Topic F

Distributive Property and Problem Solving Using Units of 2–5 and 10

3.OA.3, 3.OA.5, 3.OA.7, 3.OA.8, 3.OA.1, 3.OA.2, 3.OA.4, 3.OA.6

Focus Standard:

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.5 Apply properties of operations as strategies to multiply and divide. Examples: If 6 \times 4 = 24 is known, then 4 \times 6 = 24 is also known. (Commutative property of multiplication.) 3 \times 5 \times 2 can be found by 3 \times 5 = 15, then 15 \times 2 = 30, or by 5 \times 2 = 10, then 3 \times 10 = 30. (Associative property of multiplication.) Knowing that 8 \times 5 = 40 and 8 \times 2 = 16, one can find 8 \times 7 as 8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56. (Distributive property.)

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 \times 5 = 40, one knows 40 \div 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.¹

Instructional Days: 4

Coherence -Links from: G2–M6
-Links to: G4–M3

Foundations of Multiplication and Division
Using Place Value Understanding and Properties of Operations to Perform Multi-Digit Multiplication and Division

¹ Problem-solving limited to multiplication and division in this module.
Topic F introduces the factors 5 and 10, familiar from skip-counting in Grade 2. Students apply the multiplication and division strategies they used to mixed practice with all of the factors included in Module 1. Students model relationships between factors and decompose numbers as they further explore the relationship between multiplication and division. This culminates in Lessons 18 and 19 as students decompose the dividend in a division sentence to practice the distributive property with division. For example, students decompose $28 \div 4$ as $(20 \div 4) + (8 \div 4) = 5 + 2 = 7$. In the final lesson of the module students apply the tools, representations, and concepts they have learned to problem-solving with multi-step word problems using all four operations. They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.

### A Teaching Sequence Towards Mastery of Distributive Property and Problem Solving Using Units of 2–5 and 10

**Objective 1:** Apply the distributive property to decompose units.  
(Lesson 18–19)

**Objective 2:** Solve two-step word problems involving multiplication and division and assess the reasonableness of answers.  
(Lesson 20)

**Objective 3:** Solve two-step word problems involving all four operations and assess the reasonableness of answers.  
(Lesson 21)
Lesson 18

Objective: Apply the distributive property to decompose units.

Suggested Lesson Structure

- Fluency Practice: (15 minutes)
- Application Problem: (5 minutes)
- Concept Development: (30 minutes)
- Student Debrief: (10 minutes)
- Total Time: (60 minutes)

Fluency Practice (15 minutes)

- Add and Subtract by Five 2.NBT.5 (15 minutes)

Sprint: Add and Subtract by 5 (12 minutes)

Materials: (S) Add and Subtract by 5 Sprint

Note: This activity builds a foundation for multiplication using units of 5 through reviewing skip-counting from Grade 2. See Directions for Administration of Sprints in Lesson 2.

Between sprints, include the following group counts in place of movement exercises.

- Count by threes to 30, think/talk forward and backward.
- Count by sixes to 30, forward and backward.
- Count by fours to 40, forward and backward.

Application Problem (5 minutes)

A parking structure has 10 levels. There are 3 cars parked on each level. How many cars are parked in the structure?

3 cars 10 levels

10 x 3 = 30

There are 30 cars parked in the structure.
Note: This is the same problem that students use without context in the first problem of the concept development. Solving the problem ahead of time supports the learning below by de-emphasizing the answer in favor of focusing on the new concept of decomposing with number bonds.

**Concept Development (30 minutes)**

Materials: (S) Personal white boards

Problem 1: Use number bonds to decompose numbers and apply the distributive property.

Project an array for $7 \times 3$ with a line drawn as shown. Write $7 \times 3$ next to the array.

T: How many threes?
S: 7 threes.
T: The dotted line shows a way to break apart the array. The 7 threes are broken into...?
S: 5 threes and 2 threes.
T: Let’s draw our number bonds.
S: (Draw number bond shown to the right.)
T: Write the equation that shows how to add the two parts.
S: (Write $5 \times 3 + 2 \times 3 = 7 \times 3$.)
T: Whisper to a partner the 2 multiplication sentences you used to help you solve $7 \times 3$.
S: (Whisper $5 \times 3 = 15$ and $2 \times 3 = 6$.)
T: (Draw a second number bond using the expressions $(5 \times 3)$ and $(2 \times 3)$. The number bond is another way to show decomposition. This shows how we partitioned the array and wrote the number bond using our number sentences.
T: Let’s rewrite this as the addition of two products using my frame. (Point to the equation below.)

\[
(\_ \times 3) + (\_ \times 3) = \_ \times 3 \\
\_ + \_ = \_
\]

S: (Write.)

\[
(5 \times 3) + (2 \times 3) = 7 \times 3
\]
\[
15 + 6 = 21
\]

T: How does the number sentence show the number bond?
S: It shows the 7 broken into 5 and 2. → And the threes are shared with both parts. → Yes. 5 threes and 2 threes. → One part has 5 threes and the other part has 2 threes.
Repeat the process with $9 \times 4$.

T: Let’s call it the *break apart and distribute strategy*. The number bond helps us see that we can find the total by adding two smaller parts together.

**Problem 2: Use number bonds and the distributive property.**

T: (Write $10 \times 3$.) How many threes?
S: 10 threes.
T: What are some ways we can break apart 10?
S: 5 and 5. → 6 and 4. → 7 and 3. → 8 and 2.
T: So if we were counting apples that would be 5 apples and 5 apples, 6 apples and 4 apples?
S: Yes.
T: But we aren’t counting apples. What are we counting?
S: Threes.
T: So that would be 6 threes and...?
S: 4 threes.
T: Let’s draw our number bonds.
S: (Draw number bond shown to the right.)
T: Write the equation that shows how to add the two parts. Start with 6 threes and 4 threes.
S: (Write $6 \times 3 + 4 \times 3 = 10 \times 3$.)
T: Rewrite this as the addition of two products using my frame. (Point to the equation below.)

\[
\begin{align*}
(\_ \times 3) + (\_ \times 3) &= \_ \times 3 \\
\_ + \_ &= \_
\end{align*}
\]

S: (Write.)

\[
(6 \times 3) + (4 \times 3) = 10 \times 3 \\
18 + 12 = 30
\]

Repeat the process with $8 \times 4$.

**Problem Set (10 minutes)**

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Student Debrief (10 minutes)

Lesson Objective: Apply the distributive property to decompose units.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Compare the number bond and array models for showing the break apart and distribute strategy.
- Share work for Problem 4. Compare students’ number choices.
- Why do you think we use the number bond as a method for breaking a total into two parts? How was this strategy helpful to find the answer to a larger fact in Problem 7?
- Think about how Problem 1 in the concept development relates to today’s application problem.
- (Students use the Distributive Property with division in Lesson 19.) Do you think the break apart and distribute strategy can be used with division? What might that look like?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
### Lesson 18: Apply the distributive property to decompose units.

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### Lesson 18 Sprint

**NYS COMMON CORE MATHEMATICS CURRICULUM**

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Lesson 18: Apply the distributive property to decompose units.

Name ____________________________  Date ______________________

1. \(8 \times 10 = \) ______
   
   5 tens + _______________ = 8 tens
   
   (5 \(\times 10\)) + (_____ \(\times 10\)) =
   
   50 + _______ = _________
   
   8 \(\times 10\) = ____________

2. \(7 \times 4 = \) ______
   
   5 fours + _______________ = 7 fours
   
   (5 \(\times 4\)) + (_____ \(\times 4\)) =
   
   20 + _______ = _________
   
   7 \(\times 4\) = ____________

3. \(9 \times 10 = \) ______
   
   5 tens + _______________ = 9 tens
   
   (5 \(\times 10\)) + (_____ \(\times 10\)) =
   
   _______ + _______ = _________
   
   9 \(\times 10\) = ____________

4. \(10 \times 10 = \) ______
   
   ___________ + ___________ = 10 tens
   
   (_____ \(\times 10\)) + (_____ \(\times 10\)) =
   
   _______ + _______ = _________
   
   10 \(\times 10\) = ____________
5. There are 7 teams in the soccer tournament. 10 children play on each team. How many children are playing in the tournament?

There are __________ children playing in the tournament.

6. What is the total number of sides on 8 triangles?

7. There are 12 rows of bottled drinks in the vending machine. Each row has 10 bottles. How many bottles are in the vending machine?
Dylan used the distributive property to solve a multiplication problem. Look at his work below, write the multiplication problem Dylan solved and complete the number bond.

Dylan’s work:

\[(5 \times 4) + (1 \times 4) = \]

\[20 + 4 = 24\]

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\begin{array}{c}
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_____ × _____ = _____
Lesson 18: Apply the distributive property to decompose units.

1. Match.

   - 7 tens
     - 5 tens
     - 2 tens
   - 8 fours
     - 5 fours
     - 3 fours
   - 9 tens
     - 6 tens
     - 3 tens
   - 7 threes
     - 5 threes
     - 2 threes

   

   - (5 × 4) + (3 × 4) = 32
   - (5 × 3) + (2 × 3) = 21
   - (5 × 10) + (2 × 10) = 70
   - (6 × 10) + (3 × 10) = 90


2. 9 × 4 = __________

   

   - (_____ × 4) + (_____ × 4) =  
   - _____ + _____ = ______
   - 9 × 4 = ______
3. Lydia makes 10 pancakes. She tops each pancake with 4 blueberries. How many blueberries does Lydia use in all?

Lydia uses _______ blueberries in all.

4. Steven solves $7 \times 3$ using the distributive property. Show an example of what Steven’s work might look like below.

5. There are 7 days in 1 week. How many days are there in 10 weeks?
Lesson 19

Objective: Apply the distributive property to decompose units.

Suggested Lesson Structure

- Fluency Practice (15 minutes)
- Application Problem (5 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (15 minutes)

- Group Counting 3.OA.1 (3 minutes)
- Commutative Multiplying 3.OA.7 (3 minutes)
- Decompose and Multiply 3.OA.5 (4 minutes)
- Compose and Multiply 3.OA.5 (5 minutes)

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by threes, fours, fives, and sixes in this activity reviews multiplication with units of 3, 4, and 5 and anticipates multiplication with units of 6 in Module 3.

- Count by fours to 40, whisper and talk forward and backward.
- Count by sixes to 36 forward and backward. Emphasize the 24 to 30 transition.
- Count by threes to 30 forward and backward.
- Count by fives to 50 forward and backward.

Commutative Multiplying (3 minutes)

Note: This activity reviews the commutativity of multiplication, learned in Lessons 7, 8, and 15.

T: (Write 3 x 2 = ____) Say the multiplication sentence.
S: 3 x 2 = 6.
T: Flip it.
S: 2 x 3 = 6.

Repeat process for 5 x 2, 5 x 3, 3 x 4, 2 x 8, and 3 x 7.
Lesson 19:
Apply the distributive property to decompose units.

Date: 5/6/13

Lesson 19

Decompose and Multiply (4 minutes)

Materials: (S) Personal white boards

Note: This activity anticipates multiplication using units of 6, 7, 8, and 9 by decomposing larger facts into smaller known facts. It reviews the distributive property strategy.

T: (Write 7 fours = ___.) Write the multiplication sentence.
S: (Write 7 fours = ___)
T: (Write (5 fours) + (____ fours) = below 7 fours = ___.) 7 fours is the same as 5 fours and how many fours?
S: 2 fours.
T: (Write (5 fours) + (2 fours) =. Below it, write 20 + ____ = ____.) Complete the equation.
S: (Write 7 fours = 28. Below it, write (5 fours) + (2 fours) = 28. Below that line, they write 20 + 8 = 28.)

Repeat for possible sequence: 8 x 3, 9 x 2, and 6 x 4. Change missing numbers that students need to fill in.

Compose and Multiply (5 minutes)

Materials: (S) Personal white boards

Note: This activity anticipates multiplication using units of 6, 7, 8, and 9 by composing smaller known facts into larger unknown facts. It reviews the distributive property strategy.

T: (Write (5 x 3) + (2 x 3) = ___.) Write the number sentence on your personal white boards. Below the number sentence, write an addition sentence.
S: (Write (5 x 3) + (2 x 3) = 21. Below it, write 15 + 6 = 21.)
T: Write (5 x 3) + (2 x 3) as a single multiplication sentence.
S: (Write 7 x 3 = 21 above (5 x 3) + (2 x 3) = 21.)

Repeat for possible sequence: 8 x 2 and 9 x 4.

Application Problem (5 minutes)

Henrietta works in a shoe store. She uses 2 shoelaces to lace each pair of shoes. She has a total of 24 laces. How many pairs of shoes can Henrietta lace?
Note: This problem reviews material from Lesson 18 but intentionally previews $24 \div 2$, which is used in the first example of the concept development. Students may choose to solve the application problem with division or as an unknown factor multiplication problem. Use these variations in method to spark discussion.

**Concept Development (30 minutes)**

Materials: (S) Personal white boards

**Problem 1: Model break apart and distribute using an array as a strategy for division.**

Draw or project a $12 \times 2$ array and write $24 \div 2 = \_\_\_$ above it.

T: Let’s use the array to help us solve $24 \div 2 = \_\_\_$. There are 24 dots total. (Draw a line after the $10^{th}$ row.) This shows one way to break apart the array.

T: Write division equations that represent the part of the array above the line and the part of the array below the line.

S: (Write $20 \div 2 = 10$ and $4 \div 2 = 2$.)

T: How many twos are above the line?

S: 10 twos.

T: How many twos are below the line?

S: 2 twos.

T: Let’s rewrite this as the addition of two quotients. Use my frame.

\[
\begin{array}{c}
(\_\_\_ \div 2) + (\_\_\_ \div 2) = \_\_\_ \div 2 \\
\_\_\_ + \_\_\_ = \_\_\_
\end{array}
\]

S: (Line 1: fill in totals, Line 2: $10 + 2 = 12$)

T: Explain to your partner the process we used to solve $24 \div 2$.

S: We added the quotients of 2 smaller facts to find the quotient of a larger one.

Repeat the process with a $13 \times 2$ array to show $26 \div 2$. Break into $20 \div 2$ and $6 \div 2$. 

Lesson 19: Apply the distributive property to decompose units.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Add a challenge by asking students to think about other ways of breaking apart 27. A student will most likely choose a quotient that is not divisible by 3. This will lead to a discussion in which you help students realize that with division, the strategy relies on the decomposition being such that the dividends must be divisible by the divisor.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If appropriate, encourage the class or individual students to solve 33 ÷ 3 without using an array.

Problem 2: Break apart and distribute as a strategy for division.

T: (Write 27 ÷ 3 = ____.) What are we focused on when we break apart to divide? Breaking up the number of groups (or rows) like in multiplication or breaking up the total?
S: Breaking up the total.
T: Let’s break up 27 into 15 and what ___ 15 plus what equals 27?
S: 12.
T: Work with a partner to draw an array that shows 27 ÷ 3, where 3 is the number of columns.
S: (Draw a 9 x 3 array.)
T: Box the part of your array that shows a total of 15.
S: (Box the first 5 rows.)
T: Write a division equation for the boxed portion to the right of the array.
S: (Write 15 ÷ 3 = 5.)
T: Box the part of your array that shows a total of 12.
S: (Box the remaining 4 rows.)
T: Now write a division equation for that part of the array.
S: (Write 12 ÷ 3 = 4.)
T: Tell your partner how you will use the equations to help you solve the original problem,
27 ÷ 3 = ___.
S: I’ll add the quotients of the 2 smaller facts.
T: (Write the following.) Complete the following sequence to solve 27 ÷ 3 with your partner.

\[
27 ÷ 3 = (15 ÷ 3) + (12 ÷ 3)
\]

\[
= _____ + _____
\]

= _____

Repeat the process with 33 ÷ 3. Students can break apart 33 by using the number pair 30 and 3.
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Student Debrief (10 minutes)

Lesson Objective: Apply the distributive property to decompose units.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Compare Nell’s strategy in Problem 3, to the strategy for solving 24 ÷ 2 in the concept development.
- Yesterday we used the break apart and distribute strategy with multiplication. How is the method we learned today similar?
- How is the break apart and distribute strategy different for multiplication and division? (This strategy works for division when the totals used in the decomposition are both divisible by the divisor. For example, decomposing 33 into 25 and 8 is not effective because the divisor is 3.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 19

Name ___________________________ Date __________________

1. Label the array. Then fill in the blanks below to make statements that are true.

   a. $36 \div 3 = _____$

   \[ \begin{array}{c}
   \text{array} \\
   \hline
   \text{4 groups of 9 dots} \text{ and 6 remaining dots} \\
   \end{array} \]

   $\text{(30} \div 3) = _____$

   $\text{(6} \div 3) = _____$

   $\text{(36} \div 3) = (30} \div 3) + (6} \div 3)$

   $= 10 + _____$

   $= 12$

   b. $25 \div 5 = _____$

   \[ \begin{array}{c}
   \text{array} \\
   \hline
   \text{5 groups of 5 dots} \text{ and 0 remaining dots} \\
   \end{array} \]

   $\text{(20} \div 5) = 4$

   $\text{(5} \div 5) = _____$

   $\text{(25} \div 5) = (20} \div 5) + (5} \div 5)$

   $= 4 + _____$

   $= _____$

   c. $28 \div 4 = _____$

   \[ \begin{array}{c}
   \text{array} \\
   \hline
   \text{7 groups of 4 dots} \text{ and 0 remaining dots} \\
   \end{array} \]

   $\text{(20} \div 4) = _____$

   $\text{(_ _ _} \div 4) = _____$

   $\text{(28} \div 4) = (20} \div 4) + (_ _ _} \div 4)$

   $= _____ + _____$

   $= _____$

   d. $32 \div 4 = _____$

   \[ \begin{array}{c}
   \text{array} \\
   \hline
   \text{8 groups of 4 dots} \text{ and 0 remaining dots} \\
   \end{array} \]

   $\text{(_ _ _ _} \div 4) = _____$

   $\text{(_ _ _ _} \div 4) = _____$

   $\text{(32} \div 4) = (_ _ _ _} \div 4) + (_ _ _ _} \div 4)$

   $= _____ + _____$

   $= _____$
2. Match the equal expressions.

24 ÷ 2
36 ÷ 3
39 ÷ 3
26 ÷ 2

(30 ÷ 3) + (6 ÷ 3)
(30 ÷ 3) + (9 ÷ 3)
(20 ÷ 2) + (6 ÷ 2)
(20 ÷ 2) + (4 ÷ 2)

3. Nell draws the array below to find the answer to the division fact 24 ÷ 2. Explain Nell’s strategy.
Lesson 19 Exit Ticket

Complete the equations below to solve $22 \div 2 = ____$. 

\[
(20 \div 2) = ____ \\
(____ \div 2) = ____ \\
(22 \div 2) = (20 \div 2) + (____ \div 2) \\
= ____ + ____ \\
= ____
\]
1. Label the array. Then complete the equations to make statements that are true.

a. $18 \div 3 = ____$

\[ \begin{array}{c}
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\end{array} \]

$(9 \div 3) = 3$

$(9 \div 3) = _____$

$(18 \div 3) = (9 \div 3) + (9 \div 3)$

$= 3 + _____$

$= 6$

b. $21 \div 3 = ____$

\[ \begin{array}{c}
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\end{array} \]

$(15 \div 3) = 5$

$(6 \div 3) = _____$

$(21 \div 3) = (15 \div 3) + (6 \div 3)$

$= 5 + _____$

$= _____$

c. $24 \div 4 = ____$

\[ \begin{array}{c}
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\end{array} \]

$(20 \div 4) = _____$

$(4 \div 4) = _____$

$(24 \div 4) = (20 \div 4) + (4 \div 4)$

$= _____ + _____$

$= _____$

d. $36 \div 4 = ____$

\[ \begin{array}{c}
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\text{△} & \text{△} & \text{△} \\
\end{array} \]

$(20 \div 4) = _____$

$(16 \div 4) = _____$

$(36 \div 4) = (20 \div 4) + (16 \div 4)$

$= _____ + _____$

$= _____$
4. Match equal expressions.

- \(28 \div 2\)
- \(33 \div 3\)
- \(36 \div 3\)
- \(26 \div 2\)

- \((30 \div 3) + (3 \div 3)\)
- \((20 \div 2) + (6 \div 2)\)
- \((30 \div 3) + (6 \div 3)\)
- \((20 \div 2) + (8 \div 2)\)

5. Alex draws the array below to find the answer to \(35 \div 5\). Explain Alex’s strategy.
Lesson 20

Objective: Solve two-step word problems involving multiplication and division and assess the reasonableness of answers.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)

Fluency Practice (12 minutes)

- Skip-Count by Fives 2.NBT.2 (12 minutes)

Skip-Count by Fives (12 minutes)

Materials: (S) Skip-Count by Fives Sprint

Note: This activity builds a foundation for multiplication using units of 5 through reviewing skip-counting from Grade 2. See Directions for Administration of Sprints in Lesson 2.

Between sprints, include the following group counting in place of movement exercises:

- Count by fours to 40, hum/talk forward and backward. (Hum as you think 1, 2, 3; say 4. Hum as you think 5, 6, 7; say 8, etc.)
- Count by sixes to 42 forward and backward. Emphasize the 24 to 30 and 36 to 42 transitions.
- Count by threes to 30 forward and backward.

Application Problem (8 minutes)

Red, orange, and blue scarves are on sale for $4 each. Nina buys 2 scarves of each color. How much does she spend altogether?

Note: This problem reviews multiplication using units of four. It also leads into Problem 1 of the Concept Development.
Lesson 20

Solve two-step word problems involving multiplication and division and assess the reasonableness of answers.

Date: 5/6/13

Concept Development (30 minutes)

Materials: (S) Personal white boards

Problem 1: Model a two-step problem with a tape diagram.

Write or project the following story: Red, orange, and blue scarves are on sale for $4 each. Nina buys 2 scarves of each color. She also buys a hat that costs $4. How much does she spend altogether?

T: Compare this new problem with the application problem you just solved. What is different?

S: The question is still the same, but the new problem adds the cost of a hat to the total.

T: Turn and talk to your partner: How can we use our answer from the application problem to help solve the new problem?

S: In our application problem we found the cost of the 6 scarves. → We just have to add the cost of the hat to the total.

T: (Draw tape diagram.) This tape diagram shows the application problem.

T: Let’s call one of these boxes a unit. Tell me what 1 unit represents.

S: 1 scarf.

T: How much is 1 unit?

S: $4.

T: What do the 6 units represent?

S: 6 scarves.

T: How did you label the 6 units?

S: With a question mark.

T: What equation did you use to find the total cost of the scarves?

S: 6 x $4 = $24.

T: Now watch as I label and place a new unit and a question mark to represent our new problem.

T: (Draw and label diagram and question mark.) Now I add the cost of the hat, $4, to the total cost of the scarves, $24, (write $4 + $24 = ____), which is...

S: $28.

T: How many units did we add together to find the total of both items?

S: 7 units. → 1 unit + 6 units

T: The problem tells us the value of 1 unit, and from our diagram we can see that we will add a total of 7.
Tell your partner a multiplication sentence you use to find the total cost of the sweater and hat without finding the value of the sweater first.

S: 7 units = $28 \rightarrow 7 \times \$4 = \$28

**Problem 2: Use the tape diagram to solve a two-step problem.**

Write or project the following story: Mr. Lim buys 7 plants for his garden. Each plant costs $5. The next day he buys a rose bush that also costs $5. How much more do the 7 plants cost than the rose bush?

T: What information is known from reading the story?
S: The cost of each plant is $5. We also know the rose bush costs $5.
T: What information is unknown?
S: We don’t know the total cost of the 7 plants. So we don’t know how much more the plants cost than the rose bush.
T: Notice there are 2 unknowns in our problem. Let’s first draw and label a tape diagram to model where the unknown is the cost of the 5 plants.
S: (Draw and label tape diagram.)
T: Tell me how to find the cost of the plants.
S: We multiply 7 \times \$5.
T: The plants cost....
S: \$35.
T: Have we answered the question?
S: No.
T: What is the question we are trying to answer?
S: How much more do the plants cost than the rose bush?
T: (Label the second question mark.) Tell your partner what strategy you might use to answer the question.
S: I might subtract the cost of the rose bush from the total cost of the 5 plants. \rightarrow I might do 6 \times \$5 because the plants have 6 units more than the rose bush. \rightarrow I’ll skip-count the 6 extra fives on the plants diagram.
T: Write an equation and solve the problem on your personal white board.
S: (Possibly write: \$35 - \$5 = \$30, 6 \times \$5 = \$30, \$5 + \$5 + \$5 + \$5 + \$5 + \$5 = \$30.)
T: Reread the question. Have we answered it?
S: (Reread and confirm.)
T: Is \$30 a reasonable answer? Why or why not?
Lesson 20: Solve two-step word problems involving multiplication and division and assess the reasonableness of answers.

Date: 5/6/13

S: Yes, 7 plants are expensive! $5 is a lot less than $35, so $30 less makes sense. → I checked with addition. $30 + $5 = $35.

T: (Erase the first diagram and the $35 that marks the total value on the second diagram.) Tell your partner how this diagram represents the problem on its own.

S: It shows 1 rose bush, 5 plants and both unknowns.

T: We know that 1 unit is $5. How many units is the additional cost of the plants?

S: 6 units.

T: Given what you know, is it necessary to find the total cost of the plants? Why or why not?

S: You can just do 6 x $5 without having to know about $35.

T: Explain to your partner the difference between the two ways of solving this problem.

Problem 3: Work with a partner to model and solve a two-step problem.

Write or project the following story: 10 children equally share 40 almonds. How many almonds will three children get?

T: What information is known?

S: The total amount of almonds and the number of children.

T: What is unknown?

S: How many almonds 3 children get.

T: In order to solve, what do you need to find first?

S: The amount of almonds 1 child gets.

T: With a partner, model and solve the problem. Make sure to reread the question to see if you have answered the question. Then think about whether or not the answer makes sense. This is how we check the reasonableness of the answer.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Lesson Objective: Solve two-step word problems involving multiplication and division and assess the reasonableness of answers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Compare the structure of Problems 1 and 2 to the rest of the problem set. Problems 1 and 2 explicitly ask 2 questions to scaffold the two-step word problems. Problems 3–5 still require 2 steps, but only ask 1 question.
- Compare the Problems 3 and 5. What do the unknowns represent? How are these problems similar? How are they different?
- Have students share their models. In Problems 3 and 5, how did you show the boxes of broken cups and the bags of pears sold?
- How did you check for the reasonableness of your answers to each problem?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
### Lesson 20 Sprint

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Lesson 20: Solve two-step word problems involving multiplication and division and assess the reasonableness of answers.

Date: 5/6/13

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   a. What is the total cost of the books?

   b. How much does Ted spend altogether?

2. Seven children share 28 silly bands equally.

   a. How many silly bands does each child get?

   b. How many silly bands do 3 children get?
3. Eighteen cups are equally packed into 6 boxes. Two boxes of cups break. How many cups are unbroken?

4. There are 25 blue balloons and 15 red balloons at a party. Five children are given an equal number of each color balloon. How many blue and red balloons does each child get?

5. Twenty-seven pears are packed in bags of 3. Five bags of pears are sold. How many bags of pears are left?
1. Thirty-two jellybeans are shared by 8 students.

   a. How many jellybeans will each student get?

   b. How many jellybeans will 4 students get?

2. The teacher has 30 apple slices and 20 pear slices. Five children equally share all of the fruit slices. How many fruit slices does each child get?

   ![Diagram of pencil and marker costs]

   a. What is the total cost of the markers?

   b. How much more does David spend on 4 sets of markers than Jerry spends on a pack of pencils?

2. Thirty students are eating lunch at 5 tables. Each table has the same number of students.

   ![Diagram of students at tables]

   a. How many students are sitting at each table?

   b. How many students are sitting at 4 tables?
3. The teacher has 12 green stickers and 15 purple stickers. Three super star students are given an equal number of each color sticker. How many green and purple stickers does each student get?

4. Three friends go apple picking. They pick 13 apples on Saturday and 14 apples on Sunday. They share the apples equally. How many apples does each person get?

5. The store has 28 notebooks in packs of 4. Three packs of notebooks are sold. How many packs of notebooks are left?
Lesson 21

Objective: Solve two-step word problems involving all four operations and assess the reasonableness of answers.

Suggested Lesson Structure

- Fluency Practice (15 minutes)
- Application Problem (5 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (15 minutes)

- Group Counting 3.OA.1 (3 minutes)
- Multiply by 5 3.OA.7 (9 minutes)
- Commutative Multiplying 3.OA.7 (3 minutes)

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by threes, fours, and sixes in this activity reviews multiplication with units of 3 and 4 and anticipates multiplication with units of 6 in Module 3.

- Count by fours to 40, think/talk forward and backward. (Think 1, 2, 3; say 4. Think 5, 6, 7; say 8, etc.)
- Count by sixes to 48 forward and backward. Emphasize the 24 to 30 and 36 to 42 transitions.
- Count by threes to 30 forward and backward.

Multiply by Five (9 minutes)

Materials: (S) Multiply by 5 Sprint Set (1–5)

Note: This activity builds fluency with multiplication facts using units of 5. It works toward students knowing from memory all products of two one-digit numbers. See Directions for Administration of Multiply by in Lesson 9.

T: (Write 5 x 5 = ____.) Let’s skip-count by fives to find the answer. (Count with fingers to 5 as students count.)

S: 5, 10, 15, 20, 25.
T: (Circle 25 and write $5 \times 5 = 25$ above it. Write $5 \times 3 = ____$.) Let’s skip-count up by fives again. (Count with fingers to 3 as students count.)

S: 5, 10, 15.

T: Let’s see how we can skip-count down to find the answer, too. Start at 25. (Count down with your fingers as students say numbers.)

S: 25, 20, 15.

Repeat process for $5 \times 9$ and $5 \times 8$.

T: Let’s practice multiplying by 5. Be sure to work left to right across the page. (Distribute Multiply by Five Sprint.)

Commutative Multiplying (3 minutes)

Note: This activity reviews the commutativity of multiplication, learned in Lessons 7, 8, and 15.

T: (Write $4 \times 2 = ____$.) Say the multiplication sentence.

S: $4 \times 2 = 8$.

T: Flip it.

S: $2 \times 4 = 8$.

Repeat process for $5 \times 3$, $9 \times 2$, $4 \times 3$, $2 \times 7$, and $3 \times 8$.

Application Problem (5 minutes)

There are 4 boxes with 6 binders in each one. 3 brothers share the binders. How many binders does each brother get?

Note: This two-step problem reviews Lesson 20’s emphasis. To solve application problems, students self-select an approach and independently solve them. Practicing a two-step problem here scaffolds the difference between the structured practice in Lesson 20 and the open-ended practice in Lesson 21. Prepare students for today’s exploration by guiding them to evaluate their methods for solving and assessing the reasonableness of their answer.

Concept Development (30 minutes)

Materials: (S) Chart paper, markers, paper strips (optional for representing tape diagrams), glue

Today’s lesson is a culminating exploration that follows this process:
- Divide the class into groups no larger than four students.
- Assign each group one word problem from the Problem Set. (Cut the Problem Set so that initially each group only receives the problem they are assigned. More than one group may work on the
same problem.)

- Each group collaborates to model and solve their assigned problem.
- Each group prepares to present their problem to the class, describing their method for solving and explaining the reasonableness of their answer.

Each group needs one set of the materials listed in the materials section.

Directions (similar to RDW process):

1. Read and analyze together to determine known and unknown information.
2. Discuss how to model.
3. Model and label diagrams.
4. Discuss and agree on the steps needed to solve.
5. Write equations and solve.
6. Assess the reasonableness of the solution (ask, “Does our answer make sense? How do we know?”).
7. Write a complete sentence to answer the question.
8. Prepare a mini-presentation to explain your work on directions 1–7. Prepare to answer clarifying questions from the group.

Each group presents to the class. Audience members should be prepared to ask clarifying questions, challenge each other’s work and offer compliments. If more than one group solves the same problem, discussion might include similarities and differences between problem-solving approaches.

**Problem Set (5 minutes)**

When all groups have presented, pass out the entire Problem Set and have the students solve the problems independently. The time allotment is short, as they’ve just seen and discussed every problem.
Student Debrief (10 minutes)

Lesson Objective: Solve two-step word problems involving all four operations and assess the reasonableness of answers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

After the presentations and discussion of the problem set problems in the lesson, today’s debrief culminates the module with a celebration. Students reflect on their progress in learning to multiply and divide using units of 2, 3, 4, 5, and 10.

Students are seated with personal white boards. Select one student to stand behind someone seated. Say a fact or give a word problem. Of the pair, the first student to solve it correctly and lift their board wins the round. That student rotates one seat to the right. The goal is for a single child to work their way back to the seat they originally stood behind. The game is very fast-paced to build excitement. Given the time constraint, the game is unlikely to finish. The winner can be the student who moves the most spaces.

Sample facts or word problems:

- How many legs are there on 5 dogs?
- 4 x 3
- 6 x 2
- Write a related division fact for 5 x 3
- 18 ÷ 3
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Multiply.

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1. Jason earns $6 per week for doing all his chores. On the fifth week he forgets to take out the trash so he only earns $4. Write and solve an equation to show how much Jason earns in 5 weeks.

Jason earns $______.

2. Miss Lianto orders 4 packs of 7 markers. After passing out 1 marker to each student in her class, she has 6 left. Label the tape diagram to find how many students are in Miss Lianto’s class.

There are _______ students in Miss Lianto’s class.
3. Orlando buys a box of 18 fruit snacks. Each box comes with an equal amount of strawberry, cherry, and grape flavored snacks. He eats all of the grape flavored snacks first. Draw and label a tape diagram to find how many fruit snacks he has left.

4. Eudora buys 21 m of ribbon. She cuts the ribbon so that each piece measures 3 m in length.
   
   a. How many pieces of ribbon does she cut?

   b. If Eudora needs a total of 12 pieces of ribbon, how many more pieces of ribbon does she need?
Ms. Egeregor buys 27 books for her classroom library. She buys an equal amount of fiction, nonfiction, and poetry books. She shelves all of the poetry books first. Draw and label a tape diagram to show how many books Ms. Egeregor has left to shelve.
1. Tina eats 8 crackers for a snack each day at school. On Friday she drops 3 and only eats 5. Write and solve an equation to show the total number of crackers Tina eats during the week.

Tina eats ________ crackers.

2. Ballio has a reading goal. He checks 3 boxes of 9 books out from the library. After finishing them, he realizes that he beat his goal by 4 books! Label the tape diagrams to find Ballio’s reading goal.

Ballio’s goal is to read ________ books.
3. Mr. Nguyen plants 24 trees around the neighborhood pond. He plants equal numbers of Maple, Pine, Spruce, and Birch trees. He waters the Spruce and Birch trees before it gets dark. How many trees does Mr. Nguyen still need to water? Draw and label a tape diagram.

4. Anna buys 24 seeds and plants 3 in each pot. She has 5 pots. How many more pots does Anna need to plant all of her seeds?