Topic F
Partial Quotients and Multi-Digit Whole Number Division

5.NBT.6

Focus Standard: 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Instructional Days: 5
Coherence -Links from: G4-M3 Multi-Digit Multiplication and Division
-Links to: G6-M2 Arithmetic Operations Including Dividing by a Fraction

The series of lessons in Topic F leads students to divide multi-digit dividends by two-digit divisors using the written vertical method. Each lesson moves to a new level of difficulty with a sequence beginning with divisors that are multiples of 10 to non-multiples of 10. Two instructional days are devoted to single-digit quotients with and without remainders before progressing into two- and three-digit quotients (5.NBT.6).

A Teaching Sequence Towards Mastery of Partial Quotients and Multi-Digit Whole Number Division

Objective 1: Divide two- and three-digit dividends by multiples of 10 with single-digit quotients and make connections to a written method. (Lesson 19)

Objective 2: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method. (Lessons 20–21)

Objective 3: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients, reasoning about the decomposition of successive remainders in each place value. (Lessons 22–23)
Lesson 19

Objective: Divide two- and three-digit dividends by multiples of 10 with single-digit quotients and make connections to a written method.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (7 minutes)
- Concept Development (31 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)

Fluency Practice (12 minutes)

- Estimate and Divide 5.NBT.6 (5 minutes)
- Group Count by Multiples of 10 5.NBT.2 (3 minutes)
- Group Count by Multi-Digit Numbers 5.NBT.6 (4 minutes)

Estimate and Divide (5 minutes)

Materials: (S) Personal white boards

Note: This drill reviews content from G5-M2-Lessons 17 and 18.

Repeat the process from G5–M2–Lesson 18 with the following possible sequence: 908 ÷ 28; 152 ÷ 33; 398 ÷ 98; and 7,272 ÷ 81.

Group Count by Multiples of 10 (3 minutes)

Note: Counting by multiples of 10 helps students estimate quotients with two-digit divisors during the Concept Development.

Repeat the process from G5–M2–Lessons 16–18 for various multiples of 10.

Group Count by Multi-Digit Numbers (4 minutes)

Materials: (S) Paper

Note: This drill prepares students for G5–M2–Lesson 20’s Content Development.
Lesson 19: Divide two- and three-digit dividends by multiples of 10 with single-digit quotients and make connections to a written method.

Date: 7/4/13

T: I’m going to call out a number. I want you to skip-count by that number. You have one minute. Ready. 21.

S: (Write down multiples of 21.)

T: Stop. Let’s correct your work. (Read and write down multiples from 21 to 210 as students check their multiples.) Let’s skip count again by twenty-ones. Try not to look at the board as I guide you.

Stand away from the board. Direct the students to count by 5–10 multiples of 21 forward and backward, occasionally changing directions and attempting to avoid student frustration.

Repeat the process for 43.

Application Problem (7 minutes)

At the Highland Falls pumpkin growing contest, the prize winning pumpkin contains 360 seeds. The proud farmer plans to sell his seeds in packs of 12. How many packs can he make using all the seeds?

Note: Although the students have not yet done three-digit totals divided by a two-digit divisor, this problem has the basic fact 12 × 3 = 36 embedded in it, and it’s similar to problems encountered in G5–M2–Lesson 18’s Concept Development.

Concept Development (31 minutes)

Materials: (S) Personal white boards

Problem 1: 70 ÷ 30

T: (Write 70 ÷ 30 on the board.) The divisor is?
S: 30.

T: We need a multiple of 30 to make the division easy. How should we estimate the quotient? Turn and share with your partner.

S: I see an easy fact of 6 divided by 3 is equal to 2. → Yeah, 6 tens divided by 3 tens is 2! → I can estimate 70 to 60, because I can easily divide 30 into 60.

T: On your personal board, show me how to estimate the quotient.
S: (Show 60 ÷ 30 = 6 ÷ 3 = 2.)

T: (Write and set up the standard algorithm below 70 ÷ 30 on the board.) Our estimated quotient is 2, which means that I should be able to distribute 2 × 30. (Record 2 in the quotient.) What’s 2 × 30?
S: 60.
T: (Record 60 below the 70.) I distributed 60. The difference between 60 and 70 is?
S: 10.
T: What does this 10 mean?
S: 10 is the remainder. \(\rightarrow\) 10 is the left over from the original total of 70. \(\rightarrow\) We started with 70, made 2 groups of 30, used up 60 and were left with 10. \(\rightarrow\) We have 10 left over, but we need 20 more in order to make 1 more group of 30.
T: Can we make another group of 30 with our remainder?
S: No, 10 is not enough to make a group of 30.
T: How might we know that our quotient is correct?
S: We can check our answer to see if our quotient is correct.
T: Yes! Let’s multiply: 30 times 2. (Write 30 \(\times\) 2 = on the board.) What's the answer?
S: 60.
T: We started with 70, and 60 \(\neq\) 70. Does this mean we made an error? What else must we do? Turn and discuss.
S: Oh, no! We made a mistake because 60 doesn't equal 70. \(\rightarrow\) We have to add the remainder of 10. Then the total will be 70. \(\rightarrow\) Our thinking is correct. We could make 2 groups of 30, but there were 10 left over. They are part of the original whole. We need to add the 10 to the 60 that were put into groups.
T: Yes. (Draw number bond.) One part is made of groups of 30. The other part is the remainder.
T: What’s 60 plus 10? (Write 60 + 10 = on the board below 30 \(\times\) 2 = 60.)
S: 70.
T: Yes. We did it. We solved the division correctly. Today we got a precise answer with a quotient and remainder, while in the previous lessons, we merely estimated the quotient.

**Problem 2: 430 ÷ 60**

T: (Write 430 ÷ 60 on the board.) What’s our whole?
S: 430.
T: Again, we need a multiple of 60 to make the division easy. Show me how to estimate the quotient.
S: (Show 420 ÷ 60 = 42 ÷ 6 = 7.)
T: Let’s record this division sentence vertically. You do the same on your personal board. (Write and set up the standard algorithm below 430 ÷ 60 on the board.) Our estimate was 7, which means that there should be 7 groups of 60 in 430. Let’s divide and see if that’s true.
T: Let’s record the 7 in our quotient. (Record 7.) Why is the 7 recorded above the zero in the vertical algorithm?
S: 7 represents 7 ones, so it must be recorded in the ones place directly above the ones place in the whole. → 420 divided by 60 is just 42 tens divided by 6 tens. The answer is just 7, not 7 tens.
T: What’s 7 times 60?
S: 420.
T: (Record 420 below 430.) Was it possible to make 7 groups of 60 from 430? How do you know?
S: Yes, we distributed 420 and still have some left.
T: How many are remaining after making the groups?
S: 10.
T: What does this remainder of 10 mean?
S: 10 is what is left over after making groups from the whole. We don’t have enough to make another group of 60. We need 60 to make 1 group, so we’ll need 50 more in order to make another group of 60.
T: There are 7 units of 60 in 430 and 10 remaining. Now work with a partner and check the answer.
T: Look at your checking equation. Say the multiplication sentence starting with 60.
S: 60 × 7 = 420.
T: What does this part represent?
S: It shows the part of our whole that was put into groups of 60. (Draw number bond pictured to the right.)
T: (Write 60 × 7 = 420 on the board.) Say the equation to complete the original whole.
S: 420 + 10 = 430.
T: (Write 420 + 10 = 430 on the board below 60 × 7 = 420.) What does this part of our check represent?
S: This shows the part of the total that we could put into groups added to the part that we couldn’t put into groups. Together it is all that we had to distribute.
Problem 3: $572 \div 90$

T: (Write $572 \div 90$ on the board.) We’re trying to make groups of 90. What multiple of 90 is closest to 572 and would make this division easy? Show me how to estimate the quotient.

S: (Show $540 \div 90 = 54 + 9 = 6$.)

T: Our estimated quotient is 6. With a partner, find the actual quotient using the standard algorithm, and check the answer. When you’re finished, check your answer with another group.

T: How many nineties are there in 572? (Record the algorithm.)

S: 6.

T: Where is this recorded in the algorithm?

S: In the ones place above the ones place in the whole.

T: How many are remaining?

S: 32.

T: Is this enough to make another ninety?

S: No.

T: What are the equations for checking the problem?

S: $90 \times 6 = 540$ and $540 + 32 = 572$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Divide two- and three-digit dividends by multiples of 10 with single digit quotients and make connections to a written method.
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1(d), did anyone notice something different? Does it always make sense to use the standard algorithm?
- In Problem 2, what was Terry’s mistake? If you had to estimate the quotient, what would you have done? What could he do to correct his quotient without erasing his work so far? (Make sure students recognize that Terry’s thinking was accurate, but he stopped making groups too soon. His error can be corrected by simply making another group of 40 and subtracting it from the remaining whole.)
- What if Terry had estimated too large a quotient? What should he do?
- How was solving Problem 3 different from solving all the others? Why?
- Explain your thought process as you solved Problem 4.
- What did all our divisors have in common today? Did this make estimation easier?
- Does a divisor have to be a multiple of 10? Why do you think I chose multiples of 10 for divisors today?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 19 Problem Set

Name ________________________________ Date ____________________

1. Divide, then check. The first one is done for you.
   a. \(41 \div 30\)
      \[
      \begin{array}{c|c|c|c}
      & 1 & R 11 \\
      3 & 0 & \\
      \hline
      4 & 1 & \\
      3 & 0 & \\
      \hline
      1 & 1 & \\
      \end{array}
      \]
      Check:
      \[
      30 \times 1 = 30 \\
      30 + 11 = 41
      \]
   b. \(80 \div 30\)
   c. \(71 \div 50\)
   d. \(270 \div 30\)
   e. \(643 \div 80\)
   f. \(215 \div 90\)
2. Terry says the solution to 299 ÷ 40 is 6 R59. His work is shown below. Explain Terry’s error in thinking, and then find the correct quotient using the space on the right.

\[
\begin{array}{c}
40 \overline{\big) 299} \\
240 \\
\hline
59
\end{array}
\]

3. A number divided by 80 has a quotient of 7 with 4 as a remainder. Find the number.

\[
\frac{\text{?}}{80} = 7 \text{ R} 4
\]

4. While swimming a 2 km race, Adam changes from breaststroke to butterfly every 200 m. How many times did he switch strokes during the first half of the race?
1. Divide, then check using multiplication.

   a. \( 73 \div 20 \)

   b. \( 291 \div 30 \)
Name ___________________________________________ Date __________________________

1. Divide, then check using multiplication. The first one is done for you.

   a. \(71 \div 20\)  
      
      \[
      \begin{array}{c|c}
      & 3 \hspace{1cm} R 11 \\
      \hline
      20 & 7 \hspace{1cm} 1 \\
      \hline
      & 6 \hspace{1cm} 0 \\
      \hline
      & 1 \hspace{1cm} 1
      \end{array}
      \]
      
      Check: \(20 \times 3 = 60\)  
      \(60 + 11 = 71\)

   b. \(90 \div 40\)

   c. \(95 \div 60\)

   d. \(280 \div 30\)

   e. \(437 \div 60\)

   f. \(346 \div 80\)
2. A number divided by 40 has a quotient of 6 with a remainder of 16. Find the number.

3. A shipment of 288 textbooks has been delivered. Each of the 10 classrooms will receive an equal share of the books, with any extra books being stored in the bookroom. After the texts have been distributed to the classrooms, how many will be stored in the bookroom?

4. How many sixties are in two hundred forty-four?
Lesson 20

Objective: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (5 minutes)
- Concept Development (33 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Group Count by Multi-Digit Numbers 5.NBT.6 (3 minutes)
- Estimate and Divide 5.NBT.6 (4 minutes)
- Divide by Multiples of Ten with Remainders 5.NBT.6 (5 minutes)

Group Count by Multi-Digit Numbers (3 minutes)

Materials: (S) Paper

Note: This drill prepares students for this lesson’s Content Development.

Direct the students to count by 5–10 multiples of 21 forward and backward, occasionally changing directions and attempting to avoid student frustration.

Repeat process for 43.

Estimate and Divide (4 minutes)

Materials: (S) Personal white boards

Note: This drill reviews G5–M2–Lesson 17 content.

Repeat the process from G5–M2–Lessons 18 and 19 for the following possible sequence: 607 ÷ 19, 123 ÷ 24, 891 ÷ 96, and 5,482 ÷ 62.
Lesson 20

Divide by Multiples of Ten with Remainders (5 minutes)

Materials: (S) Personal white boards

Note: This drill reviews G5–M2–Lesson 19 content.

T: (Write 73 ÷ 50.) On your boards, solve the division problem using the standard algorithm. Check your work using multiplication and addition. Repeat process for 70 ÷ 30, 157 ÷ 30, and 432 ÷ 70.

Application Problem (5 minutes)

Billy has 2.4 m of ribbon for crafts. He wants to share it evenly with 12 friends. How many centimeters of ribbon would 7 friends get?

Note: This application problem reaches back to concepts taught in G5–M1.

Concept Development (33 minutes)

Materials: (S) Personal white boards

Problem 1: 72 ÷ 21

T: (Write 72 ÷ 21 on the board.) What is our whole?
S: 72.
T: Find a multiple of 20 close to 72 that makes this division easy. Show me how to estimate the quotient on your white board.
S: (Show 60 ÷ 20 = 3.)
T: I see you chose 60. Why not choose 80 and estimate the quotient as 4?
S: Because 4 × 20 is 80, and that’s already too big.
T: Right, so our estimate means that there are about 3 twenty-ones in 72. Let’s record that estimate. Where
should it be recorded? (Write and set up the standard algorithm below $72 \div 21$ on the board.)

S: In the ones place.

T: What is $3 \times 21$?

S: 63.

T: (Record 63 below 72.) So, we’ve distributed 3 units of 21. How many of the 72 remain? Give me the full subtraction sentence.

S: $72 - 63 = 9$.

T: Is 9 enough to make another group of 21?

S: No.

T: How did our estimate help us solve the problem? Turn and share with your partner.

S: We divided 60 by 20 to get our estimate, which was 3 ones. So, that’s what we tried first in the quotient. \( \rightarrow \) Our estimated quotient was 3, and it turned out that our actual quotient was 3 with a leftover of 9.

T: Great. Let’s check our answer. Whisper the number sentences to your partner.

T: If I have 3 groups of 21 and add 9, what should my total be?

S: 72.

T: If I have 21 groups with 3 in each and 9 more, what should my total be?

S: 72. \( \rightarrow \) It’s the same thing: 21 groups of 3 and 3 groups of 21 are both just $3 \times 21$.

T: Then that means that when using the algorithm, we can view the divisor as either the number of groups or the size of each group.

**Problem 2: $94 \div 43$**

T: (Write $94 \div 43$ on the board.) Take out your personal board. Work with a partner.

1. Round the divisor.
2. Find a multiple of the divisor that makes the division easy.
3. Estimate the quotient.
4. Solve using the standard algorithm.

T: Partner A will divide using the standard algorithm, and Partner B will check the answer. (Allow time for students to work.)

T: Partner A, say the quotient and the remainder for $94 \div 43$.

S: The quotient is 2 and the remainder is 8.

T: What does the quotient, 2, represent?

S: 2 groups of 43. \( \rightarrow \) 43 groups with 2 in each one.

T: What does the remainder of 8 represent?

S: 8 out of the 43 needed to make another group. \( \rightarrow \) 8 that couldn’t be distributed fairly into 43.
Lesson 20: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

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T: Partner B, say your number sentences for checking the problem.
S: 43 \times 2 = 86, and 86 + 8 = 94.

T: Again, let’s look at our estimated quotient and our actual quotient. Did our estimated quotient turn out to be the actual quotient?
S: Yes.

**Problem 3: 84 \div 23**

T: (Write 84 \div 23 on the board.) We need a multiple of 20 that will make this division easy. Show me how to estimate the quotient.

S: 80 \div 20 = 8 \div 2 = 4.

T: What are other ways of estimating this problem?

S: 90 \div 30 = 9 \div 3 = 3. \quad \Rightarrow 100 \div 25 = 4.

T: These are all good ideas. Let’s use our first possibility. (Write 80 \div 20 = 4 on the board.) Let’s now solve this problem using the standard algorithm. (Write and set up the standard algorithm below 84 \div 23 on the board.) Our estimated quotient was 4, so I’ll put 4 as the quotient. (Record 4 as the quotient in the ones place in standard algorithm.)

T: What are 4 units of 23?

S: 92.

T: Wait a minute! Let’s stop and think. We have 84 in our total. Do we have enough to make 4 units of 23?

S: No.

T: What’s happening here? Why didn’t our estimated quotient work this time? Turn and discuss with your partner.

S: Our estimation sentence was correct. 84 \div 23 becomes 80 \div 20 = 4. \quad \Rightarrow We rounded our divisor down from 23 to 20. When we multiply 23 times 4, the product is 92. The product of 20 times 4 is 80. The extra part came from 4 \times 3. \quad \Rightarrow I know. We made the divisor smaller. The real divisor was bigger, so that means we are going to make fewer units. \quad \Rightarrow Yeah! If the divisor was just two more, 25, we would have rounded to 30, and then 90 divided by 30 is obviously 3.

T: So if 4 ones is too big to be the quotient, what should we do?

S: Let’s try 3.

T: How much is 3 \times 23?

\[ 23 \times 3 = 69 + 15 = 84 \]

\[ 23 \times 2 = 46 + 2 = 48 \]

\[ 23 \times 1 = 23 + 3 = 26 \]

\[ 23 \times 0 = 0 \]

\[ 23 \times 4 = 92 \]

\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

\[ 23 \times 1 = 23 \]

\[ 23 \times 0 = 0 \]

\[ 23 \times 4 = 92 \]

\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

\[ 23 \times 1 = 23 \]

\[ 23 \times 0 = 0 \]

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\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

\[ 23 \times 1 = 23 \]

\[ 23 \times 0 = 0 \]

\[ 23 \times 4 = 92 \]

\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

\[ 23 \times 1 = 23 \]

\[ 23 \times 0 = 0 \]

\[ 23 \times 4 = 92 \]

\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

\[ 23 \times 1 = 23 \]

\[ 23 \times 0 = 0 \]

\[ 23 \times 4 = 92 \]

\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

\[ 23 \times 1 = 23 \]

\[ 23 \times 0 = 0 \]

\[ 23 \times 4 = 92 \]

\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

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\[ 23 \times 0 = 0 \]

\[ 23 \times 4 = 92 \]

\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

\[ 23 \times 1 = 23 \]

\[ 23 \times 0 = 0 \]

\[ 23 \times 4 = 92 \]

\[ 23 \times 3 = 69 \]

\[ 23 \times 2 = 46 \]

\[ 23 \times 1 = 23 \]
S: 69.
T: Take away those that we’ve distributed.
T: How many ones are remaining?
S: 15.
T: What does the remainder of 15 tell us?
S: We don’t have enough for a fourth group. Those 15 ones are left over. → We’ll need 8 more to make another group of 23.
T: Give me the quotient and remainder for 84 ÷ 23.
S: The quotient is 3 and the remainder is 15.
T: Whisper to your partner what these numbers represent and how we should check this problem.
S: The 3 is 3 groups of 23, and the 15 are the ones that weren’t enough to make another group. → We should multiply the quotient and the divisor, and then add the remainder.
T: Say the multiplication sentence starting with 23.
S: 23 × 3 = 69.
T: (Record 23 × 3 = 69 on the board.) Say the addition sentence starting with 69.
S: 69 + 15 = 84.
T: (Record 69 + 15 = 84 below 23 × 3 = 69 on the board.) Is 84 our original whole?
S: Yes, we solved it correctly.
T: What did we just learn about estimated quotients? Turn and discuss.
S: We should always estimate before we solve, but we may need to adjust it. → If we change the divisor or the whole a lot, it could make our estimate too big or too small.

Problem 4: 57 ÷ 29
T: (Write 57 ÷ 29 on the board.) Take out your personal board. Work on this problem independently. Remember to estimate, divide and check. Compare your work with a partner when you’re finished.
T: Tell me how you estimated.
S: 60 ÷ 30 = 6 ÷ 3 = 2.
T: Can I use the quotient of 2? Discuss with your neighbor.
S: No.
T: Why not? How much is 2 units of 29?
S: 58. 58 is greater than our whole of 57.
T: So what’s the actual quotient?
S: 1.
T: Give me the quotient and remainder for 57 ÷ 29.
S: The quotient is 1 with a remainder of 28.
T: What are the sentences for checking the problem?
Lesson Objective: Divide two- and three-digit dividends by two-digit divisors with single digit quotients and make connections to a written method.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- What pattern did you notice between 1(c) and 1(f)?
- Did your initial estimates work for every example in Problem (1)? Why or why not? What happened in 1(d)?
In Problem 2, what would you tell Linda in order to help her solve the problem? What lesson does Linda need to learn? What is another way that Linda could have estimated that would have eliminated the issue she encountered in the standard algorithm?

Explain your thought process as you set up and began to solve Problems 3 and 4. What was challenging or unique about them? (Generating a division problem with the same quotient and remainder appears on the End-of-Module Assessment. Make time to debrief the students’ thinking about Problem 4 thoroughly.)

Talk about the importance of estimation when dividing with two-digit divisors.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
1. Divide, then check with multiplication. The first one is done for you.
   a. \(65 \div 17\)
   \[
   \begin{array}{c}
   1 & \underline{7} & 14 \\
   - & 5 & 1 \\
   \hline
   1 & 4
   \end{array}
   \]
   Check:
   \[
   17 \times 3 = 51 \\
   51 + 14 = 65
   \]
   b. \(49 \div 21\)
   c. \(78 \div 39\)
   d. \(84 \div 32\)
   e. \(77 \div 25\)
   f. \(68 \div 17\)

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2. When dividing 82 by 43, Linda estimated the quotient to be 2. Examine Linda’s work and explain what she needs to do next. On the right, show how you would solve the problem.

<table>
<thead>
<tr>
<th>Linda’s estimation:</th>
<th>Linda’s work:</th>
<th>Your work:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4 0 2</td>
<td>4 3 8 2</td>
</tr>
<tr>
<td></td>
<td>- 8 6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>? ?</td>
<td>4 3 8 2</td>
</tr>
</tbody>
</table>

3. A number divided by 43 has a quotient of 3 with 28 as a remainder. Find the number. Show your work.

4. Write another division problem that has a quotient of 3 and a remainder of 28.

5. Mrs. Silverstein sold 91 cupcakes at a food fair. The cupcakes were sold in boxes of “a baker’s dozen,” which is 13. She sold all the cupcakes at $15 per box. How much money did she receive?
1. Divide, then check with multiplication.
   
a. \(78 \div 21\)
   
b. \(89 \div 37\)
Name _____________________________ Date _______________________

1. Divide, then check with multiplication. The first one is done for you.

   a. \( 72 \div 31 \) 
   
   \[ \begin{array}{c} \underline{2} \quad \frac{2}{7} \\
   \underline{6} \quad 2 \\
   1 \quad 0 \\
   \end{array} \]
   
   Check: \( 31 \times 2 = 62 \)
   
   \( 62 + 10 = 72 \)

   b. \( 89 \div 21 \)

   e. \( 79 \div 25 \)

   c. \( 94 \div 33 \)

   f. \( 83 \div 21 \)
2. A 189-square-foot rectangular office has a length of 21 feet. What is the width of the office?

3. While preparing for a morning conference, Principal Corsetti is laying out 15 dozen bagels on square plates. Each plate can hold 14 bagels.
   
a. How many plates of bagels will Mr. Corsetti have?

b. How many more bagels would be needed to fill the final plate with bagels?
Lesson 21

Objective: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (5 minutes)
- Concept Development (33 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)

Fluency Practice (12 minutes)

- Group Count by Multi-Digit Numbers 5.NBT.6 (5 minutes)
- Divide by Two-Digit Numbers 5.NBT.6 (7 minutes)

Group Count by Multi-Digit Numbers (5 minutes)

Materials: (S) Paper

Note: This drill prepares students for this lesson’s Content Development.
Repeat the process from G5–M2–Lesson 19 for 31 and 16.

Divide by Two-Digit Numbers (7 minutes)

Materials: (S) Personal white boards

Notes: This drill reviews G5–M2–Lesson 20 content.

T: (Write 61 ÷ 17.) On your boards, show me how to estimate the quotient.

S: (Write 60 ÷ 20 = 3)

T: Solve the equation.

S: (Solve and check as exemplified to the right.)

Repeat the process using the following possible sequence: 48 ÷ 21, 99 ÷ 32, and 74 ÷ 37.
Application Problem (5 minutes)

105 students were divided equally into 15 teams.

a. How many players were on each team?
b. If each team had 3 girls, how many boys were there altogether?

Note: Although yesterday’s lesson focused only on two-digit totals, the friendly divisor of 15 makes this problem manageable for students. Students who have difficulty answering Part (a) may need extra support during the Concept Development.

Concept Development (33 minutes)

Materials: (S) Personal white boards

Problem 1: 256 ÷ 47

T: (Write 256 ÷ 47 on the board.) How can we estimate the quotient? Discuss with a partner.
S: (Discuss.) We need a multiple of 50 that is close to 256. 250 ÷ 50 = 25 ÷ 5 = 5.
T: Let’s use the estimate to help us solve in the standard algorithm. (On the board, write and set up the standard algorithm below 256 ÷ 47.) Our estimated quotient is 5. I’ll record that. (Record the quotient 5 in the ones place above 256.) What is 5 × 47? You may solve it on your personal board if you like.
S: 235.
T: (Record 235 below 256.) How many are remaining?
S: 21.
T: (Record 21 in the algorithm.) Do we have enough for another group of 47 or to distribute 1 more to 47 groups?
S: No.
T: So what does the 21 represent? Whisper to your neighbor.
S: This is what is left of our whole after we made all the groups of 47 we could.
Lesson 21

Lesson 21: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

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T: How did our estimate help us solve?
S: It gave us a starting point for our quotient.
   → We estimated the quotient to be 5, and our actual quotient is 5 with a remainder of 21. The estimate was just right.
T: This time our estimate did not need to be adjusted. Why do you think that is?
S: We estimated 47 to be 50 and the whole was almost a multiple of 50. → Our divisor was smaller than 50, so we didn’t go over. → Maybe if it was 54 it wouldn’t have worked so well even though it rounds to 50, too. → Yeah, 54 would go over! It would be close to 270 (54 × 5).
T: Work with a partner to check the quotient.
T: One part is 5 complete groups of 47. The other part is the 21. What’s the whole?
S: 256.

Problem 2: 236 ÷ 39

T: (Write 236 ÷ 39 on the board.) Think on your own. How will you estimate? (Give students time to think.) Tell me how you’ll estimate.
S: 240 ÷ 40 = 6.
T: What basic fact helped you to estimate?
S: 24 ÷ 6 = 4.
T: On your personal board, solve this problem with your partner using the standard algorithm. Partner A will divide using the standard algorithm, and Partner B will check the answer.
T: Let’s go over the answer. Analyze why our estimate was perfect.
S: 39 is really close to our estimated divisor, 40. → The total was less than the rounded whole but by just a little bit. It was close! → 39 is one less than 40, so 6 groups of 39 will be 6 less than 240. The rounded quotient was 4 less than 240, so the difference is 2, our remainder!
T: What is 236 divided by 39?
Lesson 21: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

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S: The quotient is 6 with a remainder of 2.
T: Check it. How much is $39 \times 6 + 2$?
S: 236.

**Problem 3: $369 \div 46$**

T: (Write $369 \div 46$ on the board.) How will you estimate the quotient?
S: $350 \div 50 = 7. \quad 400 \div 40 = 10. \quad 360 \div 40 = 9.$
T: These are all reasonable estimates. Let’s use $350 \div 50 = (350 \div 10) \div 5 = 35 \div 5 = 7.$ (Write the estimate below the problem.)
T: (Record 7 in the ones column in the quotient.) How much is $46 \times 7$? You may solve on your personal board.
S: 322.
T: Subtract this from our whole. How many ones are remaining?
S: 47. (Record –322 and 47 in the algorithm.)
T: What do you notice about the remainder of 47 ones? Turn and discuss with your partner.
S: The remainder is larger than the group size, which means I have enough to make another group. $47$ is greater than the divisor of 46. We haven’t made enough groups. We only made 7 groups of 46, but we can make 8. Since 47 is bigger than 46, it means that the quotient of 7 is not big enough. We could try to use the quotient of 8.
T: We have 47 remaining. We agree that’s enough to make another group of 46. We can record this several ways. (Write on board.)

- Erase, start over, and use 8 as our quotient.
- Subtract one more group of 46, cross out the 7 at the top, and write in an 8.
- Subtract one more group of 46 and record a 1 above the 7 in our vertical algorithm.
T: To state our final quotient, we will need to remember to add 7 and 1.
T: (Subtract one more unit of 46.) Now, how many are remaining?
S: 1.
T: (Record this in the algorithm.) Is that enough for another group of 46?
S: No.
Lesson 21

Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

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T: How many forty-sixes are in 369?
S: 8 units of 46 with 1 one remaining.

T: Check it. Remember that we have 8 units of 46. Solve $8 \times 46 + 1$? (Write the expression on the board.)
S: 369.

T: Let’s go back and look at our original estimation. If you remember, I suggested 350 ÷ 50. Turn and talk to your partner about how we ended up with a quotient that was too small.

S: Our actual divisor was a lot smaller than the estimate. If the divisor is smaller, you can make more groups. ➔ Also, our actual whole amount was bigger than our estimate. If the whole is larger, we can make more groups! ➔ So, a smaller group size and larger whole meant our estimate was too small.

T: So what can we say about estimating quotients?
S: Sometimes when we estimate a quotient, we need to be prepared to adjust it if necessary.

Problem 4: $712 ÷ 94$

T: (Write $712 ÷ 94$ on the board.) Take out your personal board. Talk with your partner and estimate the quotient.

S: 700 ÷ 100 or just 7. ➔ 720 ÷ 90, which is just $72 ÷ 9 = 8$.

T: Both are reasonable estimates. Let’s use the estimate that divides 720 by 90. That gives us an estimated quotient of 8. (Record this estimate on the board.) Talk with your partner about this estimate. What do you notice?

S: An estimate of 8 is too much because 8 groups of 90 is already more than 712, $8 \times 90 = 720$. We’ll try 7 as our quotient.

T: What was your estimated quotient when you divided 700 by 10?
S: 7.

T: So, either estimate helped us get a starting place for our actual division. Even our imperfect estimate of 8 led us to the correct quotient. Now finish the division and check on your white board. When you’re finished, check it with a neighbor.
Lesson 21: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

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T: What’s the answer for 712 divided by 94?
S: The quotient is 7 with a reminder of 54 ones.
T: Tell me the equations that you’d use to check your answer.
S: $94 \times 7 = 658$ and $658 + 54 = 712$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- What pattern did you notice between 1(c) and 1(d)? Since the quotient was 8 with remainder 7 for both problems, does that mean the two division expressions are equal to each other? Discuss the meaning of the quotient and remainder for both problems.
- In Problem 1, did your estimate need adjusting at times? When? What did you do in order to continue dividing?
Lesson 21

Share your thought process as you solved Problem 2. Can anyone share his or her solution? How many solutions might there be to this problem? Can you create another solution to it? How did your understanding of the check process help you answer this? Explain how the expression \((n \times 8) + 11\) might be used to solve this problem.

What steps did you take as you solved Problem 3? Raise your hand if you doubled the distance (since 133 miles is just one way) before dividing. Try to find a classmate who solved this problem differently from you (one who doubled the quotient after dividing, perhaps). Compare your answers. What did you find?

- If the distance is doubled first, a quotient of 19 with no remainder is found (i.e., Mrs. Giang only needs 19 gallons of gas).
- If 133 (the one-way distance) is divided first, a quotient of 9 with 7 miles left to drive is found. Some students may interpret the remainder and conclude that 10 gallons is needed each way, and double to arrive at a total of 20 gallons. (This amount of fuel would certainly allow Mrs. Giang to arrive at her destination with extra gas in her tank.) This is good reasoning!
- Students who divide first, but are thinking more deeply may realize that if the quotient (9) is doubled, then the remainder (7 miles) must also be doubled. This yields 18 gallons of gas and 14 miles left over. This additional left over 14 miles requires 1 more gallon of gas, so Mrs. Giang needs at least 19 gallons of gas.

Discuss thoroughly the remainders in Problem 4. It might be fruitful to allow students to make a prediction about the size of the remainder in Part (b) before computing. Many students may be surprised that the teacher receives more pencils even when more students are taking pencils. Discuss how this could be possible.
- Talk about how estimating makes the process of long division more efficient.
- The estimated quotient sometimes needs to be adjusted. Talk about why this may happen.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 21 Problem Set

Name ________________________________ Date ______________________

1. Divide, then check using multiplication. The first one is done for you.
   a. 258 \div 47
      \[ \begin{array}{c|c}
          & 5 \\
        \hline
        47 & 5 \\
        \hline
        47 & 23 \\
        \hline
        \end{array} \]
      Check:
      \[ 47 \times 5 = 235 \]
      \[ 235 + 23 = 258 \]

   b. 148 \div 67

   c. 591 \div 73

   d. 759 \div 94

   e. 653 \div 74

   f. 257 \div 36
2. Generate and solve at least one more division problem with the same quotient and remainder as the one below. Explain your thought process.

\[
\begin{array}{c}
58 \longdiv{475} \\
-464 \\
\hline
11
\end{array}
\]

3. Assume that Mrs. Giang’s car travels 14 miles on each gallon of gas. If she travels to visit her niece who lives 133 miles away, how many gallons of gas will Mrs. Giang need to make the round trip?

4. Louis brings 79 pencils to school. After he gives each of his 15 classmates an equal number of pencils, he will give any leftover pencils to his teacher.

   a. How many pencils will Louis’ teacher receive?

   b. If Louis decides instead to take an equal share of the pencils along with his classmates, will his teacher receive more pencils or fewer pencils? Show your thinking.
Name ____________________________ Date __________________

1. Divide, then check using multiplication.

   a. 326 ÷ 53

   b. 192 ÷ 38
1. Divide, then check using multiplication. The first one is done for you.

   a. \( 129 \div 21 \)

   \[ \begin{array}{c|c}
   2 & 6 \\
   \hline
   12 & 12 \\
   \hline
   9 & 9 \\
   \end{array} \]

   \[ \text{R 3} \]

   Check:

   \[ 21 \times 6 = 126 \]

   \[ 126 + 3 = 129 \]

   b. \( 158 \div 37 \)

   c. \( 261 \div 49 \)

   d. \( 574 \div 82 \)

   e. \( 464 \div 58 \)

   f. \( 640 \div 9 \)
2. It takes Juwan exactly 35 minutes by car to get to his grandmother’s. The nearest parking area is a 4-minute walk from her apartment. One week he visited more often. He realized that he spent 5 hours and 12 minutes traveling to her apartment and then back home. How many round trips did he make to visit his grandmother?

3. How many eighty-fours are in 672?
Lesson 22

Objective: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients, reasoning about the decomposition of successive remainders in each place value.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (6 minutes)
- Concept Development (32 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)

Fluency Practice (12 minutes)

- Divide Decimals 5.NBT.7 (3 minutes)
- Group Count by Multi-Digit Numbers 5.NBT.6 (4 minutes)
- Divide by Two-Digit Numbers 5.NBT.6 (5 minutes)

Divide Decimals (3 minutes)

Materials: (S) Personal white boards

Note: This drill prepares students for G5–M2–Lesson 24’s Concept Development.

T: (Write 6 hundreds ÷ 2 = .) Say the division sentence in unit form.
S: 6 hundreds ÷ 2 = 3 hundreds.

Repeat process with 6 tens ÷ 2 and 6 ones ÷ 2, and 6 tenths ÷ 2.

T: On your boards, write 6 tenths ÷ 2 in decimal form.
S: (Write 0.6 ÷ 2 = 0.3.)

Repeat the process for 6 hundredths ÷ 2, 8 thousands ÷ 2, 8 ones ÷ 2, 8 tenths ÷ 2, and 8 hundredths ÷ 2.

Group Count by Multi-Digit Numbers (4 minutes)

Materials: (S) Paper

Note: This drill will prepare students for this lesson’s Content Development.

Repeat the process from G5–M2–Lessons 19 and 21 for 17 and 32.
Divide by Two-Digit Numbers  (5 minutes)

Materials: (S) Personal white boards

Note: This drill will review G5–M2–Lesson 21 content.

Repeat the process from G5–M2–Lesson 21 for the following possible sequence: 208 ÷ 37, 128 ÷ 57, and 664 ÷ 83.

Application Problem  (6 minutes)

Zenin's baby sister weighed 132 ounces at birth. How much did his sister weigh in pounds and ounces?

Note: Depending on the class, you may or may not have to remind students that there are 16 ounces in a pound. Either way, it can be used as an opportunity to interpret the remainder (i.e., what does the remainder of 4 represent in this problem?).

Concept Development  (32 minutes)

Materials: (S) Personal white boards

Problem 1: 590 ÷ 17

T: (Write 590 ÷ 17 on the board.) Can we divide 5 hundreds by 17?
S: Not without regrouping.
T: Let’s work with 59 tens, then. We can divide 59 tens into 17 groups or groups of 17. Tell me how to estimate to divide 59 tens by 17.
S: 60 tens ÷ 20 = 3 tens
T: Record 3 tens and find the remainder in the tens place. 3 tens times 17 is?
S: 51 tens.
T: (Record 51 tens below 59 tens.) Remind me why we record here. (Point to the algorithm.)
S: We record the 5 in the hundreds place, and the 1 is in the tens place because we know 51 tens is the same as 510.
T: How many tens are remaining?
S: 8 tens.
T: Can we divide 8 tens by 17?

MP.7

NOTES ON STANDARDS ALIGNMENT:
The standards specifically require students to find quotients “using strategies based on place value” (5.NBT.6). When dividing, students are decomposing units just as they have done when subtracting since Grade 2. “I don’t have enough tens to subtract, so I’ll change 1 hundred for 10 tens.” When dividing, they also change each larger unit that cannot be divided for smaller units. “I’ll change 8 remaining tens for 80 ones.”
Lesson 22: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients reasoning about the decomposition of successive remainder in each place value.

Date: 7/4/13

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Lesson 22: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients reasoning about the decomposition of successive remainder in each place value.

Date: 7/4/13

S: 54 ones.
T: How many ones remain?
S: 23 ones.
T: Can we divide 23 ones by 27?
S: No, 23 is the remainder.
T: How many groups of 27 are in 887?
S: 32 groups.
T: With how many left over?
S: 23 remaining.
T: Complete the two-part check to make sure.

Problem 3: 839 ÷ 41

T: (Write 839 ÷ 41 on the board.) Solve this problem with a partner. As you finish each step share your thinking with your partner.
S: (Work.)
T: OK. Let's share your work. How did you first estimate to begin dividing?
S: 80 tens ÷ 40 = 2 tens.
T: 2 tens times 41 equals?
S: 82 tens.
T: How many tens remain?
S: 1 ten.
T: What did you do next?
S: Regrouped the 1 ten and made 10 ones and combined them with the 9 ones in the whole to make 19 ones.
T: What is 19 ones divided by 41?
S: Zero. It can't be divided.
T: What is the quotient, then?
S: 20 remainder 19.
T: Explain how you knew that the quotient was 20 with a remainder of 19 and not 2 with a remainder of 19. Turn and talk.
S: (Share.)
T: Did you check the answer? Was it correct?
S: Yes.
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients reasoning about the decomposition of successive remainders in each place value.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1, was it ever necessary to adjust your quotient after estimating? If so, what did you do in order to continue dividing?
- While checking your work today, did anyone discover an error in his or her division? If so, how did you fix it? How did you know what to do?
- Explain your thought process as you solved Problem 1(f). What were you thinking as you recorded a digit in the ones place of your quotient and recorded the remainder? Was anyone tempted to say the answer was 4 with a remainder of 14?
- Talk to your partner about how you set up and solved Problem 2. What was your thinking like? How could you use your thinking to solve $660 \div 48$ or $661 \div 48$ or $662 \div 48$, etc.? What would the total need to be in order to have a quotient of exactly 13?
- What did you have to do in order to solve Problem 3(b)? Talk with a neighbor.
- How did estimation help you to divide today?

**Exit Ticket (3 minutes)**

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Name _____________________________ Date __________________

1. Divide, then check using multiplication. The first one is done for you.
   a. \( 580 \div 17 \)
      
      \[
      \begin{array}{c|c}
        & \quad 3 & 4 \quad \text{R}2 \\
        17 & 5 & 8 \\
        \hline
        7 & 7 & 0 \\
        \hline
        6 & 8 & 0 \\
        \hline
        2 & \quad \text{Check:} & \quad 3 \, 4 \times 17 = 578 \quad 578 + 2 = 580 \\
      \end{array}
      \]

   b. \( 730 \div 32 \)

   c. \( 940 \div 28 \)

   d. \( 553 \div 23 \)

   e. \( 704 \div 46 \)
f. $614 \div 15$

2. Halle solved $664 \div 48$ below. She got a quotient of 13 with a remainder of 40. How could she use her work below to solve $659 \div 48$ without redoing the work? Explain your thinking.

```
4 8 1 3
6 6 4
- 4 8
 1 8 4
- 1 4 4
  4 0
```

3. 27 students are learning to make balloon animals. There are 172 balloons to be shared equally among the students.
   a. How many balloons are left over after sharing them equally?
   
   b. If each student needs 7 balloons, how many more balloons are needed? Explain how you know.
Name ____________________________ Date _________________

1. Divide, then check using multiplication.
   
   a. \(413 \div 19\)

   b. \(708 \div 67\)
Name ___________________________________________ Date ____________________

1. Divide, then check using multiplication. The first one is done for you.

a. \(487 \div 21\)
   \[
   \begin{array}{c|cc}
   2 & 3 & R4 \\
   \hline
   2 & 1 & 4 \hline
   \end{array}
   \]
   \[
   \begin{array}{c|cc}
   \hline
   \hline
   - & 4 & 2 \hline
   \hline
   \hline
   6 & 7 \\
   \hline
   \hline
   6 & 3 \hline
   \end{array}
   \]
   \[
   \begin{array}{c}
   4 \\
   \end{array}
   \]
   \[
   \begin{array}{c}
   \text{Check:} \\
   21 \times 23 = 483 \\
   483 + 4 = 487
   \end{array}
   \]

b. \(485 \div 15\)
   \[
   \begin{array}{c|c}
   2 & 3 \\
   \hline
   2 & 1 \hline
   \hline
   - & 4 & 2 \hline
   \hline
   \hline
   6 & 7 \\
   \hline
   \hline
   6 & 3 \hline
   \end{array}
   \]
   \[
   \begin{array}{c}
   4 \\
   \end{array}
   \]

b. \(700 \div 21\)

b. \(399 \div 31\)

b. \(820 \div 42\)
f. \(908 \div 56\)

2. When dividing 2,458 by 51, a student finds a quotient of 48 with a remainder of 11. Check the student’s work, and use the check to find the error in their solution.

3. A baker was going to arrange 432 desserts into rows of 28. The baker divides 432 by 28 and gets a quotient of 15 with remainder 12. Explain what the quotient and remainder represent.
Lesson 23

Objective: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients, reasoning about the decomposition of successive remainders in each place value.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (5 minutes)
- Concept Development (33 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)

- Divide Decimals 5.NBT.7 (3 minutes)
- Rename Tenths and Hundredths 5.NBT.2 (4 minutes)
- Divide by Two-Digit Numbers 5.NBT.6 (5 minutes)

Divide Decimals (3 minutes)

Materials: (S) Personal whiteboards

Note: This drill will prepare students for Content Development in G5–M2–Lesson 24.

Repeat the process from G5–M2–Lesson 22 for the following possible sequence: 6 tens ÷ 3, 6 tenths ÷ 3, 6 hundredths ÷ 3, 9 thousands ÷ 3, 9 hundreds ÷ 3, 9 hundredths ÷ 3, and 9 tenths ÷ 3.

Rename Tenths and Hundredths (4 minutes)

Materials: (S) Personal whiteboards

Note: This drill will prepare students for estimating decimal quotients in G5–M2–Lesson 25.

T: I’ll say a number, and you state it as a decimal. 1 tenth.
S: 0.1.
Repeat the process for 2 tenths, 3 tenths, 8 tenths, and 9 tenths.

T:  (Write 10 tenths =.) Write the number.
S:  (Write 1.)

Repeat the process for 11 tenths, 19 tenths, 20 tenths, 30 tenths, 80 tenths, 90 tenths, 100 tenths, and 200 tenths.

Repeat the process for 1 hundredth, 2 hundredths, 3 hundredths, 8 hundredths, 9 hundredths, 10 hundredths, 20 hundredths, 30 hundredths, 90 hundredths, 100 hundredths, 200 hundredths, 900 hundredths, 1,000 hundredths, and 2,000 hundredths.

**Divide by Two-Digit Numbers (5 minutes)**

Materials: (S) Personal white boards

Note: This drill will review G5–M2–Lesson 22 content.

Repeat the process from G5–M2–Lessons 21 and 22 for the following possible sequence: 650 ÷ 16, 740 ÷ 32, and 890 ÷ 27.

**Application Problem (5 minutes)**

The rectangular room measures 224 square feet. One side of the room is 14 feet long. What is the perimeter of the room?

Note: This Application Problem builds on the previous day’s lesson involving three-digit totals divided by two-digit divisors. It also provides a review of area and is a two-step problem.

**Concept Development (33 minutes)**

Materials: (S) Personal white boards

**Problem 1: 6,247 ÷ 29**

T:  Can we divide 6 thousands by 29?
S:  Not without changing them to 60 hundreds.
T:  Ok, then work with 62 hundreds, which we can divide into 29 groups or groups of 29.
T:  Divide 62 hundreds by 29. Show me how to estimate 62 hundreds divided by 29.
S:  60 hundreds ÷ 30 = 2 hundreds.
T:  Record 2 in the hundreds place of the quotient.
T:  What is 2 hundreds x 29? Solve on your whiteboard.
Lesson 23

Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients reasoning about the decomposition of successive remainders in each place value.

Date: 7/4/13

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NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

While estimating, it is fair to assume that not all students in every class will agree to round the dividend and divisor in the same way. For example, in Problem 1, some students may want to estimate $6,300 \div 30 = 210$, while others may see $6,000 \div 25 = 240$, and the majority will probably want to estimate $6,000 \div 30$. The intent here is not to rob students of their number sense, or pigeonhole them into estimating one way, but rather to cultivate their sense of how numbers relate to one another and be able to defend why they rounded how they did. In the end, however, in order to complete the problem as a group, the teacher must decide which approximation to use for the example being done on the board.
S: When we divide 6,247 into twenty-nines we can make exactly 215 units of 29, with 12 left over. →  
Or you could think of it as sharing 6,247 into 29 groups, there are 215 in each group with 12 left over.

T: Let’s check. 215 × 29 = what? (Wait for students to solve.)

S: 6,245.

T: 6,245 + 12?

S: 6,257.

**Problem 2: 4,289 ÷ 52**

T: (Write 4,289 ÷ 52 on the board.) Let’s all complete this problem together. I’ll work on the board; you work on your personal boards.

S: (Work.)

T: First, can we divide 4 thousands by 52?

S: No, we have to decompose.

T: Yes. How many hundreds do we have?

S: 42 hundreds.

T: Can we divide 42 hundreds by 52?

S: No. We have to decompose again.

T: Ok. How many tens do we have?

S: 428 tens.

T: Good. Now, we can divide 428 tens by 52. Show me how to estimate for 428 tens divided by 29.

S: 400 tens ÷ 50 = 8 tens.

T: Record 8 in the tens place of the quotient.

T: What is 8 tens × 52?

S: 416 tens.

T: Pay attention to place value as you carefully record this.

T: (Record in algorithm.) How many tens are remaining?

S: 12 tens.

T: Decompose (regroup) those 12 tens into 120 ones plus the 9 ones in the whole. How many ones is that?

S: 129 ones.

T: Now we divide 129 ones by 52? Show me how to estimate 129 ÷ 52.

S: 100 ones ÷ 50 = 2 ones.

T: What is 2 × 52?

S: 104 ones.
Lesson 23

Divide three- and four-digit dividends by two-digit divisors resulting
in two- and three-digit quotients reasoning about the
decomposition of successive remainders in each place value.

T: 129 ones – 104 ones gives a remainder of?
S: 25 ones.

T: Are we finished or do we continue to decompose and divide? Explain.
S: We are finished. 25 is our remainder, and we don’t need to continue to decompose to the tenths
place.

T: Did you check your answer? Was it correct?
S: Yes.

Problem 3: 6,649 ÷ 63

T: (Write 6,649 ÷ 63 on the board.)
Solve this problem with a partner.
As you finish each step share your
thinking with your partner.

S: (Work while teacher circulates and
assists where necessary.)

T: OK. Let’s share your work. How
did you first estimate to begin
dividing?
S: 60 hundreds ÷ 60 = 1 hundred.
T: 1 hundred times 63 equals?
S: 63 hundreds.
T: How many hundreds remain?
S: 3 hundreds.
T: What did you do next?
S: Regrouped the 3 hundreds and made 30 tens and combined them with the 4 tens in the whole to
make 34 tens.
T: Can we divide 34 tens by 63?
S: No. We have to decompose.
T: Yes. Record 0 in the tens place of the quotient. Now we decompose; what’s 340 ones plus 9 ones?
S: 349 ones.
T: How did you estimate 349 divided by 63?
S: 300 ÷ 60 = 5.
T: What’s 5 × 63?
S: 315.
T: What’s the remainder?
S: 34.
T: Did you check the answer? Was it correct?
Problem 4: $3,164 \div 45$

T: (Write $3,164 \div 45$ on the board.) Solve this problem independently. Do all three steps, estimate, solve, and check, independently. But after you finish each step, check your answer with a partner before moving on.

Follow the questioning sequence from above. Allow students to discuss the recording of 0 ones thoroughly.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients reasoning about the decomposition of successive remainders in each place value.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.
Lesson 23

Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients reasoning about the decomposition of successive remainders in each place value.

Lesson 23: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients reasoning about the decomposition of successive remainders in each place value.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Name ____________________________   Date ______________

1. Divide, then check using multiplication.
   
   a. \(4,859 \div 23\)   b. \(4,368 \div 52\)
   
   c. \(7,242 \div 34\)   d. \(3,164 \div 45\)
   
   e. \(9,152 \div 29\)   f. \(4,424 \div 63\)
2. Mr. Riley baked 1,692 chocolate cookies. He sold them in boxes of 36 cookies each. How much money did he collect if he sold them all at $8 per box?

3. 1,092 flowers are arranged into 26 vases, with the same number of flowers in each vase. How many flowers would be needed to fill 130 such vases?

4. The elephant’s water tank holds 2,560 gallons of water. After two weeks, the zookeeper measures and finds that the tank only has 1,934 gallons of water left. If the elephant drinks the same amount of water each day, how many days will a full tank of water last?
Lesson 23 Exit Ticket

1. Divide, then check using multiplication.
   
   a. \(8,283 \div 19\)
   
   b. \(1,056 \div 37\)
Lesson 23 Homework

Name ___________________________________________ Date ________________________

1. Divide, then check using multiplication.

   a. 9,962 ÷ 41 
   b. 1,495 ÷ 45 
   c. 6,691 ÷ 28 
   d. 2,625 ÷ 32 
   e. 2,409 ÷ 19 
   f. 5,821 ÷ 62
2. A political gathering in South America held 788 people. Each of South America’s 14 countries were equally represented. The remaining people were guests from the United States. How many guests were from the United States?

3. A chocolate company is packaging 32 ounces of caramels into reusable, plastic cups. When a shipping box is filled with these caramel packages, it weighs 49 pounds 8 ounces.

   a. How many caramel filled cups are in the box?

   b. Use your remainder to find the weight of each plastic cup.