Topic C

Area of Rectangular Figures with Fractional Side Lengths

5.NF.4b, 5.NF.6

Focus Standard: 5.NF.4b  Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction.
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.6  Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Instructional Days: 6

Coherence -Links from: G4–M4  Angle Measure and Plane Figures
   -Links to: G6–M2  Arithmetic Operations Including Division of Fractions

In Topic C, students extend their understanding of area as they use rulers and right angle templates to construct and measure rectangles with fractional side lengths and find their areas. They apply their extensive knowledge of fraction multiplication to interpret areas of rectangles with fractional side lengths (5.NF.4b) and solve real world problems involving these figures (5.NF.6), including reasoning about scaling through contexts in which areas are compared. Visual models and equations are used to represent the problems through the Read-Draw-Write protocol.
### A Teaching Sequence Towards Mastery of Area of Rectangular Figures with Fractional Side Lengths

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Lesson 10

Objective: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Multiply Decimals 5.NBT.7 (4 minutes)
- Change Mixed Numbers to Fractions 4.NF.4 (4 minutes)
- Multiply Mixed Numbers and Fractions 5.NF.4 (4 minutes)

Multiply Decimals (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G5–M4–Lessons 17–18.

T: (Write 2 × 2 = ___.) Say the multiplication sentence.
S: 2 × 2 = 4.

T: (Write 2 × 0.2 = ___.) On your boards, write the number sentence.
S: (Write 2 × 0.2 = 0.4.)

T: (Write 0.2 × 0.2 = ___.) On your boards, write the number sentence.
S: (Write 0.2 × 0.2 = 0.04.)

Continue the process using the following possible suggestions: 3 × 4, 3 × 0.4, 0.3 × 0.4, 0.03 × 0.4, 5 × 7, 0.5 × 7, 0.5 × 0.7, and 0.5 × 0.07.
Change Mixed Numbers to Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for today’s lesson.

T: How many fourths are in 1?
S: 4.
T: How many fourths are in 2?
S: 8.
T: (Write $2 \frac{1}{4} =$ ____.) On your boards, write $2 \frac{1}{4}$ as an improper fraction.
S: (Write $2 \frac{1}{4} = \frac{9}{4}$.)

Continue the process for the following possible sequence: $2 \frac{3}{4}, 2 \frac{1}{2}, 4 \frac{2}{3}, 3 \frac{3}{4}, 2 \frac{5}{6}, 3 \frac{3}{8}, 4 \frac{5}{8}$, and $5 \frac{7}{8}$.

Multiply Mixed Numbers and Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for today’s lesson.

T: (Write $\frac{3}{2} \times \frac{1}{3} =$ ____ Point to $\frac{3}{2}$.) Say $\frac{3}{2}$ as a fraction.
S: $\frac{7}{2}$

T: (Write $\frac{7}{2} \times \frac{1}{3}$ Point to $\frac{1}{3}$.) Say $\frac{1}{3}$ as a fraction.
S: $\frac{7}{6}$

T: (Write $\frac{7}{2} \times \frac{7}{3}$. Beneath it, write $\times$. Beneath it, write $\times$.) Multiply the fractions. Then, write the answer as a mixed number.
S: (Write $\frac{49}{6}$. Beneath it, write $\times$. Beneath it, write $8 \frac{1}{6}$.)

Continue the process for the following possible sequence: $3 \frac{1}{3} \times 2 \frac{2}{4}$ and $3 \frac{4}{5} \times 4 \frac{2}{3}$.

Application Problem (8 minutes)

Heidi and Andrew designed two raised flowerbeds for their garden. Heidi’s flowerbed was 5 feet long by 3 feet wide, and Andrew’s flowerbed was the same length, but twice as wide. Calculate how many cubic feet of soil they need to buy in order to have soil to a depth of 2 feet in both flowerbeds.

Note: This Application Problem reviews the volume work from earlier in the module.
Lesson 10: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing and relate to fraction multiplication.

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Concept Development (30 minutes)

Materials: (T) 3 unit \(\times\) 2 unit rectangle, patty paper units for tiling, white board (5 large mystery rectangles lettered A–E (1 of each size per group), patty paper units for tiling, Problem Set

Note: The lesson is written such that the length of one standard patty paper (\(5\frac{1}{2}\)” by \(5\frac{1}{2}\)”) is one unit. Hamburger patty paper (available from big box discount stores in boxes of 1,000) is the ideal square unit for this lesson due to its translucence and size. Measurements for the mystery rectangles are given in generic units so that any size square unit may be used to tile, as long as the tiling units can be folded. Any paper may be used if patty paper is not available. Consider color-coding Rectangles A–E for easy reference.

Preparation: Each group needs one copy of Rectangles A–E. The most efficient way of producing these rectangles is to use the patty paper to measure and trace the outer dimensions of one rectangle. Then use that rectangle as a template to cut the number required for the class. Rectangles should measure as follows:

Demo Rectangle A: \(3\) units \(\times\) \(2\) units

Rectangle B: \(3\) units \(\times\) \(2\frac{1}{2}\) units

Rectangle C: \(1\frac{1}{2}\) units \(\times\) \(5\) units

Rectangle D: \(2\) units \(\times\) \(3\frac{3}{4}\) units

Rectangle E: \(\frac{3}{4}\) unit \(\times\) \(5\) units

T: We want to determine the areas of some mystery rectangles today. Find the rectangle at your table labeled A. (Allow students time to find the rectangle.)

T: If we want to find the area of this mystery rectangle, what kind of units would we use to measure it?

S: Square units.

T: (Hold up a patty paper tile.) This will be the square unit we will use to find the area of Rectangle A. Work with your partner to find the number of squares that will cover this rectangle with no space between units and no overlaps. Please start at the top left hand corner to place your first tile. (Allow students time to work.)

T: How many square units covered the rectangle?

S: 6 square units.

NOTES ON MULTIPLE MEANS REPRESENTATIONS:
Folding the square units allows students to clearly see the relationship of the fractional square unit while maintaining the relationship to the whole square unit. Consequently, if students become confused about the size of the fractional square unit, the paper may be easily unfolded as a reminder.
Lesson 10: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing and relate to fraction multiplication.

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T: Let’s sketch a picture of what our tiling looks like. Draw the outside of your rectangle first. (Model as students draw.)

T: Now show the six tiles. (Allow students time to draw.)

T: Look at the longest side of your rectangle. If we wanted to measure this side with a piece of string, how many units long would the string need to be? Explain how you know to your partner.

S: It is 3 units long. I can look at the edge of the units and count. To measure the length of the side, I’m not looking at the whole tile; I only need to count the length of each unit. There are 3 equal units on the edge, so the string would need to be 3 units long.

T: Let’s record that. (Write in the length of Rectangle A in the chart.) What is the length of the shorter side?

S: 2 units.

T: Let’s record that in our chart.

T: What is the area of Rectangle A?

S: 6 square units.

T: If we had only labeled the length and the width in our sketch, could we still know the area? Why or why not?

S: Yes. We know the square units are there even if we don’t draw them all. We still just multiply the sides together. We can imagine the tiles.

T: What would a sketch of this look like? Draw it with your partner. (Allow students time to draw.)

T: Now find Rectangle B. Compare its size to Rectangle A. Will its area be greater than or less than that of Rectangle A?

S: Greater.

T: We see that A and B are the same length. What about the width?

S: Rectangle B is wider than two tiles, but not as wide as three tiles.

T: Fold your tiles to decide what fraction of another tile we will need to cover the extra width. Work with your partner. (Allow students time to fold.)

T: What fraction of the tile do you need to cover this part of the rectangle? How do you know?

S: I need half a tile. I laid a whole tile over the extra part and it looked like half to me. After I folded up the part of the tile that was hanging off the rectangle, I could see that the fold split the tile into two equal parts. That means it is halves.
Lesson 10: Find the area of rectangles with whole-by-mixed and whole-by-
fractional number side lengths by tiling, record by drawing and relate
to fraction multiplication.

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NOTES ON MULTIPLE MEANS OF ENGAGEMENT:
The spatial and visualization skills involved in G5–M5—Lessons 10 and 11
will be quite natural for some students, and quite challenging for others.
Consequently, the time needed to accomplish the tasks will vary, but all
students should be given the opportunity to tile all the rectangles. Both
lessons offer two challenging questions at the end of the Problem
Sets for those who finish the tiling quickly.
T: Does this $7\frac{1}{2}$ unit squared area make sense given our prediction? Why or why not?

S: It does make sense. It is only a little wider than the first rectangle, and $7\frac{1}{2}$ isn’t that much more than 6. → You can see the first rectangle inside this one. There was a part that was 3 units by 2 units, and then a smaller part was added on that was 3 units by just half another unit. That’s where the extra $1\frac{1}{2}$ square units come from. → Three times two was easy, and then I know that half of 3 is $\frac{3}{2}$. By decomposing the mixed number, it was easy to find the total area.

T: Work with your partner to find the length, width, and area of Rectangles C, D, and E using the patty paper and recording with the area model. Record your findings on your Problem Set, and then answer the last two questions in the time remaining. You may record your tiling without drawing each tile if you wish.

S: (Work.)

Problem Set (5 minutes)

Students should do their personal best to complete the remainder of the Problem Set within the allotted five minutes if they have finished the tiling problems. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

Record the students’ answers to Task 1 to complete the class chart as answers are reviewed.

- What relationship did you notice between the areas of Rectangle C and Rectangle E? What accounts for this relationship?
- How was Rectangle E different from the other rectangles you tiled? Describe how you tiled it.
Lesson 10: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing and relate to fraction multiplication.

How did you determine the area of Rectangle E?
Did you count the single units? Add repeatedly? Multiply the sides?
Could you place these rectangles in order of greatest to least area by using relationships among the dimensions, but without actually performing the calculations? Why or why not?
How did you determine the area of the rectangle in Problem 6?
Analyze and compare different solution strategies for Problem 7.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 10 Problem Set

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Name _____________________________  Date ______________

Sketch the rectangles and your tiling. Write the dimensions and the units you counted in the blanks. Then use multiplication to confirm the area. Show your work. We will do Rectangles A and B together.

1. **Rectangle A:**
   - Rectangle A is
     - ________ units long  ________ units wide
   - Area = ________ units²

2. **Rectangle B:**
   - Rectangle B is
     - ________ units long  ________ units wide
   - Area = ________ units²

3. **Rectangle C:**
   - Rectangle C is
     - ________ units long  ________ units wide
   - Area = ________ units²

4. **Rectangle D:**
   - Rectangle D is
     - ________ units long  ________ units wide
   - Area = ________ units²

5. **Rectangle E:**
   - Rectangle E is
     - ________ units long  ________ units wide
   - Area = ________ units²

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Lesson 10: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing and relate to fraction multiplication.

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6. The rectangle to the right is composed of squares that measure \(2 \frac{1}{4}\) inches on each side. What is its area in square inches? Explain your thinking using pictures and numbers.

7. A rectangle has a perimeter of \(35 \frac{1}{2}\) feet. If the width is 12 ft, what is the area of the rectangle?
Emma tiled a rectangle and then sketched her work. Fill in the missing information, and multiply to find the area.

Emma's Rectangle:

_______ units long _______ units wide

Area = _________ units²
Lesson 10: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing and relate to fraction multiplication.

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1. John tiled some rectangles using square unit. Sketch the rectangles if necessary, fill in the missing information, and then confirm the area by multiplying.

   a. **Rectangle A:**

   Rectangle A is

   \[
   \begin{array}{c}
   3 \text{ units long} \\
   2\frac{1}{2} \text{ units wide}
   \end{array}
   \]

   Area = _______ units\(^2\)

   [Diagram of Rectangle A]

   b. **Rectangle B:**

   Rectangle B is

   \[
   \begin{array}{c}
   \text{units long} \\
   \text{units wide}
   \end{array}
   \]

   Area = _______ units\(^2\)

   [Diagram of Rectangle B]

   c. **Rectangle C:**

   Rectangle C is

   \[
   \begin{array}{c}
   \frac{3}{4} \text{ units long} \\
   4 \text{ units wide}
   \end{array}
   \]

   Area = _______ units\(^2\)

   [Diagram of Rectangle C]
d. **Rectangle D:**

Rectangle D is

\[
\begin{array}{c|c}
2 \text{ units long} & \frac{3}{4} \text{ units wide} \\
\end{array}
\]

Area = ________ units\(^2\)

2. Rachel made a mosaic from different color rectangular tiles. Three tiles measured \(3\frac{1}{2}\) inches \(\times\) 3 inches. Six tiles measured 4 inches \(\times\) \(3\frac{1}{4}\) inches. What is the area of the whole mosaic in square inches?

3. A garden box has a perimeter of \(27\frac{1}{2}\) feet. If the length is 9 feet, what is the area of the garden box?
Lesson 11

Objective: Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

Suggested Lesson Structure

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<td>Application Problem</td>
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Fluency Practice (12 minutes)

- Sprint: Multiply Decimals 5.NBT.7 (9 minutes)
- Multiplying Fractions 5.NF.4 (3 minutes)

Sprint: Multiply Decimals (9 minutes)

Materials: (S) Multiply Decimals Sprint

Note: This fluency reviews G5–Module 4.

Multiplying Fractions (3 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for G5–M5–Lesson 13.

T: (Write $\frac{1}{2} \times \frac{1}{3} = \_\_\_.$) Say the multiplication equation.

S: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

T: (Write $\frac{1}{2} \times \frac{3}{4} = \_\_\_.$) Say the multiplication equation.

S: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

T: (Write $\frac{2}{5} \times \frac{2}{3} = \_\_\_.$) On your boards, write the multiplication equation.

S: (Write $\frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$)
Lesson 11: Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

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Application Problem (5 minutes)

Ms. Golden wants to cover her 6.5 foot by 4 foot bulletin board with silver paper that comes in 1-foot squares. How many squares does Mrs. Golden need to cover her bulletin board? Will there be any fractional pieces of silver paper left over? Explain why or why not. Draw a sketch to show your thinking.

Note: This Application Problem reviews G5–M5–Lesson 10’s concept of tiling, now with one dimension as a decimal fraction.

Concept Development (33 minutes)

Materials: (T) Rectangles, patty paper units for tiling, white board (S) 1 demo and 5 mystery rectangles lettered A–E (1 of each size per group), patty paper units for tiling, Problem Set

Note: Today’s lesson parallels the structure of G5–M5–Lesson 10. Rectangles for each group should be prepared in advance following yesterday’s instructions. The dimensions of today’s rectangles are given below.

Rectangle A: \( \frac{3}{2} \times \frac{1}{2} \) units

Rectangle B: \( 1 \frac{3}{4} \times 3 \frac{3}{4} \) units

Rectangle C: \( \frac{3}{4} \times 1 \frac{1}{2} \) units

Rectangle D: \( \frac{3}{4} \times \frac{1}{2} \) units

Rectangle E: \( 2 \frac{1}{3} \times \frac{2}{3} \) units

The added complexities of today’s lesson involve the inclusion of two mixed number or fractional side lengths. This is an application of the fraction multiplication lessons of G5–Module 4. Students will also record partial products rather than draw individual tiles.

T: Let’s start with Rectangle A. Work with your partner to place as many whole tiles on Rectangle A as you can. Remember to start at the top left corner of the rectangle.
Lesson 11:
Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

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NOTES ON MULTIPLE MEANS OF ENGAGEMENT:
Include Rectangle E as an optional challenge. Folding and tiling Rectangle E requires students to fold thirds and to reason about another area less than 1 square tile. Recording for Rectangle E should be done on separate paper, as it is not included on the Problem Set. The last two problems on the Problem Set also offer extensions for students who finish the tiling and multiplication quickly.

NOTES ON MULTIPLE MEANS OF EXPRESSION:
For some students, it may be more effective to place a whole square unit over the last corner of the rectangle and then trace the outline or shade the corner of the rectangle on the patty paper. (Because the patty paper is translucent, the edge of the rectangle is clearly visible.) Students may then fold until only the outlined portion of the paper is visible. When the paper is unfolded, only 1 of the 4 equal parts is bordered (or shaded).

Guide students also to isolate the last corner of the rectangle and use a single piece of patty paper to model the multiplication of a fraction by a fraction to produce a double-shaded area (as in G5–Module 4). This double-shaded portion can then be laid on top of the rectangle’s corner and should fit perfectly.

S: (Place whole tiles on Rectangle A.)
T: How many whole tiles fit?
S: 8.
T: Is this the area of the rectangle?
S: No.
T: Fold some of your square units to cover the rest of the rectangle’s length. (Allow time for students to work.)
T: What fractional unit do you need to do this? How many?
S: I needed 2 half units. The unit was halves. I needed 2 of them.
T: Now, fold some units to cover the rest of the rectangle’s width. (Allow time for students to work.)
T: What fractional unit did you use this time, and how many?
S: I needed halves again. This time it was 4. It was 4 half units.
T: I see that we’ve covered almost all of the rectangle, but there seems to be a part at the bottom that is even smaller than the halves we just placed. How can we find the fractional unit that will fit here? Turn and talk.
S: I can see that if I fold a square unit in half, it fits in one direction, but it’s too long in the other direction. Maybe if I fold it again, it will fit. If I fold it in half, it fits the length. Then if I fold that half in half again, it fits perfectly in the space. The part is half the size of half a square unit. Half of a half is 1 fourth of a square unit.
T: Unfold the paper that you’ve made to fit in this part. What fraction of a whole square unit covers this part?
S: 1 fourth of a square unit.
T: Work with your partner to count the tiles to determine the area.
S: (Count tiles with partner.)
T: What is the area? How did you count it?
S: I counted the 8 squares first. I added 6 halves, or 3 more squares. Then, I added \( \frac{1}{4} \) to 11. That’s 11 \( \frac{1}{4} \) square units. I could see two rows of \( \frac{4}{2} \) units. That is 9. Then there were 4 halves and \( \frac{1}{4} \) in a row.
Lesson 11:

Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

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That is $2\frac{1}{4} + 9 = 11\frac{1}{4}$. The area is $11\frac{1}{4}$ square units.

T: Let’s record our thinking. I’ll work on the board. You record on your Problem Set. Sketch the rectangle first. Decompose the length and width into ones and fractional parts.

S: (Sketch and decompose the length and width.)

T: How did you decompose the length?

S: $4 + \frac{1}{2}$.

T: (Record in the algorithm.) The width?

S: $2 + \frac{1}{2}$.

T: (Record in the algorithm.) Let’s use multiplication to confirm the area we found with counting. Let’s start with the ones. (Point, then record each partial product in the rectangle and in the algorithm.)

2 units $\times$ 4 units equals?

S: 8 square units.

T: (Point and record.) 2 units $\times$ $\frac{1}{2}$ unit equals?

S: 2 half square units. $\rightarrow$ 1 square unit.

T: (Point and record.) $\frac{1}{2}$ unit $\times$ 4 units equals?

S: 4 half square units $\rightarrow$ 2 square units.

T: (Point and record.) $\frac{1}{2}$ unit $\times$ $\frac{1}{2}$ unit equals?

S: $\frac{1}{4}$ square unit.

T: Find the sum.

S: (Work to find $11\frac{1}{4}$ units$^2$.)

T: Was the area the same using multiplication and the area model?

S: Yes!

T: Use your tiles to determine the area and dimensions of the other rectangles. Record your findings on your Problem Set. Then multiply to confirm the area.

### Problem Set (5 minutes)

Students should do their personal best to complete the remainder of the Problem Set within the allotted 5 minutes if they have finished the tiling problems. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
**Student Debrief (10 minutes)**

**Lesson Objective:** Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Compare the rectangles we tiled today to the rectangles we tiled yesterday. What do you notice? How did that change the way we had to tile?
- Which rectangle was the easiest to tile? Which was the hardest? Why?
- Explain your strategy for tiling Rectangle D (and Rectangle E, where applicable). How was finding the area of this rectangle similar to the fraction multiplication we did in Module 4? How was it different?
- Explain your strategy for finding the areas of the rectangles in Problem 5 and how you compared them.
- How is Problem 6 in today’s Problem Set like Problem 6 in yesterday’s Problem Set (G5–M5–Lesson 10), and how is it different? Yesterday’s problem read: A rectangle has a perimeter of $35\frac{1}{2}$ feet. If the width is 12 ft, what is the area of the rectangle?
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 11: Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

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<th>Multiply</th>
<th># Correct</th>
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<td>1</td>
<td>3 x 2 =</td>
<td>23 0.6 x 2 =</td>
</tr>
<tr>
<td>2</td>
<td>3 x 0.2 =</td>
<td>24 0.6 x 0.2 =</td>
</tr>
<tr>
<td>3</td>
<td>3 x 0.02 =</td>
<td>25 0.6 x 0.02 =</td>
</tr>
<tr>
<td>4</td>
<td>3 x 3 =</td>
<td>26 0.2 x 0.06 =</td>
</tr>
<tr>
<td>5</td>
<td>3 x 0.3 =</td>
<td>27 5 x 7 =</td>
</tr>
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Lesson 11: Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

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Lesson 11 Problem Set

1. Rectangle A:
   - Draw the rectangle and your tiling.
   - Write the dimensions and the units you counted in the blanks.
   - Then, use multiplication to confirm the area. Show your work.

2. Rectangle B:

   Rectangle A is
   - _______ units long    _______ units wide
   - Area = _______ units\(^2\)

   Rectangle B is
   - _______ units long    _______ units wide
   - Area = _______ units\(^2\)

3. Rectangle C:

   Rectangle C is
   - _______ units long    _______ units wide
   - Area = _______ units\(^2\)

4. Rectangle D:

   Rectangle D is
   - _______ units long    _______ units wide
   - Area = _______ units\(^2\)
Lesson 11

Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

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5. Colleen and Caroline each built a rectangle out of square tiles placed in 3 rows of 5. Colleen used tiles that measured \(1\frac{2}{3}\) cm squares. Caroline used tiles that measured \(3\frac{1}{3}\) cm.
   a. Draw the girls’ rectangles, and label the lengths and widths of each.

   b. What are the areas of the rectangles in square centimeters?

   c. Compare the area of the rectangles.

6. A square has a perimeter of 51 inches. What is the area of the square?
1. To find the area, Andrea tiled a rectangle and sketched her answer. Sketch the rectangle, and find the area. Show your multiplication work.

Rectangle is

\[ 2 \frac{1}{2} \text{ units} \times 2 \frac{1}{2} \text{ units} \]

Area = ______________
1. Kristen tiled the following rectangles using square units. Sketch the rectangles, and find the areas. Then confirm the area by multiplying. Rectangle A has been sketched for you.
   a. **Rectangle A:**

   Rectangle A is
   \[ \frac{3}{4} \text{ unit long} \times \frac{1}{2} \text{ unit wide} \]
   Area = \[ \frac{3}{8} \text{ units}^2 \]

   b. **Rectangle B:**

   Rectangle B is
   \[ 2 \frac{1}{2} \text{ units long} \times \frac{3}{4} \text{ unit wide} \]
   Area = \[ \frac{3}{2} \text{ units}^2 \]

   c. **Rectangle C:**

   Rectangle C is
   \[ 3 \frac{1}{3} \text{ units long} \times 2 \frac{1}{2} \text{ units wide} \]
   Area = \[ 5 \frac{1}{6} \text{ units}^2 \]
Lesson 11:
Find the area of rectangles with mixed-by-mixed and fraction-by-
fraction side lengths by tiling, record by drawing, and relate to
fraction multiplication.

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2. A square has a perimeter of 25 inches. What is the area of the square?

Rectangle D:
Rectangle D is
$3\frac{1}{2}$ units long $\times 2\frac{1}{4}$ units wide

Area = ________ units$^2$
Lesson 12

Objective: Measure to find the area of rectangles with fractional side lengths.

Suggested Lesson Structure

- Fluency Practice (10 minutes)
- Application Problem (3 minutes)
- Concept Development (37 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (10 minutes)

- Multiplying Fractions 5.NF.4 (4 minutes)
- Find the Volume 5.MD.C (6 minutes)

Multiplying Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for G5–M5–Lesson 13.

T: (Write $\frac{1}{3} \times \frac{1}{4}$.) Say the multiplication number sentence.

S: $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.

T: (Write $\frac{1}{3} \times \frac{2}{5}$.) Say the multiplication number sentence.

S: $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.

T: (Write $\frac{3}{5} \times \frac{2}{3}$. Beneath it, write = __.) On your boards, write the multiplication number sentence. Then, simplify the fraction.

S: (Write $\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$. Beneath it, write $\frac{2}{5}$.)

Continue the process for the following possible sequence: $\frac{1}{2} \times \frac{1}{4}$, $\frac{1}{2} \times \frac{1}{4}$, $\frac{3}{4} \times \frac{3}{3}$, $\frac{3}{4} \times \frac{2}{6}$, $\frac{2}{3} \times \frac{2}{3}$, and $\frac{3}{4} \times \frac{7}{8}$.
Lesson 12: Measure to find the area of rectangles with fractional side lengths.

Find the Volume (6 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews volume concepts and formulas.

T: (Project a prism 5 units × 2 units × 4 units. Write V = ___ units × ___ units × ___ units.) Find the volume.
S: (Write 40 units³ = 5 units × 2 units × 4 units.)
T: How many layers of 10 cubes are in the prism?
S: 4 layers.
T: (Write 4 × 10 units³ = _____.) Four copies of 10 cubic units is...?
S: 40 cubic units.
T: How many layers of 8 cubes are there?
S: 5 layers.
T: (Write 5 × 8 units³ = _____.) Five copies of 8 cubic units is...?
S: 40 cubic units.
T: How many layers of 20 cubes are there?
S: 2 layers.
T: Write a multiplication sentence to find the volume of the prism, starting with the number of layers. (Point.)
S: (Write 2 units × 20 units² = 40 units³.)

Repeat the process with the following prisms.

Application Problem (3 minutes)

Margo is designing a label. The dimensions of the label are 3 \( \frac{1}{2} \) inches by 1 \( \frac{1}{4} \) inches. What is the area of the label? Use the RDW process.

Note: Students can use the area model used in G5–Module 4 and in G5–M5–Lessons 10–11 to solve. This bridges to today’s lesson, which extends the use of the area model.
Lesson 12: Measure to find the area of rectangles with fractional side lengths.

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NOTES ON MULTIPLE MEANS OF REPRESENTATION:
Some students will benefit from drawing each square inch as a tile, connecting back to the tiling process. Others may need to use inch tile manipulatives to understand this process. (Remember that concrete materials should be foldable.) Encourage students often to return to pictorial or concrete representations as needed during any lesson to scaffold understanding.

Concept Development (37 minutes)

Materials: (T) Ruler, projector (S) Ruler, Problem Set

Problem 1(a)

Project the first rectangle in the Problem Set.

T: We will find areas of more mystery rectangles today. What was the relationship of the areas we found using square tiles and the areas we found using multiplication?

S: We got the same answers. Tiling or finding partial products using multiplication will always give the same area, because the rectangle we are using is the same.

T: Today we’ll use a ruler to help us find area. Turn and talk to your partner about how you think a ruler might be useful in finding the area of a rectangle.

S: It’s not square units, but we can measure the edges. The ruler lets us measure the sides to find out the lengths we need to multiply.

T: Work with your partner to measure the lengths of the first rectangle of the Problem Set. Compare your measurements.

S: (Measure the first rectangle.)

T: What are the lengths of the side?

S: 2 inches and 2 \(\frac{1}{2}\) inches.

T: Estimate the area of this rectangle. Turn and talk.

S: If this was just a 2 inch square, the area would be 4 square inches. It’s a little longer than that, so it will be a little more than 4. The longer side is between 2 and 3 inches, so the area should be somewhere between 4 square inches and 6 square inches.

T: Let’s find the actual area. Decompose the longer side by marking the end of the 2 whole inches and labeling the wholes and the half inch on our rectangles. (Model on the board as shown.)

S: (Decompose and label.)

T: Now, let’s use this decomposition to find the area of smaller parts of the rectangle. Using your ruler, draw a line separating the ones from the fractional units. (Model.)

S: (Separate the ones with a line.)

T: Now, let’s multiply to find the areas of these sections. Let’s start with the ones by ones part. Talk with your partner. What is the area of the part that is 2 inches by 2 inches? If it helps, imagine or draw tiles in your rectangle.

S: There are two going across and two rows of them, so four altogether. I remember that I can multiply the sides, so 2 inches \(\times\) 2 inches is 4 square inches.

T: What is the area?
S: 4 square inches.
T: Record that.
T: Turn and talk. What is the area of the smaller part? How do you know?
S: Half of 2, so 1. \( \rightarrow \) Two times \( \frac{1}{2} \). Two halves make 1, so 1. \( \rightarrow \) 1 square inch.
T: Yes, the area is 1 square inch. Let’s write that too. (Model as shown at right.)
T: What is the total area of the rectangle? Does our answer make sense?
S: 5 square inches. \( \rightarrow \) It makes sense because we said the area should be between 4 and 6 square inches and it is.

**Problem 1(b)**

Project the second rectangle in the Problem Set.

T: Measure the next rectangle with your ruler. (Allow students time to measure.)
T: What is the length?
S: 1 \( \frac{3}{4} \) inches.
T: And the width?
S: 1 \( \frac{3}{4} \) inches. \( \rightarrow \) This is a square so the width is also 1 \( \frac{3}{4} \) inches.
T: Estimate the area with your partner.
S: It’s almost 2 inches by 2 inches. The area should be less than 4 square inches. \( \rightarrow \) The area will be between 1 square inch and 4 square inches, but closer to 4 because the sides are almost 2 inches long.
T: Decompose the sides into ones and fractional parts and record that on your Problem Set.

Circulate and assist students. Then, project a student’s work, or record on the board as shown.

T: Work with your partner to find the area of each of these four parts.
S: (Find the area of each of the four parts.)
T: What is the area of the part that is 1 inch on each side?
S: 1 square inch.
T: Then we have two parts with 1 inch on one side and \( \frac{3}{4} \) inch on the other. What is the area of each of those parts? How do you know?
S: It’s not a whole square inch. → A whole tile wouldn’t fit in either of these places. We would have to fold it to make it fit. → Three-fourths of a square inch because 3 fourths times 1 is 3 fourths.

T: (Record the measures in each part of the area model.) Now we’re left with the last little square. It is \( \frac{3}{4} \) of an inch on each side. Is this area greater or less than the other parts? How do you know?

S: It’s smaller, because both sides are shorter than the other parts. → It’s only part of an inch on each side, so it will be less area. → The area is a fraction of a fraction. We want 3 fourths of 3 fourths. It’s a fraction of an inch on each side. Three-fourths of a square inch would be like splitting a whole into four parts and taking one part off.

T: What do we need to do to find the area of this last section of our square?

S: Just like before, we need to multiply the length times the width. → We need to multiply \( \frac{3}{4} \) by \( \frac{3}{4} \).

T: What is the area of the small square?

S: \( \frac{9}{16} \) square inch.

T: How will we find the total area?

S: Add all the parts. → Add across each row and then add the rows together.

Circulate and support students as they add the partial products. Review the need for common denominators as necessary.

T: What is the total area of the square?

S: \( 3 \frac{1}{16} \) square inches!

Repeat this sequence of questioning with each problem as necessary. As students understand the concept, release them to work independently.

**Problem Set (5 minutes)**

Students should do their personal best to complete the remainder of the Problem Set within the allotted 5 minutes. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

**Student Debrief (10 minutes)**

**Lesson Objective:** Measure to find the area of rectangles with fractional side lengths.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for
misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Look back at the area model we did together in Problem 1(b) \((1 \frac{3}{4} \times 1 \frac{2}{4})\). How many squares do you see in your area model? What patterns do you see whenever you have an area model of a square?
- What is the relationship between Problem 1(e) and Problem 1(f) in the Problem Set? (Both rectangles have the same area. The length of Problem 1(f) is 5 times the length of Problem 1(e). The width of Problem 1(f) is one-fifth the width of Problem 1(e).)
- Using mental math, how can you find \(\frac{1}{2}\) times any fraction? (Double the denominator.)
- How is Problem 2(b) like the example we did together, \(1 \frac{3}{4} \times 1 \frac{3}{4}\) (Both have two factors that are the same.)

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. Measure each rectangle with your ruler, and label the dimensions. Use the area model to find each area.

a. 

b. 

c. 

d.
Lesson 12: Measure to find the area of rectangles with fractional side lengths.

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2. Find the area. Explain your thinking using the area model.

a. $1 \text{ ft} \times 1 \frac{1}{2} \text{ ft}$

b. $1 \frac{1}{2} \text{ yd} \times 1 \frac{1}{2} \text{ yd}$

c. $2 \frac{1}{2} \text{ yd} \times 1 \frac{3}{16} \text{ yd}$
3. Hanley is putting carpet in her house. She wants to carpet her living room, which measures 15 ft \times 12\frac{1}{3} ft. She also wants to carpet her dining room, which is 10\frac{1}{4} ft \times 10\frac{1}{3} ft. How many square feet of carpet will she need to cover both rooms?

4. Fred cut a 9\frac{3}{4} inch square of construction paper for an art project. He cut a square from the edge of the big rectangle whose sides measured 3\frac{1}{4} inches. (See picture below.)

   a. What is the area of the smaller square that Fred cut out?

   b. What is the area of the remaining paper?
Lesson 12: Measure to find the area of rectangles with fractional side lengths.

Date: 1/10/14

Measure the rectangle with your ruler, and label the dimensions. Find the area.

1.
Lesson 12: Measure to find the area of rectangles with fraction side lengths.

Name __________________________________________________________________________ Date _________________

1. Measure each rectangle with your ruler, and label the dimensions. Use the area model to find the area.

   a. 
   
   b. 
   
   c. 
   
   d. 
   
   e. 

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2. Find the area. Explain your thinking using the area model.
   a. $2\frac{1}{4} \text{ yd} \times \frac{1}{4} \text{ yd}$
   b. $2\frac{1}{2} \text{ ft} \times 1\frac{1}{4} \text{ ft}$

3. Kelly buys a tarp to cover the area under her tent. The tent is 4 feet wide and has an area of 31 square feet. The tarp she bought is $5\frac{1}{3}$ feet by $5\frac{3}{4}$ feet. Can the tarp cover the area under Kelly’s tent? Draw a model to show your thinking.

4. Shannon and Leslie want to carpet a $16\frac{1}{2}$ by $16\frac{1}{2}$ square foot room. They can’t put carpet under an entertainment system that juts out. (See the drawing below.)

   a. In square feet, what is the area of the space with no carpet?

   b. How many square feet of carpet will Shannon and Leslie need to buy?
Lesson 13

Objective: Multiply mixed number factors, and relate to the distributive property and area model.

Suggested Lesson Structure

- Fluency Practice (10 minutes)
- Application Problem (7 minutes)
- Concept Development (33 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (10 minutes)

- Multiplying Fractions 5.NF.4 (4 minutes)
- Find the Volume 5.MD.C (6 minutes)

Multiplying Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for today's lesson.

T: (Write \( \frac{1}{3} \times \frac{1}{5} = \_ \_ \_ \).) Say the multiplication equation.

S: \( \frac{1}{3} \times \frac{1}{5} = \frac{1}{15} \).

T: (Write \( \frac{2}{3} \times \frac{2}{5} = \_ \_ \_ \).) Say the multiplication equation.

S: \( \frac{2}{3} \times \frac{2}{5} = \frac{4}{15} \).

T: (Write \( \frac{3}{4} \times \frac{3}{3} = \_ \_ \_ \). Beneath it, write \( = \_ \_ \_ \).) On your boards, write the multiplication equation. Then, simplify the fraction.

S: (Write \( \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} \). Beneath it, write \( = \frac{1}{2} \).)

Continue the process for the following possible sequence: \( \frac{1}{2} \times \frac{3}{4}, \frac{2}{3} \times \frac{3}{5}, \frac{3}{4} \times \frac{4}{5}, \frac{5}{6} \times \frac{3}{6}, \) and \( \frac{3}{5} \times \frac{5}{6} \).
Lesson 13: Multiply mixed number factors, and relate to the distributive property and area model.

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Lesson 13: Multiply mixed number factors, and relate to the distributive property and area model.

Find the Volume (6 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews volume concepts and formulas.

T: (Project a prism 4 units $\times$ 2 units $\times$ 3 units. Write $V = \_ \_ \_ \_ units \times \_ \_ \_ \_ units \times \_ \_ \_ \_ units.$) Find the volume.
S: (Write $24 \text{ units}^3 = 4 \text{ units} \times 2 \text{ units} \times 3 \text{ units}.$)
T: How many layers of 6 cubes are in the prism?
S: 4 layers.
T: (Write $4 \times 6 \text{ units}^3$.) Four copies of 6 cubic units is...?
S: 24 cubic units.
T: How many layers of 8 cubes are there?
S: 3 layers.
T: (Write $3 \times 8 \text{ units}^3$.) Three copies of 8 cubic units is...?
S: 24 cubic units.
T: How many layers of 12 cubes are there?
S: 2 layers.
T: Write a multiplication equation to find the volume of the prism, starting with the number of layers.
S: (Write $2 \times 12 \text{ units}^3 = 24 \text{ units}^3$.)

Repeat the process for the prisms pictured.

Application Problem (7 minutes)

The Colliers want to put new flooring in a $6\frac{1}{2}$ foot by $7\frac{1}{3}$ foot bathroom. The tiles they want come in 12-inch squares. What is the area of the bathroom floor? If the tile costs $3.25 per square foot, how much will they spend on the flooring?

Note: This type of tiling applies the work from G5–M5–Lessons 10–13 and bridges to today’s lesson on the distributive property.
Lesson 13: Multiply mixed number factors, and relate to the distributive property and area model.

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Concept Development (33 minutes)

Materials: (S) Personal white boards

In this lesson, students reason about the most efficient strategy to use to multiply mixed numbers: distributing with the area model or multiplying improper fractions and canceling to simplify.

Problem 1

Find the area of a rectangle $1 \frac{1}{3}$ inches $\times$ $3 \frac{3}{4}$ inches and discuss strategies for solving.

T: (Project Rectangle 1.) How is this rectangle different from the rectangles we’ve been working with?

S: We know the dimensions of this one. The side lengths are given to us, so we don’t need to tile or measure.

T: Find the area of this rectangle. Use an area model to show your thinking.

S: (Find the area using a model.)

T: What is the area of this rectangle?

S: 5 inches squared.

T: We’ve used the area model many times in Grade 5 to help us multiply numbers with mixed units. How are these side lengths like multi-digit numbers? Turn and talk.

S: A two-digit number has two different size units in it. The ones are smaller units, and the tens are the bigger units. These mixed numbers are like that. The ones are the bigger units, and the fractions are the smaller units.

T: (Point to the model and calculations.) When we add partial products, what property of multiplication are we using?

S: The distributive property.

T: Let’s find the area of this rectangle again. This time let’s use a single unit to express each of the side lengths. What is $1 \frac{1}{3}$ expressed only in thirds?

S: 4 thirds.

T: (Record on the rectangle.) Express $\frac{3}{4}$ using only fourths.

S: 15 fourths.

T: (Record on the rectangle.) Multiply these fractions to find the area.

$$\frac{4}{3} \times \frac{15}{4} = \frac{4 \times 15}{3 \times 4} = \frac{60}{12} = 5$$

Area $= 5$ square inches
Lesson 13: Multiply mixed number factors, and relate to the distributive property and area model.

Date: 1/10/14

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:
Some students may need a quick refresher on changing mixed numbers to improper fractions or vice versa. Student should be reminded that a mixed number is an addition sentence, so when converting to an improper fraction, the whole number can be expressed in the unit of the fractional part and then both like fractions added.

Problem 2
Determine when the distributive property or the multiplication of fractions is more efficient to solve for area.

T: (Draw a rectangle with side lengths $16\frac{1}{2}$ in and $4\frac{1}{4}$ in.) Which strategy do you think might be more efficient to find the area of this rectangle? Turn and talk.

S: The fractions are pretty easy, so I think the distributive property will be quicker. → The numerators will be big. I think distribution will be easier. → I like to simplify fractions, so I think improper fractions will work easier.

T: Work with a partner to find the area of this rectangle. Partner A, use the distributive property with an area model. Partner B, express the sides using fractions greater than 1. (Allow students time to work.)

T: What is the area? Which strategy was more efficient?

S: The improper fractions were messy. When I converted to improper fractions, the numerators I got were 33 and 17, and there weren’t any common factors to help me simplify. The area is $\frac{561}{8}$ in\(^2\), which is right, but it’s weird. I had to use long division to figure out that the area was $70\frac{1}{8}$ square inches. → The distributive property was much easier on this one. The partial products were all easy to do in my head. I just added the sums of the rows and got $70\frac{1}{8}$ square inches.

T: Does the method that you choose matter? Why or why not? Turn and talk.
Lesson 13:
Multiply mixed number factors, and relate to the distributive property and area model.

Date: 1/10/14

S: Either way, we got the right answer. → Depending on the numbers, sometimes distributing is easier, and sometimes just multiplying the improper fractions is easier.

Repeat the process to find the area of a square with side length $\frac{2}{3}$ m.

T: When should you use each strategy? Talk to your partner.

S: If the numbers are small, fraction multiplication might be better, especially if some factors can be simplified. → For large mixed numbers, I think the area model is easier, especially if some of the partial products are whole numbers or have common denominators. → You can always start with one strategy and change to the other if it gets too hard.

Problem 3

An 8 inch by 10 inch picture is resting on a mat. Three-fourths inch of the mat shows around the entire edge of the picture. Find the area of the mat not covered by the picture.

T: Compare this problem to others we’ve worked. Turn and talk.

S: There are two rectangles to think about here. → We have to think about how to get just the part that is the mat and not the area of the whole thing. → It is a little bit of a mystery rectangle because they are asking about the mat, but they only gave us the measurements of the picture.

T: Work with your partner and use RDW to solve. (Allow students time to work.)

T: What did you think about to solve this problem?

S: I started by imagining the mat without the picture on top. I added the extra part of the mat (1 $\frac{1}{2}$ inches) to the picture to find the length and width of the mat. Then, I multiplied and found the area of the mat. I subtracted the picture’s area from the mat and got the answer. → I started to use improper fractions, but the numbers were really large, so I used the area model. → I used the area model for the mat’s area, because I saw the measurements were going to have fractions. Then, I just multiplied 8 $\times$ 10 to find the area of the picture. → After I figured out the area of the mat, I drew a tape diagram to show the part I knew and the part I needed to find.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Student Debrief (10 minutes)

Lesson Objective: Multiply mixed number factors, and relate to the distributive property and area model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- What are the strategies that we have used to find the area of a rectangle? Which one do you find the easiest? The most difficult? How do you decide which strategy you will use for a given problem? What kinds of things do you think about when deciding?
- In the Problem Set, when did you use the distributive property and when did you multiply improper fractions? Why did you make those choices?
- How did you solve Problem 3?
- What are some situations in real life where finding the area of something would be needed or useful?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. Find the area of the following rectangles. Draw an area model if it helps you.

   a. \( \frac{5}{4} \text{ km} \times \frac{12}{5} \text{ km} \)
   
   b. \( 16 \frac{1}{2} \text{ m} \times 4 \frac{1}{5} \text{ m} \)

   c. \( 4 \frac{1}{3} \text{ yd} \times 5 \frac{2}{3} \text{ yd} \)
   
   d. \( \frac{7}{8} \text{ mi} \times 4 \frac{1}{3} \text{ mi} \)

2. Julie is cutting rectangles out of fabric to make a quilt. If the rectangles are \( 2 \frac{2}{5} \) inches wide and \( 3 \frac{2}{3} \) inches long, what is the area of four such rectangles?
3. Mr. Howard’s pool is connected to his pool house by a sidewalk as shown. He wants to buy sod for the lawn, shown in grey. How much sod does he need to buy?
Lesson 13 Exit Ticket

Name ___________________________ Date _______________________

Find the area. Draw an area model if it helps you.

1. \( \frac{7}{2} \text{ mm} \times \frac{14}{5} \text{ mm} \)

2. \( 5 \frac{7}{8} \text{ km} \times \frac{18}{4} \text{ km} \)
1. Find the area of the following rectangles. Draw an area model if it helps you.

   a. \( \frac{8}{3} \text{ cm} \times \frac{24}{4} \text{ cm} \)

   b. \( \frac{32}{5} \text{ ft} \times \frac{3}{8} \text{ ft} \)

   c. \( 5\frac{4}{6} \text{ in} \times 4\frac{2}{5} \text{ in} \)

   d. \( \frac{5}{7} \text{ m} \times 6\frac{3}{5} \text{ m} \)

2. Chris is making a table top from some leftover tiles. He has 9 tiles that measure \( 3\frac{1}{8} \) inches long and \( 2\frac{3}{4} \) inches wide. What is the area he can cover with these tiles?
3. A hotel is recarpeting a section of the lobby. Carpet covers the part of the floor as shown below in grey. How many square feet of carpeting will be needed?
Lesson 14

Objective: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Concept Development (38 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Multiply Fractions 5.NF.4 (4 minutes)
- Find the Volume 5.MD.5c (5 minutes)
- Physiometry 4.G.1 (3 minutes)

Multiply Fractions (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G5–M4–Lessons 13–16.

T: (Write $\frac{1}{2} \times \frac{1}{4}$) Say the multiplication number sentence.
S: $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

Continue the process with $\frac{1}{2} \times \frac{1}{5}$ and $\frac{1}{2} \times \frac{1}{9}$.

T: (Write $\frac{1}{2} \times \frac{1}{8}$) Write the number sentence.
S: (Write $\frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$)

T: (Write $\frac{1}{2} \times \frac{5}{8}$) Say the multiplication sentence.
S: $\frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$.

Repeat the process with $\frac{1}{4} \times \frac{5}{3}$, $\frac{1}{3} \times \frac{9}{8}$, and $\frac{3}{4} \times \frac{1}{7}$.

T: (Write $\frac{3}{4} \times \frac{3}{5}$) Write the multiplication sentence.
S: (Write $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$)
Continue the process with $\frac{2}{3} \times \frac{3}{8}$.

T:  (Write $\frac{1}{4} \times \frac{5}{12}$.) On your boards, write the number sentence.
S:  (Write $\frac{1}{4} \times \frac{5}{8} = \frac{5}{32}$)
T:  (Write $\frac{2}{3} \times \frac{3}{2}$.) On your boards, write the number sentence.
S:  (Write $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.)

**Find the Volume** *(5 minutes)*

Materials: (S) Personal white boards

Note: This fluency reviews volume concepts and formulas.

T:  (Project a prism 3 units × 2 units × 7 units. Write $V = \_ \text{units} \times \_ \text{units} \times \_ \text{units}.$) Find the volume.
S:  (Write $42 \text{units}^3 = 4 \text{units} \times 2 \text{units} \times 3 \text{units}$)
T:  How many layers of 6 cubes are in the prism?
S:  7 layers.
T:  Write a multiplication sentence to find the volume starting with the number of layers.
S:  (Write $7 \times 6 \text{units}^3 = 42 \text{units}^3$)
T:  How many layers of 21 cubes are there?
S:  2 layers.
T:  Write a multiplication sentence to find the volume starting with the number of layers.
S:  (Write $2 \times 21 \text{units}^3 = 42 \text{units}^3$)
T:  How many layers of 14 cubes are there?
S:  3 layers.
T:  Write a multiplication sentence to find the volume starting with the number of layers.
S:  (Write $3 \times 14 \text{units}^3 = 42 \text{units}^3$)
Repeat process for the other prisms.

**Physiometry** *(3 minutes)*

Materials: (S) Personal white boards

Note: Kinesthetic memory is strong memory. This fluency prepares students for G5–M5–Lesson 16.

T:  Stand up.
Lesson 14: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Date: 1/10/14

Concept Development (38 minutes)

Materials: (S) Problem Set

Note: The Problem Set has been incorporated into the Concept Development. The problems in today’s lesson can be time intensive. It may be that only two or three problems can be solved in the time allowed. Students will approach representing these problems from many perspectives. Allow students the flexibility to use the approach that makes the most sense to them.

Suggested Delivery of Instruction for Solving Lesson 14’s Word Problems

1. Model the problem.

Have two pairs of student who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something? This may or may not be a tape diagram today. An area model may be more appropriate.
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and
respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish work on that question, sharing their work and thinking with a peer. All should write their equations and statements of the answer.

3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solution.

Problem 1

George decided to paint a wall with two windows. Both windows are $3\frac{3}{4}$ ft by $4\frac{1}{2}$ ft rectangles. Find the area the paint needs to cover.

Students must keep track of three different areas to solve Problem 1. Using a part–whole tape diagram to represent these areas may be helpful to some students, while others may find using the area model to be more helpful. Students have choices in strategy for computing the areas as well. Some may choose to use the distributive property. Others may choose to multiply improper fractions. Once students have solved, ask them to justify their choice of strategy. Were they able to tell which strategy to use from the beginning? Did they change direction once they began? If so, why? Flexibility in thinking about these types of problems should be a focus.

Problem 2

Joe uses square tiles, some of which he cuts in half, to make the figure below. If each square tile has a side length of $2\frac{1}{2}$ inches, what is total area of the figure?

The presence of the triangles in the design may prove challenging for some students. Students who understand area as a procedure of multiplying sides, but do not understand the meaning of area may need scaffolding to help them reason about mentally reassembling the 6 halves to find 3 whole tiles.
Problem 3

All-In-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three?

While this problem is a fairly straightforward, additive area problem, an added complexity occurs at finding the dimensions of Room C. The complexity of this problem also lies in the need to keep three different areas organized before finding the total area. Again, once students have had opportunity to work through the protocol, discuss the pros and cons of various approaches, including the reasoning for their choice of strategy.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Problem 3 might be extended by inviting students to research actual carpet prices from local ads or the internet and calculate what such a project might cost in real life. Comparison between the costs of using different types of flooring (hardwood versus carpet, for example) may also be made.
Lesson 14: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Date: 1/10/14

Mr. Johnson needs to buy sod for his front lawn.

a. If the lawn measures \(36\frac{2}{3}\) ft by \(45\frac{1}{6}\) ft, how many square feet of sod will he need?

b. If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

The dimensions of the yard are larger than any others in the Problem Set to encourage use of the distributive property to find the total area. Because the total area (1,656 \(\frac{1}{9}\) ft\(^2\)) is numerically closer to 1,656, students may be tempted to round down. Reasoning about the \(\frac{1}{9}\) ft\(^2\) area can provide an opportunity to discuss the pros and cons of sodding that last fraction of a square foot. In the final component of the protocol, ask the following or similar questions:

- Is it worth the extra money for such a small amount of area left to cover? While 19 cents is a small cost, what if the sod had been more expensive?
- What if the costs had been structured so that that last whole square foot of sod had lowered the price of the entire amount?
- What could Mr. Johnson do with the other 8 ninths?

Mr. Johnson needs to buy 1,657 sq ft of sod and it will cost $409.83.
Problem 5

Jennifer’s class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will lose $\frac{3}{4}$ inch.

a. Draw one way the squares could be arranged to make a rectangular quilt. Then find the perimeter of your arrangement.

b. Find the area of the quilt.

There are many ways to lay out the quilt squares. Allow students to draw their layout and then compare the perimeters. Ask the following questions:

- Does the difference in perimeter affect the area? Why or why not?
- Are there advantages to one arrangement of the blocks over another? (For example, lowering cost for an edging by minimizing the perimeter or fitting the dimensions of the quilt to a specific wall or bed size.)

Problem 5 harkens back to Problem 2, but with an added layer of complexity. Students might be asked to compare and contrast the two problems. In this problem, students must account for the seam allowances on all four sides of the quilt squares before finding the area. Students find that each quilt block becomes $6 \frac{1}{2}$ inches square after sewing and may simply multiply this area by 24.
Lesson 14: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Student Debrief (10 minutes)

Lesson Objective: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

 Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Do these problems remind you of any others that we’ve seen in this module? In what ways are they like other problems? In what ways are they different?
- What did you learn from looking at your classmates’ drawings? Did that support your understanding of the problems in a deeper way? When you checked for reasonableness, what process did they use?
- When finding the areas, which strategy did you use more often—distribution or improper fractions? Is there a pattern to when you used which? How did you decide? What advice would you give a student who wasn’t sure what to do?
- Which problems did you find the most difficult? Which one was easiest for you? Why?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. George decided to paint a wall with two windows. Both windows are $3\frac{1}{2}$ ft by $4\frac{1}{2}$ ft rectangles. Find the area the paint needs to cover.

2. Joe uses square tiles, some of which he cuts in half, to make the figure below. If each square tile has a side length of $2\frac{1}{2}$ inches, what is total area of the figure?

3. All-In-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three?
Lesson 14: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Date: 1/10/14

4. Mr. Johnson needs to buy sod for his front lawn.
   a. If the lawn measures $36\frac{2}{3}$ ft by $45\frac{1}{6}$ ft, how many square feet of sod will he need?

   b. If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

<table>
<thead>
<tr>
<th>Area</th>
<th>Price per square foot</th>
</tr>
</thead>
<tbody>
<tr>
<td>First 1,000 sq ft</td>
<td>$0.27</td>
</tr>
<tr>
<td>Next 500 sq ft</td>
<td>$0.22</td>
</tr>
<tr>
<td>Additional square feet</td>
<td>$0.19</td>
</tr>
</tbody>
</table>

5. Jennifer’s class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will lose $\frac{3}{4}$ inch.

   a. Draw one way the squares could be arranged to make a rectangular quilt. Then find the perimeter of your arrangement.

   b. Find the area of the quilt.
Lesson 14 Exit Ticket

Name ____________________________________________ Date ______________

1. Mr. Klimek made his wife a rectangular vegetable garden. The width is $5 \frac{3}{4}$ ft and the length is $9 \frac{4}{5}$ ft. What is the area of the garden?
Lesson 14: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

1. Mr. Albano wants to paint menus on the wall of his café in chalkboard paint. The grey area below shows where the rectangular menus will be. Each menu will measure 6 feet wide and $7\frac{1}{2}$ ft long.

   a. How many square feet of menu space will Mr. Albano have?
   
   b. What is the area of wall space that is not covered by chalkboard paint?

2. Mr. Albano wants to put tiles in the shape of a dinosaur at the front entrance. He will need to cut some tiles in half to make the figure. If each square tile is $4\frac{1}{4}$ inches on each side, what will the total area of the figure be?
3. A-Plus Glass is making windows for a new house that is being built. The box shows the list of sizes they must make.

   a. How many square feet of glass will they need?

   b. Each sheet of glass they use to make the windows is 9 feet long and 6 $\frac{1}{2}$ feet wide. How many sheets will they need in order to make the windows?

4. Mr. Johnson needs to buy seed for his backyard lawn.

   a. If the lawn measures $40 \frac{1}{5}$ ft by $50 \frac{2}{8}$ ft, how many square feet of seed will he need?

   b. One bag of seed will cover 500 square feet if he sets his seed spreader to its lowest setting and 300 square feet if he sets the spreader to its highest setting. How many bags of seed will he need if he uses the highest setting? The lowest setting?
Lesson 15

Objective: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Suggested Lesson Structure

- Fluency Practice (10 minutes)
- Concept Development (40 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (10 minutes)

- Divide Whole Numbers by Unit Fractions and Unit Fractions by Whole Numbers 5.NF.7 (6 minutes)
- Quadrilaterals 3.G.1 (4 minutes)

Divide Whole Numbers by Unit Fractions and Unit Fractions by Whole Numbers (6 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G5–Module 4.

T: (Write $1 \div \frac{1}{2}$.) Say the division sentence.
S: $1 \div \frac{1}{2}$

T: How many halves are in 1?
S: 2.

T: (Write $1 \div \frac{1}{2} = 2$. Beneath it, write $2 \div \frac{1}{2}$.) How many halves are in 2?
S: 4.

T: (Write $2 \div \frac{1}{2} = 4$. Beneath it, write $3 \div \frac{1}{2}$.) How many halves are in 3?
S: 6.

T: (Write $3 \div \frac{1}{2} = 6$. Beneath it, write $7 \div \frac{1}{2}$.) Write the division sentence.
S: (Write $7 \div \frac{1}{2} = 14$.)

Continue for the following possible suggestions: $1 \div \frac{1}{4}$, $2 \div \frac{1}{4}$, $9 \div \frac{1}{4}$, and $3 \div \frac{1}{5}$.

T: (Write $\frac{1}{2} \div 2$.) Say the complete division sentence.
Lesson 15:

Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations.

Date: 1/10/14

Lesson 15

S: \( \frac{1}{2} \div 2 = \frac{1}{4} \)

T: (Write \( \frac{1}{2} \div 2 = \frac{1}{4} \). Beneath it, write \( \frac{1}{2} \div 3 \).) Say the complete division sentence.

S: \( \frac{1}{2} \div 3 = \frac{1}{6} \)

T: (Write \( \frac{1}{2} \div 3 = \frac{1}{6} \). Beneath it, write \( \frac{1}{2} \div 4 \).) Say the complete division sentence.

S: \( \frac{1}{2} \div 4 = \frac{1}{8} \)

T: (Write \( \frac{1}{2} \div 9 = \_\_\_\_. \) Complete the number sentence.

S: (Write \( \frac{1}{2} \div 9 = \frac{1}{18} \).)

Continue with the following possible sequence: \( \frac{1}{5} \div 2, \frac{1}{5} \div 3, \frac{1}{5} \div 5, \) and \( \frac{1}{8} \div 4 \).

**Quadrilaterals (4 minutes)**

Materials: (T) Shapes sheet

Note: This fluency reviews Grade 3 geometry concepts in anticipation of G5–Module 6 content. The sheet can be duplicated if preferred.

T: (Project the quadrilaterals template and the list of attributes.) Take one minute to discuss the attributes of the shapes you see. You can use the list to support you.

S: Some have right angles. \( \rightarrow \) All have straight sides. \( \rightarrow \) They all have four sides. \( \rightarrow \) B and G and maybe H and K have all equal sides. I’m not really sure.

T: If we wanted to verify whether the sides are equal, what would we do?

S: Measure with a ruler.

T: What about the angles? How could you verify that they’re right angles?

S: I could compare it to something that I know is a right angle. \( \rightarrow \) I could use a set square. \( \rightarrow \) I could use a protractor to measure.

T: Now, look at the shape names. Determine which shapes might fall into each category. (Post the shape names.)

<table>
<thead>
<tr>
<th>Attributes to Consider</th>
<th>Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Sides</td>
<td>Quadrilateral</td>
</tr>
<tr>
<td>Length of Sides</td>
<td>Rhombus</td>
</tr>
<tr>
<td>Angle Measures</td>
<td>Square</td>
</tr>
<tr>
<td>Right Angle</td>
<td>Rectangle</td>
</tr>
</tbody>
</table>

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S: B and G might be squares. → All of them are quadrilaterals. → H and K might be rhombuses. It’s hard to know if their sides are equal. → D and I are rectangles. Oh yeah, and B and G are, too. → L and A look like trapezoids.
T: Which are quadrilaterals?
S: All of them.
T: Which shapes appear to be rectangles?
S: B, D, G, and I.
T: Which appear to have opposite sides of equal length but are not rectangles?
S: C, H, and K. → A and L have one pair of opposite sides that look the same.
T: Squares are rhombuses with right angles. Do you see any other shapes that might have four equal sides without right angles?
S: H and K.

**Concept Development (40 minutes)**

Materials: (S) Problem Set

Note: The Problem Set has been incorporated into and will be completed during the Concept Development. While there are only four problems, most are multi-step and will require time to solve.

**Suggested Delivery of Instruction for Solving Lesson 39 Word Problems**

1. **Model the problem.**
   
   Have two pairs of student who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:
   
   - Can you draw something? This might be a tape diagram or an area model.
   - What can you draw?
   - What conclusions can you make from your drawing?

   As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. **Calculate to solve and write a statement.**

   Give everyone two minutes to finish work on that question, sharing their work and thinking with a peer. All should write their equations and statements of the answer.

3. **Assess the solution for reasonableness.**

   Give students one to two minutes to assess and explain the reasonableness of their solution.
Lesson 15: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Date: 1/10/14

Lesson 15

Problem 1

The length of a flowerbed is 4 times as long as its width. If the width is $\frac{3}{8}$ meter, what is the area?

While this problem is quite simple to calculate, two complexities must be navigated. First, the length is not given. Second, the resulting area is less than a whole meter. Once students have arrived at a solution, ask if their result makes sense and why. If students need support, discuss what this might look like if it were tiled. The length of the bed necessitates that 2 whole tiles be used. (How is it that the area is less than 1?) Students might draw or represent the problem with concrete materials to explain their thinking. The folding for these units may prove challenging but worthwhile, as these types of problems are often done procedurally by students rather than with a deep understanding of what their answer represents. As in G5–M5–Lesson 14, continue to have students explain their choice of strategy in terms of efficiency. When students are sharing their approaches with the class, encourage those who had difficulty to ask how the presenters got started with their drawing and calculations. Also encourage students to explain any false starts they experienced when solving and how and why their thinking changed.

Problem 2

Mrs. Johnson grows herbs in square plots. Her basil plot measures $\frac{5}{8}$ yd on each side.

a. Find the total area of the basil plot.

b. Mrs. Johnson puts a fence around the basil. If the fence is 2 ft from the edge of the garden on each side, what is the perimeter of the fence?

c. What is the total area that the fence encloses?

As in Problem 1, the fraction multiplication involved in completing Part (a) is not rigorous. However, this problem offers an opportunity to explore the relationships of square yards to square feet and the importance of understanding the actual size of such units. The expression of the area as $\frac{25}{64}$ yd$^2$ may be conceptually challenging for students. They might be encouraged to relate this to a benchmark of 1 half or 1 third square yard (which is 3 square feet). Students might be asked to show what a tiling of this garden plot would look like.
It may even be helpful to use yardsticks to show the actual size of the herb plot. The area expressed as a bit more than 5 square feet may be surprising to students. Help students make connections to the shading models of G5–Module 4 and how the representation of the area model for the basil plot compares and contrasts to the representation of fraction multiplication. Part (b) offers a bit of complexity in that the dimensions of the garden are given in yards, yet the distance from the garden to the fence is given in feet. Part (c) requires that students use the fence measurements to find the total area enclosed by the fence. Multiple methods may be used to accomplish this.

Problem 3

Janet bought 5 yards of fabric 2 1/4 feet wide to make curtains. She used 1/3 of the fabric to make a long set of curtains and the rest to make 4 short sets.

a. Find the area of the fabric she used for the long set of curtains.

b. Find the area of the fabric she used for each of the short sets.

As in Problem 2, there are different units within a multi-step problem. After students have solved, allow them to share whether they converted both measurements to feet or yards and the advantages and disadvantages of both. A discussion of the relationship of the square yards to the square feet may also be fruitful. Discuss the various strategies students may have used to find the fabric left for the shorter curtains.
Lesson 15: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Problem 4

Some wire is used to make 3 rectangles: A, B, and C. Rectangle B’s dimensions are $\frac{3}{5}$ cm larger than Rectangle A’s dimensions, and Rectangle C’s dimensions are $\frac{3}{5}$ cm larger than Rectangle B’s dimensions. Rectangle A is 2 cm by $3\frac{1}{5}$ cm.

a. What is the total area of all three rectangles?
b. If a 40-cm coil of wire was used to form the rectangles, how much wire is left?

The complexity of this problem stems from students making sense of the way each rectangle increases in dimension. As always, encourage students to start by drawing each of the three rectangles. As the denominators are easily expressed as hundredths, some students may use decimals to calculate these areas. Should this occur, help make the connection for other students to that learning from G5–Module 4.

For Part (b), students must shift their thinking to perimeter and use the outer dimensions of the rectangles to find the total amount of wire used. Again, students may use decimals for the calculations. Be sure to have students compare their decimal and fraction solutions with one another for equivalence and explain why they chose each type of fraction.
Lesson 15: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Date: 1/10/14

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Student Debrief (10 minutes)

Lesson Objective: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Compare the problems for which the distributive property seems most efficient and the problems for which multiplying improper fractions (or using decimals to multiply) seems so. What influences your choice of strategy?
Lesson 15: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

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- Sort problems from yesterday’s and today’s lessons (G5–M5–Lessons 14 and 15) from simple to complex. What do the problems have in common? Have students compare their sort to a partner’s.

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. The length of a flowerbed is 4 times as long as its width. If the width is $\frac{3}{8}$ meter, what is the area?

2. Mrs. Johnson’s grows herbs in square plots. Her basil plot measures $\frac{5}{8}$ yd on each side.
   a. Find the total area of the basil plot.
   b. Mrs. Johnson puts a fence around the basil. If the fence is 2 ft from the edge of the garden on each side, what is the perimeter of the fence?
c. What is the total area that the fence encloses?

3. Janet bought 5 yards of fabric \(2\frac{1}{4}\) feet wide to make curtains. She used \(\frac{1}{3}\) of the fabric to make a long set of curtains and the rest to make 4 short sets.

a. Find the area of the fabric she used for the long set of curtains.

b. Find the area of the fabric she used for each of the short sets.
4. Some wire is used to make 3 rectangles: A, B, and C. Rectangle B’s dimensions are \( \frac{3}{5} \) cm larger than Rectangle A’s dimensions, and Rectangle C’s dimensions are \( \frac{3}{5} \) cm larger than Rectangle B’s dimensions. Rectangle A is 2 cm by 3\( \frac{1}{5} \) cm.

a. What is the total area of all three rectangles?

b. If a 40-cm coil of wire was used to form the rectangles, how much wire is left?
Wheat grass is grown in planters that are $3 \frac{1}{2}$ inch by $1 \frac{3}{4}$ inch. If there is a $6 \times 6$ array of these planters with no space between them, what is the area of the array?
Lesson 15 Homework

Name ___________________________ Date ________________

1. The width of a picnic table is 3 times its length. If the length is \( \frac{5}{6} \) yd long, what is the area in square feet?

2. A painting company will paint this wall. The homeowner gives them the following dimensions:

   Window A is \( \frac{6}{4} \) ft \( \times \) \( \frac{3}{4} \) ft
   Window B is \( \frac{3}{8} \) ft \( \times \) 4 ft
   Window C is \( \frac{9}{2} \) ft square
   Door D is 8 ft \( \times \) 4 ft

What is the area of the painted part of the wall?
Lesson 15:
Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

Date: 1/10/14

Lesson 15 Homework

3. A decorative wooden piece is made up of four rectangles as shown to the right. The smallest rectangle measures \(4 \frac{1}{2}\) inches by \(7 \frac{3}{4}\) inches. If \(2 \frac{1}{4}\) inches is added to each dimension as the rectangles get larger, what is the total area of the entire piece?