Topic C

Multiplication and Division Using Units up to 8

3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7, 3.OA.1, 3.OA.2, 3.OA.6, 3.OA.8

Focus Standard:

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?

3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Instructional Days: 4

Coherence -Links from:

G2–M3 Place Value, Counting, and Comparison of Numbers to 1000
G2–M6 Foundations of Multiplication and Division
G3–M1 Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

-Links to:

G3–M4 Multiplication and Area
G4–M3 Multi-Digit Multiplication and Division
G4–M5 Fraction Equivalence, Ordering, and Operations
G4–M7 Exploring Multiplication

Students are informally familiar with parentheses from having seen them in distributive property lessons in Module 1. In Lesson 8 they understand parentheses as tools for grouping, and learn the conventional order for performing Grade 3 operations. This practice anticipates the application of parentheses that students will make in Lesson 9 as they formally study the associative property.
In Lesson 9, students model and demonstrate how to multiplicatively compose or decompose to make problems using units up to 8 easier to solve. For example, \(8 \times 5\) may be thought of as

\[
8 \times 5 = (4 \times 2) \times 5
\]

\[
= 4 \times (2 \times 5)
\]

\[
= 4 \times 10
\]

Lessons 10 and 11 in this topic parallel Lessons 6 and 7 in Topic B. In Lesson 10, students use the \(5 + n\) pattern as a strategy for solving multiplication and division problems using units of 8 with the distributive property. They learn that multiples of 8 can be thought of as \((5 + 3) \times n\). In division problems they practice decomposing the dividend using multiples of 5. They recognize the efficacy of using this strategy when the quotient of a division equation is greater than 5, and also realize that the dividend must be decomposed into numbers that are divisible by the divisor. For example, to solve \(64 \div 8\), 64 can be decomposed as 40 and 24 because both are divisible by 8.

In Lesson 11, students analyze, model, and solve multiplication and division word problems using units of 8. They understand division as both a quantity divided into equal groups, as well as an unknown factor problem. They draw models and write equations to interpret and solve problems, using a letter to represent the unknown in various positions.

### A Teaching Sequence Towards Mastery of Multiplication and Division Using Units up to 8

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Lesson 8

Objective: Understand the function of parentheses and apply to solving problems.

Suggested Lesson Structure

- Fluency Practice: 15 minutes
- Application Problem: 5 minutes
- Concept Development: 30 minutes
- Student Debrief: 10 minutes
- Total Time: 60 minutes

Fluency Practice (15 minutes)

- Multiply by 7 3.OA.7 (6 minutes)
- Group Counting 3.OA.1 (4 minutes)
- Add 6 and 7 Mentally 2.NBT.5 (5 minutes)

Multiply by 7 (6 minutes)

Materials: (S) Multiply by 7 Pattern Sheet (6–10)

Note: This activity builds fluency with multiplication facts using units of 7. It works toward students knowing from memory all products of two one-digit numbers. See G3–M3–Lesson 5 for the directions for administration of Multiply By Pattern Sheet.

T: (Write $7 \times 6 = \underline{}$.) Let’s skip-count up by sevens to solve. I’ll raise a finger for each seven. (Count with fingers to 6 as students count.)

S: 7, 14, 21, 28, 35, 42.

T: Let’s skip-count down to find the answer, too. Start at 70. (Count down with fingers as students count.)

S: 70, 63, 56, 49, 42.

Continue with the following suggested sequence: $7 \times 8$ and $7 \times 7$, and $7 \times 9$.

T: (Distribute the Multiply by 7 Pattern Sheet.) Let’s practice multiplying by 7. Be sure to work left to right across the page.
Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes reviews multiplication using those units in Topic B. Group counting eights prepares students for multiplication in this topic, and nines anticipates multiplication using those units later in the module. Direct students to count forward and backward, occasionally changing the direction of the count.

- Sixes to 60
- Eights to 80
- Nines to 90

Add 6 and 7 Mentally (5 minutes)

Materials: (S) Personal white boards

Note: This activity reviews the make ten strategy used for skip-counting by sixes and sevens in Lessons 4 and 5.

T: (Project 6 + 6 = ___.) Say the equation.
S: 6 + 6.
T: 6 and what make ten?
S: 4.
T: (Draw a number bond beneath the second 6.) On your boards, break apart the second 6, taking out the 4.

6 + 4 = 10

S: (Write the number bond.)
T: Say the addition sentence.
S: 6 + 6 = 12.

Continue with the following possible sequence: 12 + 6, 18 + 6, 24 + 6, 30 + 6, 36 + 6, 42 + 6, 48 + 6, 54 + 6, 7 + 7, 14 + 7, 21 + 7, 28 + 7, 35 + 7, 42 + 7, 49 + 7, 56 + 7, and 63 + 7.

Application Problem (5 minutes)

Richard has 2 cartons with 6 eggs in each. As he opens the cartons, he drops 2 eggs. How many unbroken eggs does Richard have left?

Richard has 10 eggs left.

3.C.4
Lesson 8: Understand the function of parenthesis and apply to solving problems.

Date: 7/31/13

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3.C.5

Note: This problem provides context for solving equations involving multiple operations, which is central to the Concept Development.

Concept Development (30 minutes)

Materials: (S) Personal white boards

Part 1: Solve equations containing parentheses.

T: The two equations used to solve the Application Problem are
2 \times 6 = 12 and 12 – 2 = 10. (Show picture to the right.) This picture shows both. Talk to your partner: How could we include all of this information in one equation?

S: We can rewrite them as one equation. Maybe 2 \times 6 – 2 = 10?

T: Let’s check to make sure the new equation equals 10. Should we multiply first or subtract first? Does it matter?

S: I don’t think it matters. \(\rightarrow\) Before we multiplied first, so let’s do that again.

T: Let’s find out. Solve the equation twice. The first time you solve it, multiply first. The second time you solve, subtract first. (Allow time for students to calculate.)

S: When I multiplied first I still got 10, but when I subtracted first I got 8!

T: For this problem the order does matter. We can use parentheses in our equation to show what to do first. Remind me, which part of the equation do we need to do first and why?

S: 2 \times 6, because we have to find the total number of eggs Richard has in 2 cartons first.

T: Watch how I use parentheses to show that.

(Write: \((2 \times 6) – 2 = 10\).)

T: What is the product of 2 \times 6?

S: 12!

T: Rewrite 2 \times 6 as 12. What equation is left?

S: 12 – 2!

T: What does 12 – 2 equal?

S: 10!

T: In a complete sentence, how many eggs does Richard have left?

S: Richard has 10 eggs left.

Continue with the following suggested sequence:

- \(4 + 2 = 6\) and \(6 \times 6 = 36 \rightarrow (4 + 2) \times 6 = 36\)
- \(12 ÷ 3 = 4\) and \(15 – 4 = 11 \rightarrow 15 – (12 ÷ 3) = 11\)

Note: Have students refer back to the original problem, as the situation dictates the placement of the parentheses.
Part 2: Explore how moving the parentheses can change the answer in an equation.

Write or project the following equation and the picture to the right: \((25 - 10) \div 5 = 3\).

T: Check my work. Is it correct?
S: Yes, because 25 – 10 equals 15, and 15 ÷ 5 equals 3.
T: Let’s divide 10 by 5 first. What should we do with the parentheses to show that?
S: Move them over! \(\rightarrow\) Make them go around \(10 \div 5\).
T: Now the equation looks like this. \((\text{Write } 25 - (10 \div 5) = n.)\) Write the equation on your board. Why is there a letter where the 3 was before?
S: We should write 3 because the numbers didn’t change. \(\rightarrow\) We don’t know if it equals 3 anymore.
T: Really? Why not? The numbers are the same as before.
S: The parentheses moved.
T: Do the problem with your partner. Does this equation still have an answer of 3?
S: \((\text{Work and discuss.})\) No, the answer is 23!
T: Why is the answer different?
S: We divided first. \(\rightarrow\) One way we divided 15 by 5. \(\rightarrow\) The other way we subtracted 2 from 25. \(\rightarrow\) We divided and then subtracted. Before, we subtracted and then divided.
T: What does this tell you about the way we use parentheses to group the math in equations? Is it important? Why or why not?
S: The parentheses tell us what math gets done first. \(\rightarrow\) Yes it’s important, because moving the parentheses can change the answer.

Continue with the following possible suggestions:
- \((2 + 3) \times 7\) and \(2 + (3 \times 7)\)
- \((3 \times 4) \div 2\) and \(3 \times (4 \div 2)\)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Lesson 8: Understand the function of parentheses and apply to solving problems.

Lesson Objective: Understand the function of parentheses and apply to solving problems.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Look at Problem 1(j). Would the answer be the same if I solved \((12 \div 2) + (12 \div 4)\)? Why not? (Lead students to understand that they cannot distribute in this problem.)
- Look at Problem 1(l). Would the answer be the same if I solved \((9 \div 3) + (15 \div 3)\)? Why?
- Invite students to share how they discovered where the parentheses belonged in Problem 2.
- Why does moving the parentheses in an equation only change the answer sometimes?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Multiply.

\[
\begin{align*}
7 \times 1 &= \underline{\phantom{0}} & 7 \times 2 &= \underline{\phantom{0}} & 7 \times 3 &= \underline{\phantom{0}} & 7 \times 4 &= \underline{\phantom{0}} \\
7 \times 5 &= \underline{\phantom{0}} & 7 \times 6 &= \underline{\phantom{0}} & 7 \times 7 &= \underline{\phantom{0}} & 7 \times 8 &= \underline{\phantom{0}} \\
7 \times 9 &= \underline{\phantom{0}} & 7 \times 10 &= \underline{\phantom{0}} & 7 \times 5 &= \underline{\phantom{0}} & 7 \times 6 &= \underline{\phantom{0}} \\
7 \times 5 &= \underline{\phantom{0}} & 7 \times 7 &= \underline{\phantom{0}} & 7 \times 5 &= \underline{\phantom{0}} & 7 \times 8 &= \underline{\phantom{0}} \\
7 \times 5 &= \underline{\phantom{0}} & 7 \times 9 &= \underline{\phantom{0}} & 7 \times 5 &= \underline{\phantom{0}} & 7 \times 10 &= \underline{\phantom{0}} \\
7 \times 6 &= \underline{\phantom{0}} & 7 \times 5 &= \underline{\phantom{0}} & 7 \times 6 &= \underline{\phantom{0}} & 7 \times 7 &= \underline{\phantom{0}} \\
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7 \times 8 &= \underline{\phantom{0}} & 7 \times 7 &= \underline{\phantom{0}} & 7 \times 9 &= \underline{\phantom{0}} & 7 \times 7 &= \underline{\phantom{0}} \\
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7 \times 8 &= \underline{\phantom{0}} & 7 \times 9 &= \underline{\phantom{0}} & 7 \times 9 &= \underline{\phantom{0}} & 7 \times 6 &= \underline{\phantom{0}} \\
7 \times 9 &= \underline{\phantom{0}} & 7 \times 7 &= \underline{\phantom{0}} & 7 \times 9 &= \underline{\phantom{0}} & 7 \times 8 &= \underline{\phantom{0}} \\
7 \times 9 &= \underline{\phantom{0}} & 7 \times 8 &= \underline{\phantom{0}} & 7 \times 6 &= \underline{\phantom{0}} & 7 \times 9 &= \underline{\phantom{0}} \\
7 \times 7 &= \underline{\phantom{0}} & 7 \times 9 &= \underline{\phantom{0}} & 7 \times 6 &= \underline{\phantom{0}} & 7 \times 8 &= \underline{\phantom{0}} \\
7 \times 9 &= \underline{\phantom{0}} & 7 \times 7 &= \underline{\phantom{0}} & 7 \times 6 &= \underline{\phantom{0}} & 7 \times 8 &= \underline{\phantom{0}} \\
\end{align*}
\]

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1. Solve.
   a. \((12 - 4) + 6 = \) _____
   b. \(12 - (4 + 6) = \) _____
   c. _____ = \(15 - (7 + 3)\)
   d. _____ = \((15 - 7) + 3\)
   e. _____ = \((3 + 2) \times 6\)
   f. _____ = \(3 + (2 \times 6)\)
   g. \(4 \times (7 - 2) = \) _____
   h. \((4 \times 7) - 2 = \) _____
   i. _____ = \((12 \div 2) + 4\)
   j. _____ = \(12 \div (2 + 4)\)
   k. \(9 + (15 \div 3) = \) _____
   l. \((9 + 15) \div 3 = \) _____
   m. \(60 \div (10 - 4) = \) _____
   n. \((60 \div 10) - 4 = \) _____
   o. _____ = \(35 + (10 \div 5)\)
   p. _____ = \((35 + 10) \div 5\)

2. Use parentheses to show the order you would need to do the operations to make the equation true.
   
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<tbody>
<tr>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>16 (-) 4 + 7 = 19</td>
<td>16 (-) 4 + 7 = 5</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>2 = 22 (-) 15 + 5</td>
<td>12 = 22 (-) 15 + 5</td>
</tr>
<tr>
<td>e</td>
<td>f</td>
</tr>
<tr>
<td>3 + 7 \times 6 = 60</td>
<td>3 + 7 \times 6 = 45</td>
</tr>
<tr>
<td>g</td>
<td>h</td>
</tr>
<tr>
<td>5 = 10 \div 10 \times 5</td>
<td>50 = 100 \div 10 \times 5</td>
</tr>
<tr>
<td>i</td>
<td>j</td>
</tr>
<tr>
<td>26 (-) 5 \div 7 = 3</td>
<td>36 = 4 \times 25 - 16</td>
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</tbody>
</table>
3. The teacher writes $24 \div 4 + 2 = \square$ on the board. Chad says it equals 8. Samir says it equals 4. Explain how placing the ( ) in the equation can make both answers true.

4. Natasha solves the equation below by finding the sum of 5 and 12. Place the ( ) around the equation to show her thinking. Then solve.

$$12 + 15 \div 3 = \square$$

5. Find two possible answers to the expression $7 + 3 \times 2$ by placing ( ) around different numbers.
Name ____________________________________________ Date ________________

1. Use parentheses to make the equations true.
   
   a. $24 = 32 - 14 + 6$  
   b. $12 = 32 - 14 + 6$
   
   c. $2 + 8 \times 7 = 70$  
   d. $2 + 8 \times 7 = 58$

2. Marcos solves $24 \div 6 + 2 = \underline{\hspace{2cm}}$. He says it equals 6. Iris says it equals 3. Show and explain how the position of parentheses in the equation can make both answers true.
Lesson 8 Homework

Name ________________________________ Date ______________________

1. Solve.
   a. \(9 - (6 + 3) = \) ____
   b. \((9 - 6) + 3 = \) ____
   c. ____ = \(14 - (4 + 2)\)
   d. ____ = \((14 - 4) + 2\)
   e. ____ = \((4 + 3) \times 6\)
   f. ____ = \(4 + (3 \times 6)\)
   g. \((18 \div 3) + 6 = \) ____
   h. \(18 \div (3 + 6) = \) ____

2. Use parentheses to make the equations true.
   a. \(14 - 8 + 2 = 4\)
   b. \(14 - 8 + 2 = 8\)
   c. \(2 + 4 \times 7 = 30\)
   d. \(2 + 4 \times 7 = 42\)
   e. \(5 = 50 \div 5 \times 2\)
   f. \(20 = 50 \div 5 \times 2\)
   g. \(12 = 18 \div 3 \times 2\)
   h. \(3 = 18 \div 3 \times 2\)
3. Determine if the equation is true or false.

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<tbody>
<tr>
<td>a.</td>
<td>((15 - 3) \div 2 = 6)</td>
</tr>
<tr>
<td>b.</td>
<td>((10 - 7) \times 6 = 18)</td>
</tr>
<tr>
<td>c.</td>
<td>((35 - 7) \div 4 = 8)</td>
</tr>
<tr>
<td>d.</td>
<td>(28 = 4 \times (20 - 13))</td>
</tr>
<tr>
<td>e.</td>
<td>(35 = (22 - 8) \div 5)</td>
</tr>
</tbody>
</table>

4. Jerome finds that \((3 \times 6) \div 2\) and \(18 \div 2\) are equal. Explain why this is true.

5. Place parentheses in the equation below so that you solve by finding the difference between 28 and 3. Find the answer.

\[4 \times 7 - 3 =\]

6. Johnny says that the answer to \(2 \times 6 \div 3\) is 4 no matter where the parentheses are. Do you agree? Place parentheses around different numbers to show his thinking.
Lesson 9

Objective: Model the associative property as a strategy to multiply.

Suggested Lesson Structure

- Fluency Practice (11 minutes)
- Application Problems (15 minutes)
- Concept Development (24 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)

Fluency Practice (11 minutes)

- Divide by 6 and 7 3.OA.7 (3 minutes)
- Group Counting 3.OA.1 (4 minutes)
- Write In the Parentheses 3.OA.7 (4 minutes)

Divide by 6 and 7 (3 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews using a letter to represent the unknown, first taught in G3–M3–Lesson 3.

T: (Write $a \times 6 = 12$.) On your boards, write the value of $a$.
S: (Write $a = 2$.)
T: (Write $12 \div 6 = a$.) Say the division sentence.
S: $12 \div 6 = 2$.

Continue with the following suggested sequence: $a \times 6 = 30, b \times 6 = 24, c \times 6 = 36, d \times 6 = 60, e \times 6 = 54, f \times 7 = 35, g \times 7 = 28, h \times 7 = 42, j \times 7 = 70, \text{and } k \times 7 = 56$.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Group counting eights prepares students for multiplication in this topic, and nines anticipates multiplication using those units later in the module.

Direct students to count forward and backward, occasionally changing the direction of the count:

- Eights to 80
- Nines to 90
Write In the Parentheses (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews the use of parentheses, taught in G3–M3–Lesson 8.

T: (Write $10 - 5 + 3 = 8.$) On your boards, copy the equation. Then insert parentheses to make the statement true.

S: (Write $(10 - 5) + 3 = 8.$)

Continue with the following suggested sequence: $10 - 5 + 3 = 2$, $10 = 20 - 7 + 3$, $8 + 2 \times 4 = 48$, $8 + 2 \times 4 = 40$, $12 = 12 + 2 \times 2$, $3 = 12 + 2 \times 2$, $10 = 35 - 5 \times 5$, and $20 - 10 \div 5 = 2$.

Application Problems (15 minutes)

Materials: (S) Application Problems Sheet

Note: These problems give students practice solving equations with parentheses. This sequence of problems is specifically designed so that students recognize that the position of the parentheses does not change the answer in multiplication problems with more than two factors. (The same is true for addition. Problem 1 hints at this.) Debrief the Application Problems so that this is clear to students with respect to multiplication; this understanding will be critical for the Concept Development. You may choose to begin the discussion by having them analyze the difference between the problems they circled and those they did not.

Concept Development (24 minutes)

Materials: (S) Personal white boards

T: (Write $16 \times 3.$) This is a difficult problem for a third grader to solve. Let’s simplify it. Work with your partner to list factors that have a product of 16. Write them on your personal board.

S: $4$ times $4$ makes $16! \rightarrow 8$ and $2$ also works.

T: $4$, $8$, and $2$ are much friendlier factors than $16$. Let’s rewrite $16$ as $8 \times 2$. (Write $(8 \times 2) \times 3.$) Why do you think I put $8 \times 2$ in parentheses?

S: The parentheses show that when you group those numbers together and multiply, you get $16$.

A NOTE ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

One way to scaffold listing factors of $16$ is to give students $16$ beans they can put into equal groups.
T: Even with the 16 rewritten, this problem isn’t too friendly because I still have to multiply 16 × 3 in the last step. Suppose I move the parentheses to change the way the numbers are grouped. Will it completely change my answer?
S: No, we saw it’s okay to move the parentheses when it’s all multiplication in our Application Problems.
T: Write the equation on your board. Use the parentheses to group the numbers differently. Check your work with your partner’s.
S: (Write 8 × (2 × 3) and check work with a partner.)
T: (Draw array.) My array shows how I regrouped the numbers to show 8 groups of (2 × 3). Is this problem friendlier than 16 × 3?
S: Oh, it’s just 8 × 6! That’s the same as 48! That was easy!
T: So, what is the answer to 16 × 3?
S: 48!
T: Tell your partner the steps we took to simplify the problem and solve.
S: First we rewrote 16 as a multiplication problem with two easier factors. Then we grouped the numbers with parentheses to make a multiplication problem that was easy to solve.
T: When we brainstormed factors with a product of 16, some of you thought of 4 × 4. Let’s see if rewriting the 16 that way helps us simplify. Rewrite 16 × 3 using 4 × 4.
S: (Write (4 × 4) × 3.)
T: Is it easy to solve yet?
S: No!
T: Try and simplify by using the parentheses to group the numbers differently.
S: (Write 4 × (4 × 3).)
T: (Draw array.) Here is the array that shows our 4 groups of (4 × 3). Did the problem get easier?
S: Not really. It’s still 4 × 12, and that’s hard.
T: Let’s compare the two arrays. What do you notice?
S: They show 16 × 3 in different ways. → The first array shows 8 groups of 6 and the second array show 4 groups of 12. → The second array has fewer groups but multiplies a larger number. → So, both arrays still show a total of 48, but the first array breaks it up into easier numbers.
T: True. If we use repeated addition to find the answer to 4 × 12 we’ll find the answer is still 48. We didn’t do anything wrong, but rewriting the 16 as 4 × 4 and moving the parentheses didn’t do what
we wanted it to. It didn’t help us simplify. With your partner, compare the two arrays. What happened when we rewrote 16 as 4 × 4 and 8 × 2? What does the comparison tell you about this strategy?

S: It doesn’t always work. → It means you have to be careful about which numbers you choose. → Yeah, some are helpful and some aren’t. → Sometimes you might have to try more than one pair of numbers before you find the pair that helps you simplify.

Continue with 15 × 3. You may want to point out that the order in which 15 is rewritten can make a difference. For example, ask students to notice which is easier:

a. $(3 \times 5) \times 3$
   - $3 \times (5 \times 3)$
   - $3 \times 15$

b. $(5 \times 3) \times 3$
   - $5 \times (3 \times 3)$
   - $5 \times 9$

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Model the associative property as a strategy to multiply.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1, how do the problems on the bottom simplify the problems on the top?
- Invite students to share how they knew where to draw parentheses for the equations in Problem 2.
- In Problem 3, how did Charlotte simplify?

NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Learners who have not memorized sixes and sevens facts may not benefit from using the associative property to solve $14 \times 3$. Adjust the numbers, or encourage students to use a more personally efficient strategy, such as the distributive property.
- How are the commutative property and this new strategy helpful for finding unknown, larger facts?
- How did the Application Problems relate to the lesson today?
- In the Application Problems we noticed that it is okay to move the parentheses when every operation is multiplication. Is that true for the other operations too? (Provide examples for subtraction and division; students will quickly see that it is not. Provide addition examples that students can use in conjunction with Application Problem 1 to generalize that it is also true for addition.)

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 9 Application Problems

Name ____________________________ Date ________________

Solve the following pairs of problems. Circle the pairs where both problems have the same answer.

1. a. 7 + (6 + 4)       b. (7 + 6) + 4
2. a. (3 × 2) × 4       b. 3 × (2 × 4)
3. a. (2 × 1) × 5       b. 2 × (1 × 5)
4. a. (4 × 2) × 2       b. 4 × (2 × 2)
5. a. (3 + 2) × 5       b. 3 + (2 × 5)
6. a. (8 ÷ 2) × 2       b. 8 ÷ (2 × 2)
7. a. (9 − 5) + 3       b. 9 − (5 + 3)
8. a. (8 × 5) − 4       b. 8 × (5 − 4)
1. Use the array to complete the equation.

a) \( 3 \times 12 = \) 

b) \( (3 \times 3) \times 4 = \) 

\[ = ___ \times 4 \]
\[ = ___ \]

c) \( 3 \times 14 = \) 

d) \( (___ \times ___) \times 7 = \) 

\[ = ___ \times ___ \]
\[ = ___ \]
2. Place ( ) in the equations to simplify. Then solve. The first one has been done for you.

\[ 3 \times 16 = 3 \times (2 \times 8) \]
\[ = (3 \times 2) \times 8 \]
\[ = 6 \times 8 \]
\[ = 48 \]

\[ 2 \times 14 = 2 \times (2 \times 7) \]
\[ = (2 \times 2) \times 7 \]
\[ = 4 \times 7 \]

\[ 3 \times 12 = 3 \times (3 \times 4) \]
\[ = 3 \times 3 \times 4 \]
\[ = 9 \times 4 \]

\[ 3 \times 14 = 3 \times 2 \times 7 \]
\[ = 3 \times 2 \times 7 \]
\[ = 6 \times 7 \]

\[ 2 \times 14 = 2 \times (2 \times 7) \]
\[ = (2 \times 2) \times 7 \]
\[ = 4 \times 7 \]

\[ 15 \times 3 = 5 \times 3 \times 3 \]
\[ = 5 \times 3 \times 3 \]
\[ = 15 \times 3 \]

\[ 15 \times 2 = 5 \times 3 \times 2 \]
\[ = 5 \times 3 \times 2 \]
\[ = 10 \times 2 \]

3. Charlotte finds the answer to \( 16 \times 2 \) by thinking about \( 8 \times 4 \). Explain her strategy.
Lesson 9 Exit Ticket

Name ___________________________________________ Date _____________________

Simplify to find the answer to $18 \times 3$. Show your work and explain your strategy.
1. Use the array to complete the equation.

   a. \(3 \times 16 = \underline{\hspace{2cm}}\)

   b. \((3 \times \underline{\hspace{1cm}}) \times 8 = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}\)

   c. \(4 \times 18 = \underline{\hspace{2cm}}\)

   d. \((4 \times \underline{\hspace{1cm}}) \times 9 = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}\)
2. Place ( ) in the equations to simplify and solve.

\[
12 \times 4 = (6 \times 2) \times 4 \\
= 6 \times (2 \times 4) \\
= 6 \times \_8\_ \\
= \_48\_
\]

\[
3 \times 14 = 3 \times (2 \times 7) \\
= (3 \times 2) \times 7 \\
= \_\_\_7 \\
= \_\_\_\_
\]

\[
3 \times 12 = 3 \times (3 \times 4) \\
= 3 \times 3 \times 4 \\
= \_\_\_\_4 \\
= \_\_\_\_\_
\]

3. Solve. Then match the related facts.

a. \(20 \times 2 = \_40\_ = 6 \times (5 \times 2)\)

b. \(30 \times 2 = \_\_\_\_ = 8 \times (5 \times 2)\)

c. \(35 \times 2 = \_\_\_\_ = 4 \times (5 \times 2)\)

d. \(40 \times 2 = \_\_\_\_ = 7 \times (5 \times 2)\)
Lesson 10

Objective: Use the distributive property as a strategy to multiply and divide.

Suggested Lesson Structure

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
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<tbody>
<tr>
<td>Fluency Practice</td>
<td>(7 minutes)</td>
</tr>
<tr>
<td>Application Problem</td>
<td>(5 minutes)</td>
</tr>
<tr>
<td>Concept Development</td>
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</tr>
<tr>
<td>Student Debrief</td>
<td>(10 minutes)</td>
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<tr>
<td><strong>Total Time</strong></td>
<td><strong>(60 minutes)</strong></td>
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**Fluency Practice (7 minutes)**

- Group Counting 3.OA.1 (4 minutes)
- Decompose Multiples of 8 3.OA.5 (3 minutes)

**Group Counting (4 minutes)**

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes and sevens reviews multiplication using those units in Topic B. Group counting eights prepares students for multiplication in this topic, and nines anticipates multiplication using those units later in the module.

Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

**Decompose Multiples of 8 (3 minutes)**

Materials: (S) Personal white boards

Note: This activity prepares students to use the distributive property in today’s lesson.

T: (Project a number bond with a whole of 48 and a part of 16.) On your boards, fill in the missing part in the number bond.

Continue with the following suggested sequence: whole of 56 and 24 as a part, whole of 64 and 40 as a part, whole of 40 and 16 as a part, whole of 72 and 24 as a part.
Lesson 10

**Application Problem (5 minutes)**

Use the 5 plus something break apart and distribute strategy to solve 6 \times 8. Model with a tape diagram.

Note: This problem reviews modeling the break apart and distribute strategy using a tape diagram from Lesson 6. Until today’s lesson students have learned to break apart the first factor and distribute the second factor. Today’s Concept Development reverses the order using the fact in this Application Problem.

![Tape Diagram](image)

\[(5 \times 8) + (1 \times 8) \quad 40 + 8 = 48, \quad n = 48\]

\[6 \times 48 = 48\]

**Concept Development (38 minutes)**

Materials: (S) Personal white boards

**Problem 1: Multiply.**

T: When we use the break apart and distribute strategy, which factor do we break apart?

S: We break apart the number of groups.

T: Do you think our strategy would work if we broke apart the size of the groups and distributed the factor representing the number of groups instead? Think about the commutative property. Talk to your partner.

S: I’m not sure. I don’t think so. \(\rightarrow\) The commutative property says that you can switch factors around in multiplication, so maybe it would work.

T: Let’s try using the break apart and distribute strategy that way to solve 6 \times 8. Then we can compare what happens with our work on the Application Problem.

T: Take a look at my array. (Project 6 by 8 array, shown at right.) Which factor will we break apart?

S: The 8! \(\rightarrow\) The size of the groups.

T: Breaking it into 5 plus something helps us make 2 smaller facts. We can break 8 into 5 and what?

S: 3.

T: (Write 6 \times (5 + 3) under the array.) Is 8 represented by the number of columns or the number of rows in the array?

S: The columns.

T: How should I draw my line to show that we broke apart the columns?

S: Maybe an up and down line? \(\rightarrow\) You could make a vertical line after 5 columns. Then one part would show 5 columns and the other

**NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:**

Support English language learners as they engage in today’s discussion. Offer extra time for ELLs to formulate their thoughts and discuss with their partners. If appropriate, preview words such as *factor*. Conduct subtle and frequent checks for understanding. Elaborate, expand, or paraphrase the dialogue as needed.
Lesson 10: Use the distributive property as a strategy to multiply and divide.

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Date: 7/31/13

Lesson 10

6 × (5 + 3) would show 3.

T: (Draw a dotted line after the fifth column.) On your board, write the multiplication facts you would use to label each part of the array.

S: (Write 6 × 5 and 6 × 3.)

T: What did we break our 6 eights into?

S: 6 fives and 6 threes.

T: Talk to your partner about how 6 × (5 + 3) shows 6 fives and 6 threes. Use the array to help you explain.

T: Solve the problem.

S: (May use 6 × (5 + 3) or (6 × 5) + (6 × 3) to solve.)

T: What does it equal?

S: 48.

T: Look back at your work on the Application Problem. Compare it with this way of solving. Notice what is the same or different. Talk to your partner about what you see.

S: We switched around the factors that we broke apart and distributed. → In the Application Problem, the units never changed. They were always eights. The one we just did had two different units. Fives and threes, but what stayed the same was the number of fives and the number of threes.

T: Does the break apart and distribute strategy work both ways?

S: Yes!

Continue with the following suggested problem: 7 × 8.

Problem 2: Divide.

T: Let’s use the break apart and distribute strategy to solve 64 ÷ 8. Draw a number bond with 64 ÷ 8 as the whole. Leave the parts empty. (Allow time for students to draw.)

T: Let’s think about how to break apart 64 into two numbers that are easier for us to divide. Make a list with your partner. Remember that when we break apart 64, both numbers need to be divisible by 8, because we originally distributed the 8 in our Application Problem.

S: How about 60 and 4? → No, you can’t divide those by 8! → Maybe 32 and 32. → Or 40 and 24.

T: Using 32 and 32 works nicely because it’s a double. Forty and 24 also work well; 40 ÷ 8 makes 5. Five is easy to add to, so let’s try 40 and 24. Write 40 ÷ 8 as one of the parts on your number bond.

T: What division fact goes inside the other circle?

S: 24 ÷ 8.

T: How do you know?

S: 40 plus 24 equals 64. → We started with 40 and we need 24 more to make 64.

MP.7
Lesson 10

Use the distributive property as a strategy to multiply and divide.

T: Write that division fact as the other part. Our number bond shows us that 64 ÷ 8 has the same value as combining 40 ÷ 8 and 24 ÷ 8. Work with your partner to write that as an addition sentence on your board.

S: (Write 64 ÷ 8 = (40 ÷ 8) + (24 ÷ 8).)

T: Work with your partner to solve.

S: (Write 5 + 3 = 8.)

T: What is 64 ÷ 8?

S: 8!

Continue with the following suggested sequence:

- 96 ÷ 8
- 54 ÷ 6

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use the distributive property as a strategy to multiply and divide.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion:

- Describe the steps you took to solve for the missing numbers in Problem 1(a).
- How did you know what division fact to write inside the empty oval in Problem 3?
Lesson 10

What multiplication sentence is used to solve Problem 4? How did you know?

Invite students to share how to apply the break apart strategy to any of the expressions in Problem 5.

In what ways does the break apart and distribute strategy remind you of the simplifying strategy we learned yesterday?

How did our math work today help make multiplication and division with larger numbers simpler?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 10 Problem Set

1. Label the arrays. Then fill in the blanks below to make the statements true.

   a) $8 \times 8 = ____$

   $8 \times 5 = ____ | (8 \times ____ ) = ____$

   $8 \times 8 = 8 \times (5 + ____ )$

   $= (8 \times 5) + (8 \times ____ )$

   $= ____ 40 + ____$

   $= ____$

   b) $8 \times 9 = 9 \times 8 = ____$

   $8 \times 5 = ____ | (8 \times ____ ) = ____$

   $9 \times 8 = 8 \times (5 + ____ )$

   $= (8 \times 5) + (8 \times ____ )$

   $= ____ 40 + ____$

   $= ____$

2. Break apart and distribute to solve $56 \div 8$.

   $56 \div 8$

   $\begin{align*}
   40 \div 8 & \quad 16 \div 8 \\
   56 \div 8 &= (40 \div 8) + (16 \div 8) \\
   &= 5 + ____ \\
   &= ____
   \end{align*}$

3. Break apart and distribute to solve $72 \div 8$.

   $72 \div 8$

   $\begin{align*}
   40 \div 8 & \quad \quad \\
   72 \div 8 &= (40 \div 8) + (____ \div 8) \\
   &= 5 + ____ \\
   &= ____
   \end{align*}$
4. An octagon has 8 sides. Skip-count to find the total number of sides on 9 octagons.

Nine octagons have a total of _______ sides.

5. Multiply.

\[
\begin{align*}
4 \times 8 &= 32 \\
8 \times 6 &= 48 \\
3 \times 8 &= 24 \\
8 \times 10 &= 80 \\
8 \times 8 &= 64 \\
7 \times 8 &= 56
\end{align*}
\]
6. Match.

- $24 \div 8$
- $32 \div 8$
- $16 \div 8$
- $64 \div 8$
- $48 \div 8$
- $72 \div 8$

Options:
1. 16
2. 4
3. 2
4. 8
5. 32
6. 64
7. 48
8. 72
Use the break apart and distribute strategy to solve the following problem. You may or may not choose to draw an array.

\[ 7 \times 8 = \_\_] \]
1. Label the array. Then fill in the blanks to make the statements true.

   a) \( 8 \times 7 = 7 \times 8 = \)______

   \[
   \begin{array}{c|c}
   (7 \times 5) = \text{_____} & (7 \times \text{_____}) = \text{_____} \\
   \text{array} & \text{array} \\
   \end{array}
   \]

   \[
   8 \times 7 = 7 \times (5 + \text{_____}) \\
   = (7 \times 5) + (7 \times \text{_____}) \\
   = \text{35} + \text{_____} \\
   = \text{______}
   \]

2. Break apart and distribute to solve \( 72 \div 8 \).

   \[
   72 \div 8 = (40 \div 8) + (\text{______} \div 8)
   \\
   = 5 + \text{_______}
   \\
   = \text{_______}
   \]
3. Count by 8. Then match each multiplication problem with its value.

\[ \underline{8}, \underline{16}, \underline{24}, \underline{32}, \underline{40}, \underline{48}, \underline{56}, \underline{64}, \underline{72} \]

4. Divide.

\[ \begin{align*}
16 \div 8 &= \underline{2} \\
40 \div 8 &= \underline{5} \\
32 \div 8 &= \underline{4} \\
48 \div 8 &= \underline{6} \\
56 \div 8 &= \underline{7} \\
72 \div 8 &= \underline{9}
\end{align*} \]
Lesson 11

Objective: Interpret the unknown in multiplication and division to model and solve problems.

Suggested Lesson Structure

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<th>Activity</th>
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<tr>
<td>Concept Development</td>
<td>35 minutes</td>
</tr>
<tr>
<td>Student Debrief</td>
<td>10 minutes</td>
</tr>
<tr>
<td><strong>Total Time</strong></td>
<td><strong>60 minutes</strong></td>
</tr>
</tbody>
</table>

Fluency Practice (15 minutes)

- Multiply by 8 3.OA.7 (7 minutes)
- Group Counting 3.OA.1 (4 minutes)
- Decompose the Multiplication Sentence 3.OA.5 (4 minutes)

Multiply by 8 (7 minutes)

Materials: (S) Multiply by 8 Pattern Sheet (1–5)

Note: This activity builds fluency with multiplication facts using units of 8. It works toward students knowing from memory all products of two one-digit numbers. See G3–M3–Lesson 5 for the directions for administration of Multiply By Pattern Sheet.

T:  (Write 8 × 5 = ____.) Let’s skip-count by eights to find the answer. I’ll raise a finger for each eight. (Count with fingers to 5 as students count.)
S:  8, 16, 24, 32, 40.
T:  (Circle 40 and write 8 × 5 = 40 above it. Write 8 × 3 = ____.) Let’s skip-count up by eight again. (Count with fingers to 3 as students count.)
S:  8, 16, 24.
T:  Let’s see how we can skip-count down to find the answer, too. Start at 40 with 5 fingers, 1 for each eight. (Count down with your fingers as students say numbers.)
S:  40 (5 fingers), 32 (4 fingers), 24 (3 fingers).
Repeat the process for 8 × 4.

T:  (Distribute the Multiply by 8 Pattern Sheet.) Let’s practice multiplying by 8. Be sure to work left to right across the page.
Lesson 11

Lesson 11

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes and sevens reviews multiplication using those units in Topic B. Group counting nines anticipates multiplication in the next topic. Direct students to count forward and backward, occasionally changing the direction of the count.

- Sixes to 60
- Sevens to 70
- Nines to 90

Decompose the Multiplication Sentence (4 minutes)

Materials: (S) Personal white boards

Note: This activity reviews multiplying using the distributive property from G3–M3–Lesson 10.

T: (Write $8 \times 8 = (5 + \_\_) \times 8$.) On your boards, write out and complete the equation.
S: (Write $8 \times 8 = (5 + 3) \times 8$.)
T: (Write $= (\_\_ \times 8) + (\_\_ \times 8)$.) Write out and complete the equation.
S: (Write $(5 \times 8) + (3 \times 8)$.)
T: Solve the multiplication and write an addition sentence. Below it, write your answer.
S: (Write $40 + 24$ and 64 below it.)

Continue with the following suggested sequence: $7 \times 8, 6 \times 8,$ and $9 \times 8$.

Concept Development (35 minutes)

Materials: (S) Personal white boards

Problem 1: Interpret the unknown in multiplication.

Write the following problem: Asmir buys 8 boxes of 9 candles for his dad’s birthday. After putting some candles on the cake, there are 28 candles left. How many candles does Asmir use?

T: Model the problem. Then tell your partner the steps you’ll need to take to solve.
S: (Model.) First you have to find out how many candles Asmir has. $\rightarrow$ After that you could subtract 28 from the total to see how many he used.
T: Write an equation to find the total number of candles. Instead of using a question mark, use letter c to represent the unknown.
T: Read your equation out loud.
S: 8 times 9 equals c.

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Lesson 11
Interpret the unknown in multiplication and division to model and solve problems.

T: What does c represent?
S: The product. \( \rightarrow \) The total number of candles.
T: Choose a strategy and find the value of c. (Possible strategies: known from memory, skip-count, distributive property, associative property.)
T: Use a complete sentence to tell what c equals.
S: c equals 72. \( \rightarrow \) He bought 72 candles.
T: Did we solve the problem?
S: No, we have to find how many candles Asmir uses.
T: Solve the second step of the problem, this time using letter a to represent the unknown.
S: \( 72 - 28 = a \).
T: Find the value of a. This is a good problem to practice your mental math strategies. (Allow time for solving.) What is the value of a?
S: 44 candles.
T: Answer the question in a complete sentence.
S: Asmir uses 44 candles.

Problem 2: Interpret the unknown in division.

Write the following problem: The fabric store sells 1 meter of cloth for $8. Maria buys some cloth that costs a total of $56. She then uses 3 meters to sew a dress. How many meters of cloth does she have left?

T: Draw a model to represent the problem. Choose letters to represent the unknowns.
T: What is unknown in this problem?
S: The total meters of cloth Maria buys \( \rightarrow \) There’s something else too. We don’t know how many meters of cloth Maria has left.
T: Tell your partner why you need to know how many meters of cloth Maria buys.
S: First you have to find out how many meters of cloth Maria buys. \( \rightarrow \) After that you could subtract 3 meters from the total to see how many meters she has left.
T: What will be your first step to solving this problem?
S: Finding the total meters of cloth Maria buys.
T: Whisper to your partner how you’ll do that, then write an equation using a letter for the unknown.
S: I’m going to do the total cost divided by the cost of 1 meter of fabric. So, \( \$56 / \$8 = t \).
T: Tell your partner why you picked the letter you used to represent the unknown. How does it relate to the problem?

NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Many learners will benefit from this step-by-step guidance from planning a strategy to finding a solution.

Students above grade level and others may be motivated by more choice and autonomy. In addition, welcome various strategies, plans for solving, and modeling.
Lesson 11
Interpret the unknown in multiplication and division to model and solve problems.

S: (Possible response: I chose letter \( t \) to stand for the total meters of cloth Maria buys.)

T: Whisper what \( t \) equals.

S: (Possible response: \( t \) equals 7 meters.)

T: Tell your partner your next step for solving. Then, write an equation using a letter for the unknown.

S: Now that I know that Maria bought a total of 7 meters, I’ll do \( 7 - 3 = n \). Letter \( n \) stands for the number of meters she has left.

T: Is your letter the same as the one you used for the first step? Why or why not?

S: It’s different because it represents something different.

→ Oh yeah, I need to change mine!

T: Finish solving, and then answer the question using words.

S: (Solve to find \( n \) is 4 meters. Write: Maria has 4 meters of cloth left.)

T: Does Maria have enough cloth to sew another dress? Why or why not?

S: Yes, she has 4 meters left and she only needs 3 meters.

→ So, even after making a second dress, she will still have 1 meter of cloth left.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.
Lesson 11

Student Debrief (10 minutes)

Lesson Objective: Interpret the unknown in multiplication and division to model and solve problems.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1, did you need to solve to find the number of groups or the number of items in each group?
- What equations can be used to solve Problem 1?
- In Problem 4, how many parts did each pack need to be split into in order for each boy to get 1 part? (Two equal parts.) Could we use that fact to solve the problem without first finding the total number of cards?
- Problems 4–6 are multiple-step problems. Why is it useful to use different letters to represent two unknowns in the same problem?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Multiply.

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Lesson 11 Problem Set

Name ____________________________________________ Date __________________

1. Ms. Santor divides 32 students into 8 equal groups for a field trip. Draw a tape diagram and label the number of students in each group as $n$. Write an equation and solve for $n$.

2. Tara buys 6 packs of printer paper. Each pack of paper costs $8. Draw a tape diagram and label the total amount she spends as $m$. Write an equation and solve for $m$.

3. Mr. Reed spends $24 on coffee beans. How many kilograms of coffee beans does he buy? Draw a tape diagram and label the total amount of coffee beans he buys as $c$. Write an equation and solve for $c$.

$8$ for $1$ kg
4. Eight boys equally share 4 packs of baseball cards. Each pack contains 10 cards. How many cards does each boy get?

5. There are 8 bags of yellow and green balloons. Each bag contains 7 balloons. If there are 35 yellow balloons, how many green balloons are there?

6. The fruit seller packs 72 oranges into bags of 8 each. He sells all the oranges at $4 a bag. How much money did he receive?
1. Erica buys some packs of rubber bracelets. There are 8 bracelets in each pack.
   a. How many packs of rubber bracelets does she buy if she has a total of 56 bracelets? Draw a tape diagram and label the total number of packages as $p$. Write an equation and solve for $p$.
   
   \[ 8p = 56 \]
   \[ p = \frac{56}{8} \]
   \[ p = 7 \]

   b. After giving some bracelets away, Erica has 18 bracelets left. How many did she give away?
   
   \[ 8p - x = 18 \]
   \[ x = 8p - 18 \]
   \[ x = 8(7) - 18 \]
   \[ x = 56 - 18 \]
   \[ x = 38 \]
Lesson 11 Homework

Name ________________________ Date __________________

1. Jenny bakes 10 cookies. She puts 7 chocolate chips on each cookie. Draw a tape diagram and label the total of amount of chocolate chips as $c$. Write an equation and solve for $c$.

2. Mr. Lopez arranges 48 dry erase markers into 8 equal groups for his math stations. Draw a tape diagram and label the number of dry erase markers in each group as $v$. Write an equation and solve for $v$.

3. There are 35 computers in the lab. Five students each turn off an equal number of computers. How many computers does each student turn off? Label the unknown as $m$, then solve.
4. There are 9 bins of books. Each bin has 6 comic books. How many comic books are there altogether?

5. There are 8 trail mix bags in one box. Clarissa buys 5 boxes. She gives an equal number of bags of trail mix to 4 friends. How many bags of trail mix does each friend receive?

6. Leo earns $8 a week for doing chores. After 7 weeks, he buys a gift and has $38 left. How much does he spend on the gift?