720 \text{ inches.}
\]
\[
720 \times \frac{1}{144} = 5 \text{ inches.}
\]

Is he correct? Why or why not?

Jake is correct. He took the original scale drawing length and divided by the original scale drawing to get the actual length, 720 inches. To get the new scale drawing length he takes the actual length, 720 and multiplies by the new scale factor, 1/144 to get 5 inches.

2. What is the scale factor of the new scale drawing to the original scale drawing (SD2 to SD1)? 5/8

3. If the length of the pool measures 10 cm on the new scale drawing:
   a. What is the actual length of the pool in meters? 14.40 m
   b. What is the surface area of the actual pool? 207.36 m²
   c. If the pool has a constant depth of 4 feet, what is the volume of the pool? 252.81 m³
   d. If 1 cubic meter of water is equal to 264.2 gallons, how much water will the pool contain when completely filled? 66,792.40 gal.

4. Complete a new scale drawing of your dream room from Lesson 20’s problem set by either reducing by ¼ or enlarging it by 4.

Scale Drawings will vary.

Original Scale Drawing Length: 6 \frac{1}{2} in

New Scale Drawing Length: 5 in. \(6 \frac{1}{2} \times x = 5\)

\[
\frac{13}{2} \times x = 5
\]

\[
x = \frac{10}{13} \text{ in. But because there are 16 units in one inch, } \frac{10}{13} \times 16 = 12.3, \text{ which is } \approx \frac{12.3}{16}
\]
Lesson 21
An Exercise in Changing Scales

Date: 7/9/13

Pending final editorial review

Equivalent Fraction Computations

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Result</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{13} \times \frac{1}{16} )</td>
<td>( \frac{1}{16} \approx 0.06 )</td>
<td>( \frac{1}{16} \approx 0.06 )</td>
</tr>
<tr>
<td>( \frac{1}{13} \times \frac{2}{13} )</td>
<td>( \frac{2}{13} \approx 0.15 )</td>
<td>( \frac{2}{13} \approx 0.15 )</td>
</tr>
<tr>
<td>( \frac{3}{13} \times \frac{3}{16} )</td>
<td>( \frac{3}{16} \approx 0.19 )</td>
<td>( \frac{3}{16} \approx 0.19 )</td>
</tr>
<tr>
<td>( \frac{4}{13} \times \frac{4}{16} )</td>
<td>( \frac{4}{16} \approx 0.25 )</td>
<td>( \frac{4}{16} \approx 0.25 )</td>
</tr>
<tr>
<td>( \frac{5}{13} \times \frac{5}{16} )</td>
<td>( \frac{5}{16} \approx 0.31 )</td>
<td>( \frac{5}{16} \approx 0.31 )</td>
</tr>
<tr>
<td>( \frac{6}{13} \times \frac{6}{16} )</td>
<td>( \frac{6}{16} \approx 0.38 )</td>
<td>( \frac{6}{16} \approx 0.38 )</td>
</tr>
<tr>
<td>( \frac{7}{13} \times \frac{7}{16} )</td>
<td>( \frac{7}{16} \approx 0.44 )</td>
<td>( \frac{7}{16} \approx 0.44 )</td>
</tr>
<tr>
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<td>( \frac{8}{16} \approx 0.50 )</td>
</tr>
<tr>
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<td>( \frac{9}{16} \approx 0.56 )</td>
</tr>
<tr>
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<td>( \frac{10}{16} \approx 0.62 )</td>
<td>( \frac{10}{16} \approx 0.62 )</td>
</tr>
<tr>
<td>( \frac{11}{13} \times \frac{11}{16} )</td>
<td>( \frac{11}{16} \approx 0.68 )</td>
<td>( \frac{11}{16} \approx 0.68 )</td>
</tr>
<tr>
<td>( \frac{12}{13} \times \frac{12}{16} )</td>
<td>( \frac{12}{16} \approx 0.75 )</td>
<td>( \frac{12}{16} \approx 0.75 )</td>
</tr>
</tbody>
</table>

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## Conversions (inches)

<table>
<thead>
<tr>
<th>Entire Room</th>
<th>Windows</th>
<th>Doors</th>
<th>Desk</th>
<th>Seating</th>
<th>Storage</th>
<th>Bed</th>
<th>Shelf</th>
<th>Side Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Drawing Length</td>
<td>See above</td>
<td>$\frac{2}{5} \times \frac{10}{13} = \frac{20}{65}$ = $\frac{4}{13} \approx 0.31$</td>
<td>$\frac{1}{2} \times \frac{10}{13} = \frac{5}{13} \approx 0.38$</td>
<td>$\frac{1}{2} \times \frac{10}{13} = \frac{5}{13} \approx 0.38$</td>
<td>$\frac{10}{13} \times \frac{1}{2} = \frac{10}{26} = \frac{5}{13} \approx 0.38$</td>
<td>$\frac{10}{13} \times \frac{1}{2} = \frac{10}{26} = \frac{5}{13} \approx 0.38$</td>
<td>$\frac{10}{13} \times \frac{1}{2} = \frac{10}{26} = \frac{5}{13} \approx 0.38$</td>
<td>$\frac{10}{13} \times \frac{1}{2} = \frac{10}{26} = \frac{5}{13} \approx 0.38$</td>
</tr>
<tr>
<td>Scale Drawing Width</td>
<td>$\frac{10}{13} \times \frac{5}{13} = \frac{50}{169} \approx 0.30$</td>
<td>$\frac{10}{13} \times \frac{5}{13} = \frac{50}{169} \approx 0.30$</td>
<td>$\frac{10}{13} \times \frac{5}{13} = \frac{50}{169} \approx 0.30$</td>
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</tr>
</tbody>
</table>

SD1 Example for students who were unable to create their own from Lesson 20

**SCALE FACTOR:** $\frac{1}{120}$
Lesson 22: An Exercise in Changing Scales

Student Outcomes

- Given a scale drawing, students produce a scale drawing of a different scale.
- Students recognize that the scale drawing of a different scale is a scale drawing of the original scale drawing.
- For the scale drawing of a different scale, students compute the scale factor for the original scale drawing.

Classwork

Reflection on Scale Drawings (15 minutes): Ask students to take out the original scale drawing and new scale drawing of their dream rooms they completed as part of Lesson 20 and 21 problem sets. Have students discuss their answers with a partner. Discuss as a class:

- How are the two drawings alike?
- How are the two drawings different?
- What is the scale factor of the new scale drawing to the original scale drawing?

Direct students to fill-in-the blanks with the two different scale factors. Allow pairs of students to discuss the posed question, “What is the relationship?” for 3 minutes and share response for 4 minutes. Summarize the Key Idea with students.

Using the new scale drawing of your dream room, list the similarities and differences between this drawing and the original drawing completed for Lesson 20.

<table>
<thead>
<tr>
<th>Similarities</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same room shape</td>
<td>one is bigger than the other</td>
</tr>
<tr>
<td>Placement of furniture</td>
<td>different scale factors</td>
</tr>
<tr>
<td>Space between furniture</td>
<td></td>
</tr>
<tr>
<td>Drawing of the original room</td>
<td></td>
</tr>
<tr>
<td>Proportional</td>
<td></td>
</tr>
</tbody>
</table>

Original Scale Factor: \( \frac{1}{120} \)  New Scale Factor: \( \frac{1}{30} \)

What is the relationship between these scale factors? \( \frac{1}{4} \)
Example 1 (10 minutes): Building a Bench

Students are given the following information- the scale factor of Taylor’s scale drawing to the actual bench is 1/12, Taylor’s scale drawing and the measurements of the corresponding lengths (2 in. and 6 in. as shown). Ask the students the following questions:

- What information is important in the diagram?
  - The scale factor of Taylor’s reproduction.

- What information can be accessed from the given scale factor?
  - The actual length of the bench can be computed from the scale length of Taylor’s drawing.

- What are the processes used to find the original scale factor to the actual bench?
  - Take the length of the new scale drawing, 6 inches, and divide by the scale factor, $\frac{1}{12}$, to get the actual length of the bench, 72 inches. The original scale factor, $\frac{1}{36}$, can be computed by dividing the original scale length, 2 inches, by the actual length, 72 inches.

- What is the relationship of Taylor’s drawing to the original drawing?
  - Taylor’s drawing is 3 times as big as her father’s drawing. The lengths corresponding to the actual length which is 72 inches are 6 inches from Taylor’s drawing and 2 inches from the original drawing. 6/2 is 3 so the scale factor is 3.

To surprise her mother, Taylor helped her father build a bench for the front porch. Taylor’s father had the instructions with drawings but Taylor wanted to have her own copy. She enlarged her copy to make it easier to read. Using the following diagram, fill in the missing information.

The pictures below show the diagram of the bench shown on the original instructions and the diagram of the bench shown on Taylor’s enlarged copy of the instruction.

<table>
<thead>
<tr>
<th>Scale Factors</th>
<th>Bench</th>
<th>Father’s Diagram</th>
<th>Taylor’s Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bench</td>
<td>1</td>
<td>36</td>
<td>12</td>
</tr>
<tr>
<td>Father’s Diagram</td>
<td>$\frac{1}{36}$</td>
<td>1</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>Taylor’s Diagram</td>
<td>$\frac{1}{12}$</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
Exercise 1 (5 minutes)

Allow students to work problem with partners for 3 minutes. Discuss for 2 minutes:

- How did you find the original scale factor?
  - Divide the Carmen’s map distance, 4 cm, by the scale factor, 1/563,270 to get the actual distance, 2,253,080 cm. Take the distance from Jackie’s map, 26 cm and divide by the actual distance to get the original scale factor 1/ 86,656.92.

- What are the steps to find the scale of new to original scale drawing?
  - Divide the new scale distance, 4 cm, to the corresponding original scale distance, 26 cm, to get 2/13.

- What is the actual distance in miles?
  - 2,253,080 cm divided by 2.54 cm gives 887,039.37 inches. Divide 887,039.37 by 12 to get 73,919.95 feet. Then divide 73,919.95 by 5280 to get around 14 miles.

- Would it make more sense to answer in centimeters or miles?
  - Although both are valid units, miles would be a more useful unit to describe the distance driven in a car.

Carmen and Jackie were driving separately to a concert. Jackie printed a map of the directions on a piece of paper before the drive and Carmen took a picture of Jackie’s map on her phone. Carmen’s map had a scale factor of 1/563,270. Using the pictures, what is the scale of Carmen’s map to Jackie’s map? What was the scale factor of Jackie’s printed map to the actual distance?

Jackie’s Map                                    Carmen’s Map:

\[
\begin{align*}
\text{26 cm} & \quad \text{4 cm} \\
\end{align*}
\]

Scale Factor of SD2 to SD1: \( \frac{4}{26} = \frac{2}{13} \)

Scale Factor of SD1 to actual distance: \( \frac{1}{563,270} \times \frac{13}{2} = \frac{1}{112,6540} \)
Exercise 2 (10 minutes)

Allow students to work in pairs to find the solutions. Ask:

- What is another way to find the scale factor of the toy set to the actual boxcar?
  - Take the length of the toy set and divide it by the actual length.
- What is the purpose of question c?
  - To take notice of the relationships between all the scale factors.

Ronald received a special toy train set for his birthday. In the picture of the train on the package, the box car has the following dimensions: length – 4 5/16 inches, width - 1 1/8 inches and height- 1 5/8 inches. The toy box car that Ronald received has dimensions \( l = 17.25 \) inches, \( w = 4.5 \) inches, \( h = 6.5 \) inches. If the actual boxcar is 50 feet long:

a. Find the scale factor of the picture on the package to the toy set.

\[
\frac{\frac{4\frac{5}{16}}{17\frac{1}{4}}}{\frac{4\frac{5}{16}}{16}} = \frac{\frac{69}{16}}{\frac{4}{69}} = \frac{1}{4}
\]

b. Find the scale factor of the picture on the package to the actual boxcar.

\[
\frac{\frac{4\frac{5}{16}}{50\times12}}{\frac{4\frac{5}{16}}{17\frac{1}{4}}} = \frac{\frac{69}{600}}{\frac{4}{69}} = \frac{23}{3200}
\]

c. Use these two scale factors to find the scale factor between the toy set and the actual boxcar.

\[
\frac{\frac{4\frac{5}{16}}{600}}{\frac{4\frac{5}{16}}{17\frac{1}{4}}} = \frac{\frac{23}{3200}}{\frac{4}{69}} = \frac{23}{800}
\]

d. What are the width and height of the actual boxcar?

\[
W:\frac{4}{2} \div \frac{23}{800} = \frac{9}{2} \times \frac{800}{23} = 156 \frac{12}{23} \text{ in.} \quad H:\frac{6}{2} \div \frac{23}{800} = \frac{13}{2} \times \frac{800}{23} = 226 \frac{2}{23} \text{ in.}
\]

Closing (5 minutes)

- What is the relationship between the scale drawing of a different scale to the original scale drawing?
  - The scale drawing at a different scale is scale drawing of the original scale. If the scale factor of one of the drawings is known, the other scale factor can be computed.
- Describe the process of computing the scale factor for the original scale drawing from the scale drawing at a different scale.
  - Find corresponding known lengths and compute the actual length from the given scale factor using the new scale drawing. To find the scale factor for the original drawing, write a ratio to compare a drawing length from original drawing to its corresponding actual length from the second scale drawing.
Lesson Summary:
The scale drawing at a different scale is a scale drawing of the original scale drawing.
To find the scale factor for the original drawing, write a ratio to compare a drawing length from original drawing to its corresponding actual length from the second scale drawing.
Refer to the example below where we compare drawing length from Original Scale drawing to its corresponding actual length from the New Scale drawing:
6 inches/12 feet, or 0.5 feet/12 feet converting to the same units
This gives an equivalent ratio of 1/24 for the scale factor of the original drawing.

Exit Ticket (5 minute)
Lesson 22: An Exercise in Changing Scales

Exit Ticket

The school is building a new wheelchair ramp for one of the remodeled bathrooms. The original drawing was created by the contractor, but the principal drew another scale drawing to see the size of the ramp relative to the walkways surrounding it. Find the missing values on the table.

<table>
<thead>
<tr>
<th>Actual Ramp</th>
<th>Original Scale Drawing</th>
<th>Principal’s Scale Drawing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Ramp</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Original Scale Drawing</td>
<td>1</td>
<td>175</td>
</tr>
<tr>
<td>Principals’ Scale Drawing</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

New Scale Factor of SD2 to the actual ramp: 1/700
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. The school is building a new wheelchair ramp for one of the remodeled bathrooms. The original drawing was created by the contractor but the principal drew another scale drawing to see the size of the ramp relative to the walkways surrounding it. Find the missing values on the table.

   **Original Scale Drawing**
   - 12 in.

   **Principal's Scale Drawing:**
   - New Scale Factor of SD2 to the actual ramp: 1/700
   - 3 in.

   **Scale Factor Table**

<table>
<thead>
<tr>
<th>Actual Ramp</th>
<th>Original Scale Drawing</th>
<th>Principal's Scale Drawing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Ramp</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Original Scale Drawing</td>
<td>1/175</td>
<td>1</td>
</tr>
<tr>
<td>Principal's Scale Drawing</td>
<td>1/700</td>
<td>1/4</td>
</tr>
</tbody>
</table>

Problem Set Sample Solutions

1. For the scale drawing, the actual lengths are labeled onto the scale drawing. Measure the lengths of the scale drawing and draw anew scale drawing with a scale factor (SD2 to SD1) of ½.

   - New scale factor: 1/2
   - 10 feet → 5 feet
   - 5 cm → 2.5 cm
   - 2 ft → 1 ft
   - 4 feet → 2 feet
   - 1 cm → 0.5 cm
   - 2 cm → 1 cm
2. Use the measurements on the diagrams below to identify whether each would be scale drawings of a garden. The garden contains a rectangular portion measuring 24 ft by 6 ft and two circular fountains each with a diameter of 5 ft.

   a. 10 in
   b. 2 1/2 ft
   c. 3 1/3 cm
   d. 1 2/3 cm

   3 in
   12 in
   3cm
   12cm
   4 cm
   2 cm
   6 cm

   b and c

3. Compute the scale factor of the new scale drawing (SD2) to original scale drawing (SD1) using information from the given scale drawing.

   a. Original Scale Factor: 6/35
      New Scale Factor: 1/280
      Scale Factor: \( \frac{6}{35} \)

   b. Original Scale Factor: 1/12
      New Scale Factor: 3
      Scale Factor: \( 36 \)

   c. Original Scale Factor: 20
      New Scale Factor: 25
      Scale Factor: \( \frac{5}{4} \)