Topic B
Angle Measurement

4.MD.5, 4.MD.6

Focus Standard: 4.MD.5
Recognize angles as geometric shapes that are formed whenever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.

b. An angle that turns through $n$ one-degree angles is said to have an angle measure of $n$ degrees.

4.MD.6
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Instructional Days: 4

Coherence - Links from: G2–M8 Time, Shapes, and Fractions as Equal Parts of Shapes

In Topic B, students explore the definition of degree measure. Beginning in Lesson 5 with a circular protractor, students divide the circumference of a circle into 360 equal parts, treating each part as representing 1 degree (4.MD.5). Students apply this understanding as they discover that a right angle measures 90 degrees and, in turn, that the angles they know as acute measure less than 90 degrees, and obtuse angles measure more than 90 degrees. The idea that an angle measures the amount of “turning” in a particular direction is explored, giving students the opportunity to recognize familiar angles in varied positions (4.G.1, 4.MD.5).

Through experimentation with circles of various sizes and angles constructed to varying specifications in Lesson 6, students discover that although the size of a circle may change, an angle spans an arc representing a constant fraction of the circumference. This reasoning forms the basis for the understanding that degree measure is not a measure of length. For example, as shown at right, the 45° angle spans $\frac{1}{8}$ of the circumference of the circle, whether we choose the small circle or the large one.

Armed with this understanding of the degree as a unit of measure, students use various protractors in Lesson 7, including standard 180° protractors, to measure angles to the nearest degree and construct angles of a given measure (4.MD.6).
The topic wraps up in Lesson 8 with students further exploring angle measure as an amount of turning. This provides a link to Grade 3 work with fractions, as students reason that a $\frac{1}{4}$ turn is a right angle and measures $90^\circ$, a $\frac{1}{2}$ turn measures $180^\circ$, and a $\frac{3}{4}$ turn measures $270^\circ$. They go on to identify these angles in their environment.

A Teaching Sequence Towards Mastery of Angle Measurement

**Objective 1:** Use a circular protractor to understand a 1-degree angle as $1/360$ of a turn. Explore benchmark angles using the protractor.  
(Lesson 5)

**Objective 2:** Use varied protractors to distinguish angle measure from length measurement.  
(Lesson 6)

**Objective 3:** Measure and draw angles. Sketch given angle measures and verify with a protractor.  
(Lesson 7)

**Objective 4:** Identify and measure angles as turns and recognize them in various contexts.  
(Lesson 8)
Lesson 5

Objective: Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

Suggested Lesson Structure

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Fluency Practice (11 minutes)

- Divide Using the Standard Algorithm 4.NBT.6 (3 minutes)
- Identify Two-Dimensional Figures 4.G.1 (4 minutes)
- Physiometry 4.G.1 (4 minutes)

Divide Using the Standard Algorithm (3 minutes)

Materials: (S) Personal white boards

Note: This reviews G4–M3–Lesson 16 content.

T: (Write 48 ÷ 2.) On your boards, solve the division problem using the vertical method.

Continue with the following possible sequence: 49 ÷ 2, 69 ÷ 3, 65 ÷ 3, 55 ÷ 5, 58 ÷ 5, 88 ÷ 4, and 86 ÷ 4.

Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white board, rulers

Note: This fluency reviews terms learned in G4–M4–Lessons 1–4.

T: (Project $\overline{AB}$. Point to the A.) Say the term for what I’m pointing to?
S: Point A.
T: (Point to the B.) Say the term.
S: Point B.
T: (Point to $\overline{AB}$.) Say the term.
S: Line AB.
Lesson 5: Use a circular protractor to understand a 1\degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

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Physiometry (4 minutes)

Materials: (S) Personal white boards

Note: Kinesthetic memory is strong memory. This fluency reviews terms from G4–M4–Lessons 1–4.

T: Stand up.
S: (Stand up.)
T: Show me a point.
S: (Clench one hand in a fist and extend arm forward.)
T: Show me a line.
S: (Extend arms straight so that they are parallel with the floor. Open both hands.)
T: Show me a ray.
S: (Extend arms straight so that they are parallel with the floor. Clench one hand in a fist and leave the other hand open.)
T: Show me a ray pointing in the other direction.
S: (Open clenched hand and clench open hand.)
T: Show me a line segment.
S: (Extend arms straight so that they are parallel with the floor. Clench both hands into fists.)
T: Show me a right angle.
S: (Stretch one arm up directly at the ceiling. Stretch another arm directly towards a wall, parallel to the floor.)
T: Show me a different right angle.
S: (Stretch the arm pointing towards a wall directly up towards the ceiling. Move the arm pointing towards the ceiling so that it points directly towards the opposite wall.)
T: Show me an obtuse angle.
S: (Make an obtuse angle with arms.)
T: Show me an acute angle.
Lesson 5: Use a circular protractor to understand a 1-degree angle as 1/360 of a turn.

Explore benchmark angles using the protractor.

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S: (Make an acute angle with arms.)
Continue with the following possible sequence: point, right angle, line segment, acute angle, line, right angle, and obtuse angle.

T: (Stretch one arm up directly at the ceiling. Stretch another arm directly towards a wall, parallel to the floor.) What type of angle am I making?

S: Right angle.
T: What is the relationship of the lines formed by my arms?

S: Perpendicular lines.

T: (Point to the classroom’s back wall.) Point to the walls that run perpendicular to the wall I’m pointing to.

S: (Point to the side walls.)
T: (Point to the front wall.)
S: (Point to the side walls.)

Continue pointing to one side wall, the back wall, the other side wall, and the front wall.

T: (Point to the back wall.) Point to the wall that runs parallel to the wall I’m pointing to.

S: (Point to the front wall.)

Continue pointing to one side wall, the front wall, and the other side wall.

Application Problem (6 minutes)

Materials: (S) 1 paper circle from the Concept Development

Place right angle templates on top of the circle to determine how many right angles can fit around the center point of the circle. If necessary, team up with other students to share templates. (Overlaps are not allowed.)

How many right angles can fit?

Note: This Application Problem bridges from G4–M4–Lesson 2. Students will use the right angle templates that they made in class in order to build understanding as they measure right angles around the center point of a circle.

Concept Development (33 minutes)

Materials: (T) 2 paper circles with 5” diameter (same size, one red, one white, with a radius delineated by a strong, straight black segment) (S) 2 paper circles with 5” diameter (same size, one red, one white, with a radius cut into each one), circular
Lesson 5

NYS COMMON CORE MATHEMATICS CURRICULUM

Use a circular protractor to understand a 1-degree angle as 1/360 of a full turn. Explore benchmark angles using the protractor.

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Lesson 5: Use a circular protractor to understand a 1-degree angle as 1/360 of a full turn. Explore benchmark angles using the protractor.

protractor with 4” diameter on paper or transparency

Directions for Constructing a Paper Protractor:

a. Label and cut a radius into one red and one white paper circle.

b. Stack the red circle on top of the white circle with the radii aligned and parallel to the desk’s edge.

c. Pinch the corner of the white circle directly below the slit, as shown above.

d. To illustrate an angle, turn the segment given by the edge of the white region counterclockwise.

Problem 1: Reason about the number of turns necessary to make a full turn with different fractions of a full turn.

T: What do you see as you turn this segment to the left?

S: The white part is getting larger. The red part is getting smaller.

T: Do you see an angle?

S: Yes.

T: Let’s agree that the white region is the interior of the angle we are focusing on.

T: (Demonstrate a quarter-turn.) Now show a quarter-turn of the segment to the left. (Expect some confusion.)

S: (Show.)

T: Make another quarter-turn of the segment to the left. What fraction of the circular region is white now?

S: One half.  Two fourths.

Continue the same process until the 360-degree turn is complete.

T: (List the following fractions on the board.)

\[
\frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad \frac{4}{4}
\]

T: (Point to each fraction as you speak, pausing as students manipulate the turns.) Show \(\frac{1}{4}\) turn, \(\frac{2}{4}\) turn, now a \(\frac{3}{4}\) turn, a \(\frac{4}{4}\) turn. Is the angle getting larger or smaller?

S: Larger!

T: How many fourth-turns did it take to make one full turn?

S: Four.
Lesson 5:

Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

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T: Now I want to break up a turn into eight equal parts. Count eighths with me.

T: Will one eighth-turn be less than or greater than one fourth-turn?
S: Less than.
T: One fourth-turn is the same as two eighth-turns (point to the listed fractions). Show me what you think would be one eighth-turn.

Repeat the same process of pointing to each eighth in order as the students open the angle.

T: Did it take more fourth-turns or eighth-turns to get all the way around?
S: Eighths.
T: How many eighth-turns did it take to make a whole turn?
S: Eight!
T: How many \( \frac{1}{100} \) turns would it take to make a whole turn?
S: 100!
T: Would \( \frac{1}{360} \) turn be smaller or larger than \( \frac{1}{100} \) turn?
S: Smaller.
T: We have a special name for \( \frac{1}{360} \) of a whole turn. It is called a degree! How many degrees are in one whole turn?
S: 360.
T: Yes!
T: Here is a tool that has been partitioned and marked off to show 360 degrees. It is called a protractor. Take a moment to analyze it with your partner. What do you notice?
Problem 2: Use a circular protractor to determine that a quarter-turn or a right angle measures 90 degrees, a half turn measures 180 degrees, a three quarter-turn measures 270 degrees, and a full rotation measures 360 degrees.

T: Show me a quarter-turn with your circles. Keep the base segment of your angle parallel to your desk.

T: Put the zero line, or base line, on top of the bottom segment of your angle. Align the center point of the protractor with the vertex of the angle to the best of your ability.

T: Adjust the circle’s angle to match your right angle template. (Pause.) Remove the template and place the protractor to measure that angle. What do you notice?

S: The quarter-turn matches the bold lines of the protractor. → It’s 90 degrees! → One fourth-turn is 90 degrees. → A right angle measures 90 degrees.

T: Do a half-turn and see how many degrees your angle is?

S: 180 degrees.

T: Turn another quarter- or fourth-turn.

S: 270 degrees.

T: And one last quarter- or fourth-turn?

S: 360 degrees. → Zero degrees.

T: What does your angle look like right now?

S: It’s all white.

T: A zero-degree angle is when we have not turned at all. We have made one full turn of 360 degrees. There are 360 degrees in a full turn.

T: How many 90-degree angles or right angles are there in a full turn?

S: Four right angles.

T: How do you know?

S: Because we made four quarter-turns and each one was 90 degrees. → It is easy to see them because of the bold perpendicular lines on the protractor.
Lesson 5:

Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

T: Using your white circle, position your protractor with the zero or base line on top of the black segment, matching up the center point of the circle with the center point of the protractor.

T: Estimate to make a point at 90 degrees. Draw a line segment from the center point to that point. What have you drawn?

S: A right angle. \(\rightarrow\) A 90-degree angle. \(\rightarrow\) Perpendicular lines.

T: Now make a point at 45 degrees. Draw a line segment from the center point to the point you just made. What have you made?

S: A 45-degree angle.

T: Yes! What do you notice?

S: The 45-degree angle is half as big as the 90-degree angle. \(\rightarrow\) Two forty-five degree angles are the same as one 90-degree angle. \(\rightarrow\) \(2 \times 45 + 90\).

Problem 3: Measure and draw benchmark angles with the protractor.

T: Now let’s work to measure and draw benchmark angles using your circles and protractors.

T: We have already started Set A, using your white circle. Continue turning your circle, aligning the zero or base line with each last segment drawn. Be sure to keep your protractor’s center point on the center point of the circle. Draw new 45-degree angles until you have gone a whole turn. Let me demonstrate. (Demonstrate silently.)

T: Draw Set B on your red circle just as you did Set A, but this time draw 30-degree angles. This full turn will be made of 30-degree angles. Draw 30-degree angles until you have made a whole turn.

Set A

45 degrees

Set B

30 degrees

T: Place the center point of the protractor on the shared endpoints of the segments on your white circle. Align the zero line with the black segment. What are the measurements of the angles you have drawn?

S: 0 degrees, 45 degrees, 90 degrees, 135 degrees, 180 degrees, etc.
Lesson 5: Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

**Student Debrief (10 minutes)**

**Lesson Objective:** Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- When you listed the benchmark angles, did you notice any numerical patterns?
- You listed some measures of acute and obtuse angles. What would be some measurements of other acute angles? Obtuse angles?
- A full turn is 360 degrees. What could you do to find the degree measure of an angle that takes 10 turns to make a whole turn?
- How did you respond to the final question?
Lesson 5: Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

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- If you were to draw a tape diagram to represent one whole-turn and the benchmark angles of Set A, what would you do? Set B?
- Shade in the region of a 45-degree angle on your white circle. What fraction of the whole turn is that? Do the same for your 30-degree angle.
- What if you shaded in a region defined by a 120-degree angle on your red circle? What fraction of the whole circle is that?
- Use your protractor to explain to your partner what a degree is.
- Use your protractor to trace some benchmark angles.
- If you didn’t have a protractor, how could you construct one?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 5 Problem Set

Name __________________________________________ Date ______________________

1. Make a list of the measures of the benchmark angles you drew starting with Set A. Round each angle measure to the nearest 5 degrees. Both sets are started for you.

   a. Set A: 45 degrees, 90 degrees,

   b. Set B: 30 degrees, 60 degrees

2. Circle any angle measures that appear on both lists. What do you notice about them?

3. List the angle measures from Problem 1 that are acute. Trace each angle with your finger as you say its measurement.

4. List the angle measures from Problem 1 that are obtuse. Trace each angle with your finger as you say its measurement.
5. We found out today that 1 degree is $\frac{1}{360}$ of a whole turn. It is 1 out of 360 degrees. That means a 2-degree angle is $\frac{2}{360}$ of a whole turn. What fraction of a whole turn is each of the benchmark angles you listed in Problem 1?

6. How many 45-degree angles does it take to make a full turn?

7. How many 30-degree angles does it take to make a full turn?

8. If you didn’t have a protractor, how could you reconstruct the quarter of it from 0 degrees to 90 degrees?
Lesson 5 Exit Ticket

Name _______________________________  Date ____________________

1. How many right angles make a full turn?

2. What is the measurement of a right angle?

3. What fraction of a full turn is 1 degree?

4. Name at least four benchmark angle measurements.
Lesson 5: Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

Name ____________________________ Date ____________________

1. Identify the measures of the following angles.

   a. 
   
   b. 
   
   c. 
   
   d. 

2. If you didn’t have a protractor, how could you construct one? Use words, pictures, and numbers to explain in the space below.
Lesson 6

Objective: Use varied protractors to distinguish angle measure from length measurement.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (5 minutes)
- Concept Development (37 minutes)
- Student Debrief (6 minutes)

Total Time (60 minutes)

Fluency Practice (12 minutes)

- Divide Using the Area Model 4.NBT.6 (4 minutes)
- Draw and Identify Two-Dimensional Figures 4.G.1 (4 minutes)
- Physiometry 4.G.1 (4 minutes)

Divide Using the Area Model (4 minutes)

Materials: (S) Personal white boards

Note: This drill reviews G4–M3–Lesson 20 content.

T: (Project area model that shows 68 ÷ 2.) Write a division expression for this area model.
S: (Write 68 ÷ 2.)
T: Label the length of each rectangle in the area model.
S: (Write 30 above the 60 and 4 above the 8.)
T: Solve using the standard algorithm.

Students do so.

Continue with the following possible suggestions: 69 ÷ 3, 78 ÷ 3, and 76 ÷ 4.

Draw and Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white boards, straightedge

Note: This fluency reviews terms introduced in G4–M4–Lessons 1–5.
T:  (Project \( \overline{AB} \). Point to the A.) Say the term for what I’m pointing to?
S:  Point A.
T:  (Point to the B.) Say the term.
S:  Point B.
T:  (Point to \( \overline{AB} \)) Say the term.
S:  Line segment \( \overline{AB} \).
T:  Use your rulers to construct \( \overline{CD} \) on your boards.
S:  (Draw \( \overline{CD} \).)
T:  Beneath \( \overline{CD} \), draw \( \overline{EF} \) that is parallel to \( \overline{CD} \).
S:  (Beneath \( \overline{CD} \), draw \( \overline{EF} \) that is parallel to \( \overline{CD} \).)
T:  Draw \( \overline{GH} \) that begins on \( \overline{EF} \) and runs perpendicular through \( \overline{CD} \).
S:  (Draw \( \overline{GH} \) that begins on \( \overline{EF} \) and runs perpendicular through \( \overline{CD} \).)
T:  What’s the relationship between \( \overline{GH} \) and \( \overline{CD} \)?
S:  \( \overline{GH} \) is perpendicular to \( \overline{CD} \).
T:  Draw \( \overline{IJ} \) that is perpendicular to \( \overline{KL} \).
S:  (Draw \( \overline{IJ} \). Draw \( \overline{KL} \) that is perpendicular to \( \overline{IJ} \).)
T:  Draw \( \overline{MN} \) that is perpendicular to \( \overline{IJ} \) and parallel to \( \overline{KL} \).
S:  (Draw \( \overline{MN} \) that is perpendicular to \( \overline{IJ} \) and parallel to \( \overline{KL} \).)
T:  (Project a right angle \( \angle ACB \).) Name the angle.
S:  \( \angle ACB \).
T:  What type of angle is it?
S:  Right angle.
T:  What’s the relationship of \( \overline{CA} \) and \( \overline{CB} \)?
S:  They’re perpendicular.
T:  How many degrees are in \( \angle ACB \)?
S:  90 degrees.
T:  (Project an acute angle \( \angle DFE \).) Name the angle.
S:  \( \angle DFE \).
T:  (Beneath \( \angle DFE \), write 30° or 150°.) Estimate. Is the measure of \( \angle DFE \) 30° or 150°?
S:  30°.
T:  How do you know?
S:  Acute angles are less than 90°.

Continue with the other given angles.
Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency reviews terms from G4–M4–Lessons 1–5.

T: Stand up.
S: (Stand up.)
T: Show me a right angle.
S: (Stretch one arm up directly at the ceiling. Stretch another arm directly towards a wall, parallel to the floor.)
T: Show me a different right angle.
S: (Stretch the arm pointing towards a wall directly up towards the ceiling. Move the arm pointing towards the ceiling so that it points directly towards the opposite wall.)
T: Show me an obtuse angle.
S: (Make an obtuse angle with arms.)
T: Show me an acute angle.
S: (Make an acute angle with arms.)
T: Make a right angle.
S: (Make a right angle with arms.)
T: Make an angle that measures approximately 30°.
S: (Move arms closer together, lessening the space between their arms, so that it is approximately 30°.)
T: Make an angle that measures approximately 60°.
S: (Open arms further apart to approximately 60°.)

Continue with the following possible sequence: 90°, 120°, 150°, 50°, 170°, 70°, and 180°.

T: What is the term for a 180° angle?
S: Line.
T: Make a line segment.
S: (Close fists.)
T: (Point at the classroom’s back wall.) Point to the walls that run perpendicular to the wall I’m pointing to.
S: (Point to the side walls.)
T: (Point to the front wall.)
S: (Point to the side walls.)

Continue pointing to one side wall, the back wall, the other side wall, and the front wall.

T: (Point to the back wall.) Point to the wall that runs parallel to the wall I’m pointing to.
S: (Point to the front wall.)

Continue pointing to one side wall, the front wall, and the other side wall.
Lesson 6

Use varied protractors to distinguish angle measure from length measurement.

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NOTES ON MULTIPLE MEANS OF REPRESENTATION:
Check that English language learners and others understand the meaning of the new math term arc. If necessary and possible, offer explanations in students’ first language. Link arc to more familiar words or phrases such as golden arches.
S: 360 degrees.
T: How many total degrees in this circle? (Point to Circle B.)
S: 360 degrees.
T: So if I divide Circle A into 360 degrees, each arc length will be a little longer than the arc lengths in Circle B. I’m still measuring a quarter turn in each circle, and each arc is one fourth of the total distance around the circle.
T: Think of it also like taking the arc lengths from each circle and stretching them out into a line. (Model two wires that wrap the circumference of each circle stretched out in a line.) I can chop each wire into 360 equal-size pieces. Which arc will have smaller pieces?
S: The arc from Circle B.
T: Right! 90 degrees is one quarter of 360 degrees. (Cut each wire into four equal parts. Show one part from each wire is the same length as the arc of each circle.) Which arc is longer?
S: Circle A has a longer arc.
T: So does the length of the arc determine the measure of a given angle? Discuss this with your partner.
S: No! The arcs might be longer or shorter, but they could be measuring the same size angle. → No matter where the arc is, I just have to remember that arc is part of 360 degrees. → Right, because I could have a super tiny circle or a really big circle, but still the right angles measure 90 degrees.
T: Place Circle B on top of Circle A to show the length of the arc does not determine the degree measure.

Problem 2: Use a 180° protractor to verify angle measure.

T: (Project ∠C and ∠D from the Practice Sheet.) What type of angle do you see?
S: Acute!
T: Discuss what you notice about the arc length in each angle.
S: The arc length in ∠C is longer than the one in ∠D. → The arcs are different lengths, but the angles look like they might be the same. → It looks like ∠C came from a larger circle than ∠D did.
T: Let’s measure to find out if the angles turn the same number of degrees.
T: (Distribute and display a 180° protractor.) What do you notice about this protractor?
S: It’s half a protractor. → It’s only a piece of a circular protractor. → It’s got a straight edge.
T: Just like you measured angles with a circular protractor, you can measure angles with this 180° protractor. Protractors sometimes have two sets of numbers. We determine which number to read based off the side of the angle that touches zero. (Show a 40-degree angle as pictured to the right, aligning both sides to zero and discussing...
which set of numbers to read.)

T: (Model. Place the middle notch on the vertex of the angle. Line up a side with the zero or base line on the protractor. Read the number the second side length touches.)

T: With your partner, measure $\angle C$.

S: 60 degrees! No wait, 120 degrees. $\rightarrow$ It can’t measure 120 degrees. It’s an acute angle. 60 degrees.

$\rightarrow$ Remember, we count up from the side of the angle at zero, so we are using the outside numbers for this angle.

T: Measure $\angle D$.

S: 60 degrees!

T: What did you discover? Discuss with your partner.

S: The arc lengths are different, but the degrees are the same. $\rightarrow$ Both angles are 60 degrees, but $\angle D$ looks different because the sides of the angle are shorter.

T: What would happen if we placed the angles on top of each other? Turn and talk. (Allow time for brief discussion.) Let’s try! (Model.)

S: They match up! $\rightarrow$ The angles are the same size!

T: Imagine a circle drawn with the vertex of $\angle D$ as its center point, the end of one segment being the length to the arc and another circle drawn in the same way around $\angle C$.

T: What could you say about the two circles?

S: The circles would be different sizes. $\rightarrow$ The lengths of the sides of $\angle C$ would make a larger circle than the sides of $\angle D$. $\rightarrow$ The arcs and sides will be different lengths, but the angle will measure the same because each angle represents a fraction of 360 degrees.

Problem 3: Use multiple protractors to measure the same angle.

T: Look at the different protractors in front of you. What do you notice about them?

S: Some are 360° protractors and some are 180° protractors. $\rightarrow$ Some have just one set of numbers; others have two sets. $\rightarrow$ They are all different sizes. $\rightarrow$ The base line on this one is on the bottom of the protractor, but the base line on this one is above the plastic.

T: Line up your protractors using the center point, just like we did with our two circles at the beginning of the lesson. Do you see how these different protractors have different arcs?

S: Yes, some are small, and some are big.

T: Yes, but they all measure 360 degrees of a circle.

S: But some only measure 180 degrees.

T: That’s because it is representing half a circle. Notice the tick marks on all of the different protractors.

S: Some are really close together!

T: Why is that?

S: It’s on the smallest protractor, so that means the arc length is shorter than those of the other protractors.

T: Let’s use at least three different protractors to measure $\angle F$.

Allow time for students to measure individually, in partners or in small groups, depending on the variety of
protractors available in the classroom.

S: All three protractors showed this is a 120° angle!
T: What does that tell you about the side lengths of an angle?
S: The side lengths can be any length. → No matter where you measure on the circle, the number of degrees will always be the same. → We aren’t measuring the sides of angles. The different sizes of protractors pick a different point on each segment where a circle could be and measures that.
T: Let’s look at Problem 1(a) of the Problem Set together. Measure the angle that is shown.
S: I can’t measure that angle. The image is too small! → I know what to do! We can make the segments of the angle longer. We just found out that the angle measure stays the same no matter what the side length is.

T: Use your straightedge to extend the sides of the angle until they are long enough for you to use the protractor to measure the angle. (Model.)
S: Now I can measure the angle!

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted time frame. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first.

Student Debrief (6 minutes)

Lesson Objective: Use varied protractors to distinguish angle measure from length measurement.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.
You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1 which angle had the same measure as $\angle G$? $\angle I$?
- In Problem 1 which angles had the same angle measures but different side length measures?
- Discuss your experience of measuring with different protractors (Problem 2).
- How many degrees did the angles in Problem 3 measure? What type of angle is Part (a)? We know a straight angle forms a straight line. Points $A$, $B$, and $C$ create $\angle ABC$ and $\overline{ABC}$. When three or more points are found on a line, we call them collinear points. Are points $D$, $E$, and $F$ collinear? Why not?
- Take a look at your 180° protractor. Find pairs of numbers that label the two scales, such as 150° and 30°. Name another pairs of numbers. What do you notice about the pairs of numbers?
- How did the Application Problem help you to understand angle measure remains constant and is not a length measure?

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 6: Use varied protractors to distinguish angle measure from length measurement.

Date: 10/16/13
Lesson 6: Use varied protractors to distinguish angle measure from length measurement.

Date: 10/16/13

1. Use a protractor to measure the angles and then record the measurements in degrees.
   a. 
   b. 
   c. 
   d. 

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Lesson 6: Use varied protractors to distinguish angle measure from length measurement.

Date: 10/16/13
2.  
   a. Use three different-size protractors to measure the angle. Extend the lines as needed using a straightedge.
      
      Protractor #1: ________°
      Protractor #2: ________°
      Protractor #3: ________°

   b. What do you notice about the measurement of the above angle using each of the protractors?

3. Use a protractor to measure each angle. Extend the length of the lines if you need to. When you extend the lines, does the angle measure stay the same? Explain how you know.

   a. 
   
   b. 

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1. Use any protractor to measure the angles and then record the measurements in degrees.

a. 

b. 

c. 

d.
1. Use a protractor to measure the angles and then record the measurements in degrees.
   a. 
   b. 
   c. 
   d. 

Use varied protractors to distinguish angle measure from length measurement.
Lesson 6: Use varied protractors to distinguish angle measure from length measurement.

Date: 10/16/13

- e.
- f.
- g.
- h.
- i.
- j.
2. Using the green and red circle cutouts from today’s lesson, explain to someone at home how the cutouts can be used to show that the angle measures are the same even though the circles are different sizes. Write words to explain what you told him/her.

3. Use a protractor to measure each angle. Extend the length of the lines if you need to. When you extend the lines, does the angle measure stay the same? Explain how you know.

   a.

   b.
Lesson 7

Objective: Measure and draw angles. Sketch given angle measures and verify with a protractor.

Suggested Lesson Structure

- Fluency Practice: (12 minutes)
- Application Problem: (5 minutes)
- Concept Development: (33 minutes)
- Student Debrief: (10 minutes)

Total Time: (60 minutes)

Fluency Practice (12 minutes)

- Break Apart 90, 180, and 360 4.MD.7 (4 minutes)
- Physiometry 4.G.1 (4 minutes)
- Identify Angle Measures 4.MD.6 (4 minutes)

Break Apart 90, 180, and 360 (4 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for unknown angle problems in G4–M4–Lessons 10–11.

T: (Project a number bond with a whole of 90. Fill in 10 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 90 and 10 and 80 as parts.)

Continue the process for the following possible sequence: 50, 40, and 45.

T: (Project a number bond with a whole of 180. Fill in 80 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 180 and 80 and 100 as parts.)

Continue the process for the following possible sequence: 90, 120, 140, and 35.

T: (Project a number bond with a whole of 360. Fill in 300 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

S: (Draw a number bond with a whole of 360 and 300 and 60 as parts.)

Continue with the following possible sequence: 100, 90, 180, 120, and 45.
Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency reviews terms from G4–M4–Lessons 1–5.

T:  Stand up.
S:  (Stand up.)
T:  Show me an acute angle.
S:  (Make an acute angle with arms.)
T:  Show me an obtuse angle.
S:  (Make an obtuse angle with arms.)
T:  Make a right angle.
S:  (Make a right angle with arms.)
T:  Make an angle that measures approximately 80°.
S:  (Move arms closer together, lessening the space between their arms, so that it’s approximately 80°.)
T:  Make an angle that measures approximately 10°.
S:  (Close arms more to approximately 10°.)

Continue with the following possible suggestions: 90°, 100°, 170°, 150°, 60°, 140°, 70°, and 180°.

T:  What is the term for a 180° angle?
S:  Straight angle.
T:  Make a line segment.
S:  (Close fists.)
T:  Make a ray.
S:  (Open one hand while keeping the other hand clenched.)
T:  Partner up with a classmate next to you. Decide who is Partner A and who is Partner B.
S:  (Pair up with a partner. Decide who is Partner A and who is Partner B.)
T:  Partner A, point at a side wall.
S:  (Point at a side wall.)
T:  Partner B, point at the walls that are perpendicular to the wall Partner A is pointing to.
S:  (Point at front and back walls.)
T:  Partner B, point to any wall in the room.
S:  (Point at a wall.)
T:  Partner A, point at the wall that is parallel to the wall Partner B is pointing to.
S:  (Point at wall parallel to the wall Partner B is pointing to.)

Identify Angle Measures (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G4–M4–Lesson 5.
Lesson 7

Measure and draw angles. Sketch given angle measures and verify with a protractor.

T: How many degrees are in a right angle?
S: 90 degrees.
T: (Project a right angle DEF.) Name the angle.
S: \(\angle DEF\).
T: What type of angle is it?
S: Right angle.
T: What’s the relationship of \(\overrightarrow{ED}\) and \(\overrightarrow{EF}\)?
S: They’re perpendicular.
T: How many degrees are in \(\angle DEF\)?
S: 90 degrees.

T: (Project an acute angle GIH.) Name the angle.
S: \(\angle GIH\).
T: (Beneath \(\angle GIH\), write 40° or 140°.) Estimate. Is the measure of \(\angle GIH\) 40° or 140°?
S: 40°.
T: How do you know?
S: Acute angles are less than 90°.

Continue with the following possible suggestions: obtuse angle measuring 130° or 50°, acute angle measuring 75° or 105°, and obtuse angle measuring 92° or 88°.

Application Problem (5 minutes)

Predict the measure of \(\angle XYZ\) using your right angle template. Then find the actual measure of \(\angle XYZ\) using a circular protractor and a 180° protractor. Compare with your partner when you are finished.

Note: This Application Problem reviews the practice of measuring angles from G4–M4–Lesson 6 and leads up to the Concept Development of today’s lesson where students measure and draw angles. This figure can be found on the Practice Sheet, Figure 1.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Provide protractor alternatives for students, if necessary. Some students may work more efficiently with large-print protractors that include a clear, moveable wand. Others may find using an angle ruler easier.

For students with low vision and others, outline angles and shapes to be measured with glue to make the activity tactile.
Lesson 7

Measure and draw angles. Sketch given angle measures and verify with a protractor.

Date: 10/16/13

Concept Development (33 minutes)

Materials: (T) Circular protractor, 180° protractor, Practice Sheet (S) Circular protractor, 180° protractor, Practice Sheet

Problem 1: Measure angles less than 180° using a circular and a 180° protractor.

T: In completing the Application Problem, what was your prediction for the measure of $\angle XYZ$?
S: I predicted $\angle XYZ$ to be about 100°. $\Rightarrow$ I know that $\angle XYZ$ is an obtuse angle because it is greater than a right angle so I predicted it to be about 110°.

T: How did you use the circular and 180° protractor to find the measure of $\angle XYZ$?
S: I lined up one side of the angle with the base line on the circular protractor. Then, I saw where the other side of the angle touched on the arc. $\Rightarrow$ First, I put the center hole of the 180° protractor at the vertex, Y, of the angle. Next, I lined up $\overline{YZ}$ with the zero line on the protractor. Then I read where $\overline{XY}$ measured on the protractor.

T: Lining up the protractor correctly is very important. Let’s practice measuring $\angle CAB$ using the circular protractor. Measure $\angle CAB$. (Practice Sheet, Figure 2.)

T: Now, with your partner, take the 180° protractor and measure the same angle.

T: What do you notice?
S: Both protractors say 45 degrees. $\Rightarrow$ The angle measure is the same no matter which protractor we use.

T: Look at Figure 3 on your Practice Sheet. Using either protractor, find the measure of $\angle DEF$.
S: With the circular protractor, $\angle DEF$ measures 120°. $\Rightarrow$ With the 180° protractor, $\angle DEF$ measures 120°.

Problem 2: Measure an angle greater than 180° by subtracting from 360°.

T: Look at Figure 4 on your Practice Sheet. Use either protractor to measure $\angle QRS$.
S: I am going to use the circular protractor because the 180° protractor doesn’t fit right. $\angle QRS$ measures 230°. $\Rightarrow$ I want to use a 180° protractor, but I am not sure how. It isn’t big enough to measure the angle.

T: Let’s figure out how to use the 180° protractor. The arc close to the vertex symbolizes the angle we want to measure.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

As they measure angle A, guide students working below grade level to adjust the paper rather than the protractor.

Challenge students working above grade level to predict the measure of angle A before measuring. Invite students to explain their reasoning.

Also, extend the task as time permits by having students measure angle A using each side of the angle as a base. Ask, “What do you notice?”
Lesson 7

Measure and draw angles. Sketch given angle measures and verify with a protractor.

Date: 10/16/13

T: What happens if we extend the drawing of the arc? Show me.
S: (Extend arc.) We have a circle with point R in the middle.
T: There are two angles represented. Talk to your partner about them.
S: One angle is shown by the arc that was already there. The other angle is shown by the arc that we just drew. \( \rightarrow \) The two angles go together to represent a whole turn.
T: Which angle is easier to measure with the 180° protractor?
S: The smaller angle.
T: What is the measure of that angle? (Pause.)
S: 130°.
T: What is the total angle measure around point R?
S: 360°.
T: If there are 360° in the whole and 130° in one of the parts, figure out the measure of the other part. Talk to your partner about your strategy.
S: We could subtract. \( \rightarrow \) We know that the whole minus a part equals the other part. \( 360 - 130 = 230 \). \( \rightarrow \) I counted up 2 hundreds from 130 to 330 and then added 30 more, \( \angle QRS \) is 230°. \( \rightarrow \) That’s the same as when we measured with the circular protractor!

Problem 3: Measure an angle greater than 180° by adding on to 180°.

T: Let’s explore another way to find the measure. Erase the arc that you just drew. Now, use your straightedge to extend \( QR \) to the right.
S: (Extend one of the rays.)
T: What happened to \( \angle QRS \), the larger angle?
S: Now it’s chopped into two smaller angles.
T: What is the angle measure of this straight line?
S: 180°.
T: Measure the new acute angle. (Pause.)
S: It’s 50°.
T: Label each angle with its measure. What do you notice?
S: When I add the two angles together, I get the measure of the whole thing. \( 180° + 50° = 230° \). Hey, it’s the same!

Problem 4: Draw an angle less than 180° using a 180° protractor.

T: Now let’s practice drawing angles. Draw a ray that we can line up to our 0° line.
T: Watch as I draw my ray and label my endpoint with the letter A. Now, you draw. (See Step 1
Lesson 7

Measure and draw angles. Sketch given angle measures and verify with a protractor.

Date: 10/16/13

NOTES ON USING A PROTRACTOR:
Help students measure accurately using a protractor with the following tips:
1. Place the center notch of the protractor on the vertex.
2. Put the pencil point through the notch and move the straightedge into alignment.
3. When measuring angles, it is sometimes necessary to extend the sides of the angle so that they intersect with the protractor’s scale.

Step 1

Step 2

Step 3

Step 4

Step 5
Lesson 7

Lesson Objective: Measure and draw angles. Sketch given angle measures and verify with a protractor.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1, how did you draw the angles with a 180° protractor?
- In Problem 1, which were the most challenging angles to draw? Explain.
- Why is it important to be precise when drawing angles? Tell your partner how you can be precise when drawing angles.
- Why do we verify our sketches with a protractor?
- It is important to learn to use the 180° protractor because it is the one you will see everywhere. Explain to your partner how to measure an angle greater than 180° with a 180° protractor.

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 7 Practice Sheet

Lesson 7: Measure and draw angles. Sketch given angle measures and verify with a protractor.

Name ___________________________ Date ________________

Figure 1

Figure 2

Figure 3

Figure 4
Lesson 7 Problem Set

1. Construct angles that measure the given number of degrees. For (a)–(d), use the ray shown as one of the rays of the angle with its endpoint as the vertex of the angle. Draw an arc to indicate the angle that was measured.

   a. 30°
   b. 65°
   c. 115°
   d. 135°
Lesson 7 Problem Set

Measure and draw angles. Sketch given angle measures and verify with a protractor.

Date: 10/16/13

e. 5°  
f. 175°

g. 27°  
h. 117°

i. 48°  
j. 132°
1. Construct angles that measure the given number of degrees. Draw an arc to indicate the angle that was measured.

   a. 75°
   b. 105°
   c. 81°
   d. 99°
1. Construct angles that measure the given number of degrees. For (a)–(d), use the ray shown as one of the rays of the angle with its endpoint as the vertex of the angle. Draw an arc to indicate the angle that was measured.

   a. 25°  
   b. 85°  
   c. 140° 
   d. 83°
Lesson 7 Homework

NYS COMMON CORE MATHEMATICS CURRICULUM

Measure and draw angles. Sketch given angle measures and verify with a protractor.

Date: 10/16/13

108°  f.  72°

g.  25°  h.  155°

i.  45°  j.  135°
Lesson 8

Objective: Identify and measure angles as turns and recognize them in various contexts.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (5 minutes)
- Concept Development (33 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Count by 90° 4.MD.7 (2 minutes)
- Break Apart 90, 180, and 360 4.MD.7 (4 minutes)
- Physiometry 4.G.1 (2 minutes)
- Sketch Angles 4.MD.6 (4 minutes)

Count by 90° (2 minutes)

Note: This fluency prepares students for G4–M4–Lesson 8. If students struggle to connect counting groups of 9, groups of 9 tens, and groups of 90, write the counting progressions on the board.

Direct students to count forward and backward:

- Nines to 36
- 9 tens to 36 tens
- 90 to 360
- 90 degrees to 360

<table>
<thead>
<tr>
<th>9</th>
<th>18</th>
<th>27</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 tens</td>
<td>18 tens</td>
<td>27 tens</td>
<td>36 tens</td>
</tr>
<tr>
<td>90</td>
<td>180</td>
<td>270</td>
<td>360</td>
</tr>
<tr>
<td>90°</td>
<td>180°</td>
<td>270°</td>
<td>360°</td>
</tr>
</tbody>
</table>
Break Apart 90, 180, and 360 (4 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for unknown angle problems in G4–M4–Lessons 10–11.

T: (Project a number bond with a whole of 90. Fill in 20 for one of the parts.) On your boards, write the number bond, filling in the missing part.

S: (Draw a number bond with a whole of 90 and 20 and 70 as parts.)

Continue breaking apart 90 with the following possible sequence: 60, 40, 50, and 45.

T: (Project a number bond with a whole of 180. Fill in 70 for one of the parts.) On your boards, write the number bond, filling in the missing part.

S: (Draw a number bond with a whole of 180 and 70 and 110 as parts.)

Continue to break apart 180 with the following possible suggestions: 90, 130, 40, and 135.

T: (Project a number bond with a whole of 360. Fill in 50 for one of the parts.) On your boards, write the number bond, filling in the missing part.

S: (Draw a number bond with a whole of 360 and 50 and 310 as parts.)

Continue to break apart 360 with the following possible suggestions: 200, 190, 180, 90, 120, and 45.

Physiometry (2 minutes)

Note: Kinesthetic memory is strong memory. This fluency reviews terms from G4–M4–Lessons 1–7.

T: Stand up.

S: (Stand up.)

T: Show me an acute angle.

S: (Make an acute angle with arms.)

T: Show me an obtuse angle.

S: (Make an obtuse angle with arms.)

T: Make a right angle.

S: (Make a right angle with arms.)

T: Make an angle that measures approximately 100°.

S: (Move arms further apart, increasing the space between their arms, so that it is approximately 100°.)

T: Make an angle that measures approximately 150°.

S: (Move arms further apart to approximately 150°.)

Continue with the following possible suggestions: 90°, 80°, 30°, 20°, 120°, 40°, 110°, and 180°.

T: What’s another name for a 180° angle?

S: A line.

T: (Point at one of the classroom’s side walls.) Point to the walls that run perpendicular to the wall I’m pointing to.
Lesson 8: Identify and measure angles as turns and recognize them in various contexts.

Date: 10/16/13

4.4

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

If you have observed that students do not recognize that the middle letter of the angle (for example, B of angle ABC) denotes the vertex, quickly review. Then, guide students to set a goal for the Sketch Angles fluency. An appropriate goal may be to consistently label the vertex as the middle letter of the angle.

In addition, some students may benefit from sketching a 90° angle as a reference point for each angle.

S: (Point to the front and back wall.)
T: (Point at the front wall.)
S: (Point to the side walls.)

Continue pointing to the other side wall and the back wall.

T: (Point at the back wall.) Point to the wall that runs parallel to the wall I’m pointing to.
S: (Point at the front wall.)

Continue pointing to one side wall, the back wall, and the other side wall.

Sketch Angles (4 minutes)

Materials: (S) Personal whiteboards

Note: This fluency reviews terms from G4–M4–Lesson 7.

T: On your boards, show me \( \angle ABC \) that measures about 90°.
S: (Sketch \( \angle ABC \) that measures approximately 90°.)
T: What do we call an angle that measures 90°?
S: Right angle.
T: On your boards, show me \( \angle DEF \) that measures about 80°.
S: (Sketch \( \angle DEF \) that measures approximately 80°.)
T: What type of angle did you draw?
S: Acute.

Continue with the following possible sequence: 10°, 150°, 50°, 120°, and 45°.

Application Problem (5 minutes)

Draw a series of clocks that show 12:00, 3:00, 6:00, and 9:00. Use an arc to identify an angle and estimate the angle created by both hands on the clock.

Note: This Application Problem reviews the sketching of angles from G4–M4–Lesson 7 and leads up to the
Concept Development of today’s lesson where students will further explore angle measure on a circle. Some students may identify 3:00 as a 270° angle, and 9:00 as a 90° angle. Confirm with the arcs if the estimated measurements are accurate.

Concept Development (33 minutes)

Materials: (T) Analog clock (S) Clock template

Problem 1: Explore angle measure as turning in relation to the hour hand on a clock.

T: Use your straightedge to draw a line segment that starts at the tick mark representing the hour of 12 and ends at the tick mark representing the hour of 6. Fold along the line that you just drew. What fractional units have you just created?
S: Halves!
T: Next, fold your clock template in half again. Unfold and trace along the second fold. What is the new fractional unit you have created?
S: Fourths. → Quarters.
T: At 12:00, the hour hand points at the 12. Point at the 12. At 3:00, the hour hand points at the 3. Use your finger to trace along the edge of the circle from the 12 to the 3 to represent the movement of the hour hand. What fraction of the arc is that?
S: One fourth.
T: How many degrees did you just move?
S: 90°.
T: At 6:00, the hour hand points at the 6. Trace along the edge of the circle from the 3 to the 6. How many degrees did you just move?
S: 90°.
T: At 9:00, the hour hand points at the 9. Trace along the edge of the circle from the 6 to the 9. How many degrees did you just move?
S: 90°.
T: Point to the 9 and trace another quarter of the way around the clock. Where does your finger stop?
S: At the 12.
T: Talk to your partner about the total number of degrees and the number of quarter turns we just made.
S: One quarter of the way around the clock plus one quarter plus one quarter plus one quarter. That’s 90° + 90° + 90° + 90°. 360°! → Four quarter-turns. → 4 × 90 = 360. → 4 × 9 tens, 36 tens or 360 degrees.
T: Talk to your partner about moving from the 12 to the 6 along the arc.
S: That would be half way around the clock. → 180° because 90 + 90 = 180.
T: How about from the 12 to the 9?
S: That’s three quarter-turns. → 270° because 90 + 90 + 90 = 270.
Problem 2: Explore angle measure as turning in relation to the room.

T: Everyone stand up and face the front of the room. Let’s represent turns by using our bodies. Stay in same spot you are and show me a complete turn if you can.
S: (Attempt to do so.)
T: How many degrees did you turn?
S: 360. A full turn is 360°. \( \rightarrow \) It’s just like what we showed on the clock.
T: Face the front of the room again. This time, make a half-turn. Where are you facing?
S: The back of the room.
T: How many degrees did you turn when you made a half-turn?
S: 180°. \( \rightarrow \) 180° is half of 360°. \( \rightarrow \) 90° + 90° = 180°.
T: What is another turn that we can show?
S: We can show a quarter-turn. That would be 90°.
T: Everyone face the front of the room again. Show me where you will face when you make a quarter-turn.
T: Why are people facing in different directions?
S: I turned to the left. \( \rightarrow \) I turned to the right.
T: Who is correct? The students who turned to the left or right? Take a moment to discuss with your neighbor.
S: We are both correct. \( \rightarrow \) We both made a quarter-turn. We just turned in different directions. \( \rightarrow \) Whether you turn to the left or right, you are still turning 90°. No one said which way, just that it had to be a quarter-turn.
T: Face the front of the room. Make two quarter-turns in the same direction.
S: We are all facing the back of the room! \( \rightarrow \) Two quarter-turns is the same as a half-turn. Some of us started off going to the left and some started off going to the right, but we all ended up facing the back of the classroom.
T: We can say that we all did a 180. We were facing in one direction and then we were facing in the opposite direction.

Problem 3: Recognize turning angles in various contexts.

T: When a skateboarder does a 180, what does she do?
S: She spins around to face the other way.
T: When a car loses control on an icy road and does a 360 what does the car do?
S: It spins all the way around in a circle.
T: Turn your pencil a quarter-turn.
S: (Do so.)
T: With your partner, come up with an example of something that might turn. Identify the turn using degrees or turns and then be prepared to report back to the class.
Lesson 8

Identify and measure angles as turns and recognize them in various contexts.

Student Debrief (10 minutes)

Lesson Objective: Identify and measure angles as turns and recognize them in various contexts.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Why was there confusion with turning 90° but not with turning 180° or 360°? How can the terms clockwise and counterclockwise be used in Problem 7?
- Why is there more than one answer for Problem 7?
- Does it matter in Problem 8 if you turned 180° to the right or 180° to the left? Explain.
- What do you notice about the terms used to tell time? (All of the benchmark angles have terms, i.e., half-past, quarter of, quarter past.)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

S: My mom turned up the heat on the stove, so she moved the knob a quarter-turn. → To find the library, walk down to the end of this hall and turn 90° to the right. → The earth does a 360 every day. → When the plug didn’t fit into the iPad to charge it, I flipped the charger a half-turn.

Notes on Multiple Means of Action and Expression:

Scaffold the Problem Set with the following options:
- Put a dot in the center of the circle to assist student drawing in Problem 5.
- Guide students to count by 90 degrees or by fourths up to the desired turn.
- Clarify for English language learners that quarters and fourths are interchangeable terms.
- For Problem 7, encourage students to actually turn the Problem Set paper and count the quarter turns to make the picture upright.
Stand face to face with your partner. Ask your partner to turn to the left. Why does it appear to you that she turned to the right? In each problem in this lesson, when someone turns to the right or left, it is from her perspective. What does this mean?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 8 Problem Set

Name ___________________________________________ Date ______________________

1. Joe, Steve, and Bob stood in the middle of the yard and faced the house. Joe turned 90° to the right. Steve turned 180° to the right. Bob turned 270° to the right. To what was each boy now facing?

   Joe ____________________

   Steve ____________________

   Bob ____________________

2. Monique looked at the clock at the beginning of class and at the end of class. How many degrees did the minute hand turn from the beginning of class until the end?

3. The skater jumped into the air and did a 360. What does that mean?

4. Mr. Martin drove away from his house without his wallet. He did a 180. Where was he heading now?
5. John turned the knob of the shower 270° to the right. Draw a picture showing the position of the knob after he turned it.

Before

After

6. Barb used her scissors to cut out a coupon from the newspaper. How many quarter-turns does she need to turn the paper in order to stay on the lines?

SAVE
$1.00

7. How many quarter-turns does the picture need to be rotated in order for it to be upright?

8. Meredith faced north. She turned 90° to the right and then 180° more. In what direction was she now facing?
1. Marty was doing a handstand. Describe how many degrees his body will turn to be upright again.

2. Jeffrey started riding his bike at the $\star$. He travelled north for 3 blocks, then turned 90° to the right and rode for 2 blocks. What direction was he headed? Sketch his route on the grid below. Each square unit represents 1 block.
Name ________________________________ Date __________________

1. Jill, Shyan, and Barb stood in the middle of the yard and faced the barn. Jill turned $90^\circ$ to the right. Shyan turned $180^\circ$ to the left. Barb turned $270^\circ$ to the left. To what was each girl now facing?

   Jill ____________________________

   Shyan __________________________

   Barb ____________________________

2. Allison looked at the clock at the beginning of class and at the end of class. How many degrees did the minute hand turn from the beginning of class until the end?

   Beginning

   End

3. The snowboarder went off a jump and did a $180$. In which direction was the snowboarder facing when he landed? How do you know?

4. As she drove down the icy road, Mrs. Campbell slammed on her brakes. Her car did a $360$. What does this mean?
Lesson 8 Homework

5. Jonah turned the knob of the stove two quarter-turns. Draw a picture showing the position of the knob after he turned it.

Before  

After

6. Betsy used her scissors to cut out a coupon from the newspaper. How many total quarter-turns will she need to rotate the paper in order to cut out the entire coupon?

7. How many quarter-turns does the picture need to be rotated in order for it to be upright?

8. David faced north. He turned 180° to the right and then 270° degrees to the left. In what direction was he now facing?

Before  

After
Lesson 8: Identify and measure angles as turns and recognize them in various contexts.

Date: 10/16/13