# Topic C: Ratios and Rates Involving Fractions

**7.RP.1, 7.RP.3, 7.EE.4a**

<table>
<thead>
<tr>
<th>Focus Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.RP.1</td>
<td>Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently $2$ miles per hour.</td>
</tr>
<tr>
<td>7.RP.3</td>
<td>Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</td>
</tr>
<tr>
<td>7.EE.4a</td>
<td>Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</td>
</tr>
<tr>
<td></td>
<td>a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is $54$ cm. Its length is $6$ cm. What is its width?</td>
</tr>
</tbody>
</table>

**Instructional Days: 5**

- Lessons 11–12: Ratios of Fractions and Their Unit Rates (P)¹
- Lesson 13: Finding Equivalent Ratios Given the Total Quantity (P)
- Lesson 14: Multistep Ratio Problems (P)
- Lesson 15: Equations and Graphs of Proportional Relationships Involving Fractions (P)

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¹ Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson
In the first two lessons of Topic C, students’ knowledge of unit rates for ratios and rates are extended by considering applications involving fractions, such as a speed of $\frac{1}{2}$ mile per $\frac{1}{4}$ hour. Students continue to use the structure of ratio tables to reason through and validate their computations of rate. In Lesson 13, students continue to work with ratios involving fractions as they solve problems where a ratio of two parts is given along with a desired total quantity. Students can choose a representation that most suits the problem and their comfort levels, such as tape diagrams, ratio tables, or possibly equations and graphs, as they solve these problems, reinforcing their work with rational numbers. In Lesson 14, students solve multistep ratio problems, which include fractional markdowns, markups, commissions, and fees. In the final lesson of the topic, students focus their attention on using equations and graphs to represent proportional relationships involving fractions, reinforcing the process of interpreting the meaning of points on a graph in terms of the situation or context of the problem.
Lesson 11: Ratios of Fractions and Their Unit Rates

Student Outcomes

- Students use ratio tables and ratio reasoning to compute unit rates associated with ratios of fractions in the context of measured quantities such as recipes, lengths, areas, and speed.
- Students work together and collaboratively to solve a problem while sharing with the class their thinking process, strategies, and solutions.

Classwork

Example 1 (25 minutes): Who is Faster?

During their last workout, Izzy ran 2 ¼ miles in 15 minutes and her friend Julia ran 3 ¾ miles in 25 minutes. Each girl thought they were the faster runner. Based on their last run, which girl is correct? Use any approach to find the solution.

Even if one of the approaches were not taken or a student took a different approach go through ALL possible ways as a class as shown below (bar models, Equations, Number Line, Clocks). Each approach reviews and teaches different concepts that are needed for the “big” picture. Starting with tables will not only reinforce all of the previous material but also review and address concepts required for the other possible approaches.

Note: Time can be represented in either hours or minutes, the solutions show both.

<table>
<thead>
<tr>
<th>Izzy</th>
<th></th>
<th></th>
<th>Julia</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time (minutes)</strong></td>
<td><strong>Time (hours)</strong></td>
<td><strong>Distance (miles)</strong></td>
<td><strong>Time (minutes)</strong></td>
</tr>
<tr>
<td>15</td>
<td>¼</td>
<td>2 ¼</td>
<td>25</td>
</tr>
<tr>
<td>30</td>
<td>½</td>
<td>4 ½</td>
<td>50</td>
</tr>
<tr>
<td>45</td>
<td>¾</td>
<td>6 ¾</td>
<td>75</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>9</td>
<td>100</td>
</tr>
<tr>
<td>75</td>
<td>1 ¼</td>
<td>11 ¼</td>
<td></td>
</tr>
</tbody>
</table>

Scaffolding:

- It may be helpful to draw a clock or continually refer to a clock. Many students have difficulty telling time with the new technology available to them.
- Also, may be helpful to do an example similar to the first example but uses whole numbers.
When looking and comparing the tables, it appears that Julia went further so that would mean she went faster. Is that assumption correct? Explain your reasoning.

By creating a table of equivalent ratios for each runner showing the elapsed time and corresponding distance run, it may be possible to find a time or distance that is common to both tables. The student can see if one girl had a greater distance for a given time or if one girl had a lesser time for a given distance. In this case, at \( t = 75 \) min both girls would have run 11 \( \frac{1}{4} \) miles assuming constant speed.

How can we use the tables produced to determine the unit rate?

Since we assumed distance is proportional to time you can find the unit rate or constant of proportionality by dividing the distance by the time. If you used time in hours then you will find the unit rate to be 9 miles/hr. If you used time in minutes, then the unit rate would be \( \frac{3}{20} \) miles per minute.

Discuss: Some students may have chosen to calculate the unit rates for each of the girls. To calculate the unit rate for Izzy, students divided the distance run by the elapsed time which gives us the ratio \( \frac{2.75}{15/60} \) which is 9. To find the unit rate for Julia, students divided \( \frac{3.75}{25/60} \) and arrived at a unit rate of 9 as well, leading the students to conclude that neither girl was the fastest.

We all agree that the girls ran at the same rate; however, some members of the class identified the unit rate as 9 while others gave a unit rate of 3/20. How can both groups of students be correct?

Time can be represented in minutes; however, in the real-world, most people are comfortable with distance measured by hours. It is easier for a person to visualize 9 miles per hour compared to 3/20 miles per minute. It is an acceptable answer provided the labels are correct.

Scaffolding:

Review how to divide fractions using a bar model.

How can you divide fractions with a picture, using a bar model?

Make 2 whole blocks and a third whole block broken into fourths. Then, divide the wholes into fourths and count how many fourths there are in the original 2 \( \frac{3}{4} \) blocks. The answer would be 9.

1. Green Blocks are the original 2 \( \frac{3}{4} \) blocks (1st diagram)
2. Divide the whole blocks into \( \frac{1}{4} \) (2nd diagram)
3. How many blocks are there? (green blocks = 9)

More practice if needed with bar models

1. Make 1 \( \frac{3}{4} \) blocks, represented by green blocks
2. Divide the blocks into groups of \( \frac{1}{2} \)
3. The number of \( \frac{1}{2} \) that are shaded are 3 \( \frac{1}{2} \)

We all agree that the girls ran at the same rate; however, some members of the class identified the unit rate as 9 while others gave a unit rate of 3/20. How can both groups of students be correct?

Time can be represented in minutes; however, in the real-world, most people are comfortable with distance measured by hours. It is easier for a person to visualize 9 miles per hour compared to 3/20 miles per minute. It is an acceptable answer provided the labels are correct.

1. Make 2 \( \frac{1}{3} \) blocks, represented by green blocks
2. Divide the whole blocks into groups of \( \frac{1}{6} \)
3. The number of \( \frac{1}{6} \) that are shaded is 14.
Equations:

\[
\begin{align*}
\text{Izzy} & \\
\frac{d}{t} &= r \\
2 \frac{1}{4} &= r \cdot \frac{1}{4} \\
4 \frac{2}{4} &= r \cdot \frac{4}{4} \\
9 &= r \\
9 \text{ miles/hour}
\end{align*}
\]

\[
\begin{align*}
\text{Julia} & \\
\frac{d}{t} &= r \\
3 \frac{3}{4} &= r \cdot \frac{25}{60} \\
60 \frac{3}{4} &= r \cdot \frac{25}{60} \\
9 &= r \\
9 \text{ miles/hour}
\end{align*}
\]

- What assumptions are made when using the formula \( d = rt \) in this problem?
  - We are assuming the distance is proportional to time and that Izzy and Julia run at a constant rate. This means they run the same speed the entire time, not slower at one point and faster at another.

Picture:
- Some students may decide to draw a clock.
- Possible student explanation:
  - For Izzy, every 15 minutes will give 2 \( \frac{1}{4} \) miles. So, if you divide the clock into 15 minute intervals, then you can add the distance for each 15 minute interval until you get 60 minutes which is your unit rate since 60 minutes = 1 hour. For Julia, you are given 3 \( \frac{3}{4} \) miles for 25 minutes so I divided the clock into 25 minute intervals. This will give 50 minutes. For the remaining 10 minutes I broke it up into 5 minute intervals. To figure out the amount for each 5 minute interval I found how much one five minute interval was when given 25 minutes = 3 \( \frac{3}{4} \). I found 1 five minute interval to be \( \frac{3}{4} \) mile.

Total Distance for 1 hour (unit rate)

Izzy: \( 2 \frac{1}{4} + 2 \frac{1}{4} + 2 \frac{1}{4} + 2 \frac{1}{4} = 9 \)

Julia: \( 3 \frac{3}{4} + 3 \frac{3}{4} + \frac{1}{4} + \frac{1}{4} = 9 \)
Double Number Line Approach:

Izzy          Julia

- How do you find the value of a 5-minute time increment? What are you really finding?
  - To find the value of a 5-minute increment, you need to divide 3 ¾ by 5 since 25 minutes is 5 – 5-minute increments. This is finding the unit rate for a 5-minute increment.

- Why were 5-minute time increments chosen?
  - 5-minute time increments were chosen for a few reasons. First, a clock is broken into 5 minute intervals so it may be easier to visualize what fractional part of an hour one has when given in a 5 minute interval. Also, 5 is the greatest common factor of the two given times.

- What if the times had been 24 and 32? How about 18 and 22?
  - If it were 24 and 32 minutes then the time increment would be 8-minute intervals. This is because 8 is the greatest common factor of 24 and 32.
  - If the times were 18 and 22 then the comparison should be broken into 2-minute intervals since the greatest common factor of 18 and 22 is 2.

Exercises (10 minutes)

1. A turtle walks ⅞ of a mile in 50 minutes. What is the unit rate in miles per hour?
   a. To find the turtle’s unit rate, Meredith wrote and simplified the following complex fraction. Explain how the fraction \( \frac{7}{6} \) was obtained.

      \[
      \frac{\frac{7}{6}}{\frac{24}{24}} = \frac{7 \cdot 3}{5 \cdot 4} = \frac{21}{20}
      \]
      \[
      \left( \frac{\frac{3}{8}}{\frac{24}{24}} \right) = \frac{21}{20}
      \]

      Since the unit rate is in miles per hour the 50 minutes needs to be converted to hours. Since 60 minutes is equal to 1 hour, 50 minutes can be written as \( \frac{50}{60} = \frac{5}{6} \)

   b. Did Meredith simplify the complex fraction correctly? Explain how you know.

      Yes she multiplied the fraction by 24/24 which doesn’t change the value but simplifies to 21/20.
2. For Anthony’s birthday his mom is making cupcakes for his 12 friends at his daycare. The recipe calls for 3 ⅓ cups of flour. This recipe makes 2 ½ dozen cookies. Anthony’s mom has only 1 cup of flour. Is there enough flour for each of his friends to get a cupcake? Explain and show your work.

\[
\frac{\text{cups}}{\text{dozen}} \quad \frac{3 \frac{1}{3}}{2 \frac{1}{2}} = 10 \times \frac{2}{5} = \frac{20}{1} = \frac{1}{3} \quad \text{cups/dozen}
\]

OR

\[
\frac{\text{cups}}{\text{dozen}} \quad \frac{3 \frac{1}{3}}{2 \frac{1}{2}} = \frac{10}{3} \times \frac{6}{5} = \frac{20}{15} = \frac{1}{3} \quad \text{cups/dozen}
\]

No. Since Anthony has 12 friends, he would need 1 dozen cupcakes. This means you need to find the unit rate. Finding the unit rate will tell us how much flour his mom needs for 1 dozen cupcakes. Upon finding the unit rate Anthony’s mother would need 1 ⅔ cups of flour so she does not have enough flour to make cupcakes for all his friends.

Scaffolding:
- For advanced learners: Ask students to calculate how many cupcakes his mother would be able to make with 1 cup of flour? Remind students that there are 12 items in a dozen.

3. Sally is making a painting where she is mixing red paint and blue paint. The table below shows the different mixtures used.

<table>
<thead>
<tr>
<th>Red Paint (Quarts)</th>
<th>Blue Paint (Quarts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \frac{2}{5} )</td>
<td>4</td>
</tr>
<tr>
<td>( \frac{3}{4} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>1.2</td>
<td>2</td>
</tr>
<tr>
<td>1.8</td>
<td>3</td>
</tr>
</tbody>
</table>

What are the unit rates for the values?

\[ \frac{5}{3} = 1 \frac{2}{3} \]

a. Is the amount of blue paint proportional to the amount of red paint?

Yes, blue paint is proportional to red paint because there exists a constant, \( 1 \frac{2}{3} \), such that when each amount of red paint is multiplied by the constant, the corresponding amount of blue paint is obtained.

b. Describe, in words, what the unit rate means in the context of this problem.

For every \( 1 \frac{2}{3} \) quarts of blue paint, she must use 1 quart of red paint.
Lesson Summary:

A fraction whose numerator or denominator is itself a fraction is called a complex fraction.

Recall: A unit rate is a rate which is expressed as \( \frac{A}{B} \) units of the first quantity per 1 unit of the second quantity for two quantities \( A \) and \( B \).

For example: If a person walks \( 2 \frac{1}{2} \) miles in \( 1 \frac{3}{4} \) hours at a constant speed, then the unit rate is \( \frac{2 \frac{1}{2}}{1 \frac{3}{4}} = \frac{\frac{5}{2}}{\frac{7}{4}} \).

\[
\frac{5 \cdot 4}{2 \cdot 5} = 2. \quad \text{The person walks 2 mph.}
\]
Lesson 11: Ratios of Fractions and Their Unit Rate

Exit Ticket

Which is the better buy? Show your work and explain your reasoning.

3 ⅓ lbs of turkey for ten and one-half dollars

2 ⅖ lbs of turkey for six and one-quarter dollars
Exit Ticket Sample Solutions

The following responses indicate an understanding of the objectives of this lesson:

Which is the better buy? Show your work and explain your reasoning.

3 ⅓ lbs of turkey for ten and one-half dollars

\[ \frac{10 \frac{1}{2}}{3 \frac{1}{3}} = 3.15 \]

2 ⅔ lbs of turkey for six and one-quarter dollars

\[ \frac{6 \frac{1}{4}}{2 \frac{1}{2}} = 2.50 \]

2 ⅔ lb is the better buy because the price per pound is cheaper.

Problem Set Sample Solutions

1. Simplify: \( \frac{2}{7} + \frac{3}{6} \)

\[ \frac{5}{7} \]

2. One lap around a dirt track is \( \frac{1}{3} \) mile. It takes Bryce \( \frac{1}{9} \) hour to ride one lap. What is Bryce’s unit rate around the track?

3 miles / hour

3. Mr. Gengel wants to make a shelf with boards that are \( \frac{1}{3} \) feet long. If he has an 18 foot board, how many pieces can he cut from the big board?

13 ½ boards

4. The local bakery uses 1.75 cups of flour in each batch of cookies. The bakery used 5.25 cups of flour this morning.

a. How many batches of cookies did the bakery make?

3 batches

b. If there are 5 dozen cookies in each batch, how many cookies did the bakery make yesterday?

5(12) = 60 cookies per batch

60(3) = 180 cookies in 3 batches

5. Jason eats 10 ounces of candy in 5 days.

a. How many pounds will he eat per day? (16 ounces = 1 pound)

\( \frac{1}{8} \) lb each day

b. How long will it take Jason to eat 1 pound of candy?

8 days
Lesson 12: Ratios of Fractions and Their Unit Rates

Student Outcomes

- Students use ratio tables and ratio reasoning to compute unit rates associated with ratios of fractions in the context of measured quantities such as recipes, lengths, areas, and speed.
- Students use unit rates to solve problems and analyze unit rates in the context of the problem.

Classwork

Example 1 (25 minutes) Time to Remodel

Students are given the task of determining the cost of tiling a rectangular room. The students are given the dimensions of the room, the area, in square feet, of one tile, and the cost of one tile.

You have decided to remodel your bathroom and put tile on the floor. The bathroom is in the shape of a rectangle and the floor measures 14 feet 8 inches long, 5 feet 6 inches wide. The tile you want to use costs $5 each and each tile covers \( \frac{4}{3} \) square feet. If you have $100 to spend, do you have enough money to complete the project?

- Make a Plan: Decide what the necessary steps are to finding the solution and complete the chart.
- If students are unfamiliar with completing a chart like this one, guide them in completing the first row.

<table>
<thead>
<tr>
<th>What I Know</th>
<th>What I Want to Find</th>
<th>How to Find it</th>
</tr>
</thead>
</table>
| **Dimensions of the Floor** | **Area** | \( 1 \) – Convert inches to feet as a fraction over 12  
\( 2 \) – Area = length x width |
| **Square Foot of 1 Tile** | **Number of Tiles Needed** | Area divided by the area of 1 tile |
| **Cost of 1 Tile** | **Total Cost of all tiles** | Multiply the total amount of tiles by the cost of one (\$5). |
| **Have $100** | **Is this enough?** | \( \$100 \) – total money to spend  
If the total cost is more than \$100 then there is not enough money |

Compare your plan with a partner. Using your plans, work together to determine how much money you will need to complete the project and if you have enough money.
### Lesson 12: Ratios of Fractions and Their Unit Rates

**Pending Final Editorial Review**

<table>
<thead>
<tr>
<th>1: Area</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dimensions</strong></td>
</tr>
<tr>
<td>5 ft 6 in = 5 ( \frac{1}{2} )</td>
</tr>
<tr>
<td>14 ft 8 in = 14 ( \frac{2}{3} )</td>
</tr>
<tr>
<td><strong>Area (sq. feet)</strong></td>
</tr>
<tr>
<td>( A = lw )</td>
</tr>
<tr>
<td>( A = \left( 5 \frac{1}{2} \right) \left( 14 \frac{2}{3} \right) )</td>
</tr>
<tr>
<td>( A = \left( 5 \frac{1}{2} \right) \left( 14 \frac{2}{3} \right) )</td>
</tr>
<tr>
<td>( A = \frac{11}{2} \left( \frac{44}{3} \right) )</td>
</tr>
<tr>
<td>( A = \frac{242}{3} = 80 \frac{2}{3} \text{ sq ft} )</td>
</tr>
</tbody>
</table>

**Number of tiles:**

\[
\frac{80\frac{2}{3}}{42} = \frac{242}{14} = 17 \frac{2}{7}
\]

*Can’t buy part of a tile so you will need to purchase 18 tiles*

**Total Cost:** \( 18(5) = $90 \)

**Enough Money?:** Yes since the total is less than $100, there is enough money.

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**Discussion Questions:**

- **Why was the mathematical concept of area and not perimeter or volume used?**
  - *Area was used because we were “filling” in the rectangular space of the floor. Area is 2-dimensional and we needed two dimensions, length and width of the room, to calculate the area of the floor. If we were just looking to put trim around the outside then we would use perimeter. If we were looking to fill the room from floor to ceiling then we would use volume.*

- **Why would 5.6 inches and 14.8 inches be incorrect representations for 5 feet 6 inches and 14 2/3 feet?**
  - *The relationship between feet and inches is 12 inches = 1 foot. To convert you need to figure out what fractional part 6 inches is of a foot or 12 inches. If you just wrote 5.6 then you would be basing the inches out of 10 not 12. The same holds true for 14 feet 8 inches.*

- **How is the unit rate useful?**
  - *The unit rate is given, 4 2/3 square feet per one tile. We can find the total number of tiles needed by dividing the total square footage by the unit rate.*

- **Can I buy 17 2/7 tiles?**
  - *No you have to buy whole tiles and cut what you may need.*

- **How would rounding to 17 tiles compared to 18 tiles affect the job?**
  - *Even though the rules of rounding would say round down to 17 tiles we would not in this problem. If we round down then the entire floor would not be covered, we would be a little short if we round up to 18 tiles the entire floor would be covered with a little extra.*
Exercises (10 minutes)

1. Which car can travel further on 1 gallon of gas?

   **Blue Car:** Travels \( \frac{18}{2} \) miles using 0.8 gallons of gas
   
   **Red Car:** Travels \( \frac{17}{2} \) miles using 0.75 gallons of gas

   \[
   \text{Blue Car:} \quad \frac{18}{2} = \frac{92}{4} = 23
   \]
   
   \[
   \text{Red Car:} \quad \frac{17}{2} = \frac{87}{3} = 23 \frac{1}{5}
   \]

   \(23 \text{ miles} / \text{ gallon} \)

   \(23 \frac{1}{5} \text{ miles/gallon}\)

   **The red car traveled 1/5 mile further on one gallon of gas.**

Closing (5 minutes)

- How can unit rates with fractions be applied in the real world?

Exit Ticket (5 minutes)

**Scaffolding:**

Since the students are a young age they may not be familiar with cars, distance, miles per gallon relationships. Students may select the car with the lower unit rate because they may be confused with the better buy and lower unit prices. Further clarification may be needed to explain the more miles per gallon the better.
Lesson 12: Ratios of Fractions and Their Unit Rates

Exit Ticket

If \( \frac{3}{4} \) lb of candy cost $20.50, how much would 1 pound of candy cost?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

If \( \frac{3}{4} \) lb of candy cost $20.50, how much would 1 pound of candy cost?

\[
27 \frac{1}{3} = 27.3.
\]

Students may find the unit rate by first converting 20.50 to \( \frac{41}{2} \) and dividing it by \( \frac{3}{4} \). Students may also divide 20.50 by 3 because it represents 3 parts of the total. Once finding the quotient, the student may then add this to the 20.50 to get \( 27.3 \) per pound.

Problem Set Sample Solutions

1. You are getting ready for a family vacation. You decide to download as many movies as possible before leaving for the road trip. If each movie takes \( 1 \frac{2}{5} \) hours to download and you downloaded for 5 \( \frac{3}{4} \) hours, how many movies did you download?
   
   \( 3 \frac{3}{4} \) movies

2. The area of a blackboard is \( 1 \frac{1}{3} \) square yards. A poster's area is \( \frac{8}{9} \) square yards. Find a unit rate and explain, in words, what the unit rate means in the context of this problem. Is there more than one unit rate that can be calculated? How do you know?
   
   \( 1 \frac{1}{2} \) - The area of the blackboard is \( 1 \frac{1}{2} \) time the area of the poster. Another possible answer: \( 2/3 \) the area of the poster is \( 2/3 \) the area of the blackboard.

3. A toy remote control jeep is 12 \( \frac{1}{2} \) inches wide while an actual jeep is pictured to be 18 \( \frac{3}{4} \) feet wide. What is the value of the ratio of the width of the remote control jeep to width of the actual jeep?
   
   \( \frac{1}{18} \), convert 18 \( \frac{3}{4} \) feet to 225 inches.

4. \( \frac{1}{3} \) Cup of flour is used to make 5 dinner rolls.
   
   a. How many cups of flour are needed to make 3 dozen dinner rolls?
      
      \( 2 \frac{2}{5} \) cups
   
   b. How many rolls can you make with \( \frac{2}{3} \) cups of flour?
      
      \( 85 \) rolls
   
   c. How much flour is needed to make one dinner roll?
      
      \( \frac{1}{15} \) cup
Lesson 13: Finding Equivalent Ratios Given the Total Quantity

Student Outcomes

- Students use tables to find an equivalent ratio of two partial quantities given a part-to-part ratio and the total of those quantities, in the third column, including problems with ratios of fractions.

Classwork

Example 1 (25 minutes)

Have students work in partners to complete the chart below. NOTE: Teacher may allow students to utilize a calculator to assist in the multiplication step of converting mixed numbers to improper fractions.

**Scaffolding:**
Review 16 oz. = 1 lb.

A group of 6 hikers are preparing for a one-week trip. All of the group's supplies will be carried by the hikers in backpacks. The leader decided that it would be fair for each hiker to carry a backpack that is the same fraction of his weight as all of the other hikers'. In this set-up, the heaviest hiker would carry the heaviest load. The table below shows the weight of each hiker and the weight of his/her backpack.

Complete the table. Find the missing amounts of weight by applying the same ratio as the first 2 rows.

<table>
<thead>
<tr>
<th>Hiker's Weight</th>
<th>Backpack Weight</th>
<th>Total Weight (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>152 lbs 4 oz</td>
<td>14 lbs 8 oz</td>
<td>166 3/4</td>
</tr>
<tr>
<td>152 1/4</td>
<td>141/2</td>
<td></td>
</tr>
<tr>
<td>107 lbs 10 oz</td>
<td>10 lbs 4 oz</td>
<td>117 7/8</td>
</tr>
<tr>
<td>107 5/8</td>
<td>10 1/4</td>
<td></td>
</tr>
<tr>
<td>129 lbs 15 oz</td>
<td>12 lbs 3 oz</td>
<td>142 5/16</td>
</tr>
<tr>
<td>129 15/26</td>
<td>12 3/8</td>
<td></td>
</tr>
<tr>
<td>68 lbs 4 oz</td>
<td>8 lbs 12 oz</td>
<td>100 5/8</td>
</tr>
<tr>
<td>68 1/4</td>
<td>8 3/4</td>
<td></td>
</tr>
<tr>
<td>91 7/8</td>
<td>8 3/4</td>
<td></td>
</tr>
<tr>
<td>10 lbs</td>
<td>10 lbs</td>
<td>115</td>
</tr>
</tbody>
</table>

**Scaffolding:**
May need to review abbreviations for pounds (lbs.) and ounces (oz.)
What challenges did you encounter when calculating the missing values?

- Remembering the conversions of ounces to pounds, and also dividing of fractions.

How is a third column representing the total quantity found and how is it useful?

- To find the third column you need to add the total weight of both the hiker and the backpack. The third column giving the total allows one to compare the overall quantities. Also if the total and ratio is known then you can find the all the quantities.

When a table is given with the 3rd column, how do you fill in the missing pieces?

- In order to find the third column you need the first two columns or the ratio of the first two columns. If the third column is the total then add the first two columns.

When a table is given and either of the first two columns is missing, how do you complete the table?

- If one of the first two columns is missing, you need to look at the rest of the table to determine the constant rate or ratio. You can either set up proportions (if students recall this from 6th grade), write an equation of the relationship then substitute in or write an equivalent ratio of the unknown to the constant of proportionality.

Based on the given values and found values is the backpack weight proportional to the hikers weight? How do you know?

- The table shows the backpack weight proportional to the hiker’s weight because there exists a constant, 2/21 that when each measure of the hiker’s weight is multiplied by the constant gives the corresponding weight of the backpack.

Would these two quantities always be proportional to each other?

- Not necessarily. The relationship between backpack weight and hiker’s weight will not always be in the ratio 2/21 but these 6 hikers were.

Describe how to use different approaches to finding the missing values of either quantity.

- Writing equations, setting up proportions or writing equivalent ratios can be used.

Describe the process of writing and using equations to find the missing values of a quantity.

- First find the constant of proportionality or unit rate.
- Once that is found then set up an equation in the form \( y = kx \), replacing \( k \) with the constant of proportionality.
- Substitute the known value in for the variable and solve for the unknown.
PENDING FINAL EDITORIAL REVIEW

- When writing equations to find the missing value(s) of a quantity, are we restricted to using the variables \(x\) and \(y\)? Explain.
  - *No any variable can be used. Often using a variable to represent the context of the problem makes it easier to know which variable to replace with the known value. For instance if the two quantities are hours and pay, one may use the variable \(p\) to represent pay instead of \(y\) and \(h\) to represent hours instead of \(x\).*

- Describe the process of using proportions to find the missing value of a quantity.
  - *Find the constant of proportionality or unit rate. Set up a proportion comparing the two quantities. One side of the proportion is the unit rate and the other includes the given value and the unknown variable.*

- Describe the process of writing equivalent ratios to find the missing value(s) of a quantity. How is this method similar and different to writing proportions?
  - *Start with the unit rate or constant of proportionality. Determine what variable is known and determine what you must multiply by to obtain the known value. Multiply the remaining part of the unit rate by the same number to get the value of the unknown value.
  - *Writing proportions is writing two equivalents ratios. The difference between the second and third methods is the process of solving. Students may recall to “cross multiply” from previous knowledge as in the 2nd approach or multiplying by a common factor of the unknown.*

- What must be known in order to find the missing value(s) of a quantity regardless of what method is used?
  - *The ratio of the two quantities must be known.*

- If the ratio of the two quantities and the total amount are given, can you find the remaining parts of the table?
  - *Yes, once the ratio is determined or given find an equivalent ratio to the given ratio that also represents the total amount.*

Have students extend the table. Direct them to create a total amount and instruct them to find the two missing quantities.

**Example 2**

When a business buys a fast food franchise, it is buying the recipes used at every restaurant with the same name. For example, all Pizzeria Specialty House Restaurants have different owners but they must all use the same recipes for their pizza, sauce, bread, etc. You are now working at your local Pizzeria Specialty House restaurant and listed below are the amounts of meat used on one meat-lovers pizza.

\[
\begin{align*}
\frac{1}{4} \text{ cup of sausage} \\
\frac{1}{3} \text{ cup of pepperoni} \\
\frac{1}{6} \text{ cup of bacon} \\
\frac{1}{8} \text{ cup of ham} \\
\frac{1}{8} \text{ cup of beef}
\end{align*}
\]

*Scaffolding:*

May need to review solving a one-step equation requiring using the multiplicative inverse to solve.
What is the total amount of toppings used on a meat-lovers pizza?  ____1____ cups

The meat must be mixed using this ratio to ensure that customers will receive the same great tasting meat-lovers pizza from every Pizzeria Specialty House Restaurant nationwide. The table below shows 3 different orders for meat-lovers pizza on Superbowl Sunday. Using the amounts and total for one pizza given above, fill in every row and column of the table so the mixture tastes the same.

<table>
<thead>
<tr>
<th></th>
<th>Order 1</th>
<th>Order 2</th>
<th>Order 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sausage (cups)</td>
<td>1</td>
<td>1 1/2</td>
<td>2 1/4</td>
</tr>
<tr>
<td>Pepperoni (cups)</td>
<td>1 1/3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Bacon (cups)</td>
<td>2/3</td>
<td>1</td>
<td>1 1/2</td>
</tr>
<tr>
<td>Ham (cups)</td>
<td>1/2</td>
<td>3/4</td>
<td>1 1/8</td>
</tr>
<tr>
<td>Beef (cups)</td>
<td>1/2</td>
<td>3/4</td>
<td>1 1/8</td>
</tr>
<tr>
<td>TOTAL (cups)</td>
<td>4</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

What must you calculate or know to complete this table?
- You need to know how many pizzas are being made for each order.

How many pizzas were made for Order 1? Explain how you obtained and used your answer?
- There were 4 pizzas ordered in this order. The amount of sausage increased from 1/4 cup to 1 cup, which is 4 times as big. Knowing this, each ingredient can now be multiplied by 4 and the answer can be reduced to determine how much of each ingredient is needed for Order 1.

A bar model can be utilized as well:

The amount of sausage is represented by the green in the bar model. This represents 1/4 of a cup.

If the amount of sausage becomes 1 cup, then the model should represent 1 whole. (New green).

The amount of 1/4’s in 1 whole is 4.
• How many pizzas were made for Order 2? Explain how you obtained and used your answer?
  □ There were 6 pizzas ordered in this order. The amount of bacon increased from 1/6 to 1, which is 6 times as big. Each ingredient can then be multiplied by 6 and the answer reduced to determine how much is needed for the order.
  
  Bar Model:
  The amount of bacon, 1/6, is represented by the green portion in the model.

  ![Bar Model Image]

  The amount of bacon became 1 cup, so the model should represent 1 whole. (New Green)

  The number of 1/6’s in 1 whole is 6.

• How many pizzas were made for Order 3? Explain how you obtained and used your answer?
  □ There were 9 pizzas ordered in this order. The amount of pepperoni increased from 1/3 to 3 which is 9 times as big. The other ingredients can then be multiplied by 9 and the answers reduced to determine how much of each ingredient is needed for the order.

  Bar Model:
  The amount of pepperoni 1/3 is represented by the green portion in the model.

  ![Bar Model Image]

  The amount of pepperoni becomes 3 or 3 wholes so we need to draw 3 whole models, broken in thirds.

  The amount of thirds in the total models is 9.

• Is it possible to order 1 ½ or 2 ½ pizzas? If so, describe the steps to determine the amount of each ingredient necessary.
  □ Yes, pizzas can be sold by the halves. This may not be typical but it is possible. Most pizza places can put the ingredients on only half of a pizza. To determine the amount of each ingredient necessary multiply the ingredients original amount by the number of pizzas ordered.
Exercises (10 minutes)

1. The table below shows 6 different-sized pans of the same recipe for macaroni and cheese. If the recipe relating the ratio of ingredients stays the same, how might it be altered to account for the different sized pans?

<table>
<thead>
<tr>
<th>Noodles (cups)</th>
<th>Cheese (cups)</th>
<th>Pan Size (number of cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>3/4</td>
<td>3 3/4</td>
</tr>
<tr>
<td>1</td>
<td>1/4</td>
<td>1 1/4</td>
</tr>
<tr>
<td>2/3</td>
<td>1/6</td>
<td>5/6</td>
</tr>
<tr>
<td>5 1/3</td>
<td>1 1/3</td>
<td>6 2/3</td>
</tr>
<tr>
<td>4 1/2</td>
<td>1 1/8</td>
<td>5 5/8</td>
</tr>
</tbody>
</table>

Sample responses to the questions:

Method 1: Equations

Find the constant rate. To do this use the row that gives both quantities, not the total. To find the unit rate:

\[
\frac{3}{3} = \frac{1}{4}
\]

Write the equation of the relationship. \( c = \frac{1}{4}n \) where \( c \) = cups of cheese, \( n \) = cups of noodles

\[
c = \frac{1}{4}n \\
\frac{1}{4} = \frac{1}{n} \\
1 = n
\]

\[
c = \frac{1}{4}n \\
\frac{1}{4} = \frac{1}{3}n \\
1 = \frac{3}{4}n
\]

\[
c = \frac{1}{4}n \\
\frac{1}{4} = \frac{2}{3}n \\
1 = \frac{6}{3}n
\]

\[
c = \frac{1}{4}n \\
\frac{1}{4} = \frac{5}{11}n \\
1 = \frac{16}{3}n
\]

\[
c = \frac{1}{3}n \\
\frac{1}{3} = \frac{4}{1}n \\
1 = n
\]
Method 2: Proportions

Find the constant rate as describe in Method 1.

Set up proportions.

\[ y = \text{cups of cheese and } x = \text{cups of noodles} \]

\[
\frac{\frac{1}{4}}{x} = \frac{1}{4} \\
x = 1
\]

\[
y = \frac{1}{3} \\
4y = \frac{2}{3} \\
y = \frac{2}{3} \\
y = \frac{2}{3} \\
y = \frac{2}{12} = \frac{1}{6}
\]

\[
\frac{\frac{1}{4}}{y} = \frac{1}{4} \\
\frac{5}{2} = \frac{y}{4} \\
4y = \frac{5}{3} \\
y = \frac{5}{4} = \frac{16}{3} \cdot \frac{1}{4} = \frac{4}{3}
\]

Method 3: Writing Equivalent Ratios

Multiply both the numerator and denominator of the original fraction by a common fraction that will give the known value. For example, what is multiplied by 1 to get \(\frac{1}{4}\)? Multiply both the numerator and denominator by that fraction. Likewise, what is multiplied to 4 to get \(\frac{2}{3}\)? Multiply both the numerator and denominator by that fraction.

Closing (3 minutes)

- How is the 3rd column representing the total quantity used?
- Describe how you can fill in the missing information in a table that includes the total quantity.

Lesson Summary:

To find missing quantities in a ratio table where a total is given, determine the unit rate from the ratio of two given quantities and use it to find the missing quantities in each equivalent ratio.

Exit Ticket (5 minutes)
Lesson 13: Finding Equivalent Ratios Given the Total Quantity

Exit Ticket

The table below shows the combination of dry prepackaged mix and water to make concrete. The mix says for every 1 gallon of water stir 60 pounds of dry mix. We know that 1 gallon of water is equal to 8lbs. Using the information provided in the table, complete the remaining parts of the table.

<table>
<thead>
<tr>
<th>Dry Mix (pounds)</th>
<th>Water (pounds)</th>
<th>Total (pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>10</td>
<td>14 1/6</td>
</tr>
<tr>
<td>4 1/2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

The table below shows the combination of dry prepackaged mix and water to make concrete. The mix says for every 1 gallon of water stir 60 pounds of dry mix. We know that 1 gallon of water is equal to 8lbs. Using the information given in the table, complete the remaining parts of the table.

<table>
<thead>
<tr>
<th>Dry Mix (pounds)</th>
<th>Water (pounds)</th>
<th>Total (pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>8</td>
<td>68</td>
</tr>
<tr>
<td>75</td>
<td>10</td>
<td>85</td>
</tr>
<tr>
<td>12½</td>
<td>1 2/3</td>
<td>14 1/6</td>
</tr>
<tr>
<td>4 1/2</td>
<td>3 5/8</td>
<td>5 1/10</td>
</tr>
</tbody>
</table>

Problem Set Sample Solutions

1. Students in 6 classes displayed below ate the same ratio of cheese pizza slices to pepperoni pizza slices. Complete the following table which represents the number of slices of pizza students in each class ate.

<table>
<thead>
<tr>
<th>Slices of Cheese Pizza</th>
<th>Slices of Pepperoni Pizza</th>
<th>Total Pizza</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>5 1/2</td>
<td>3 3/4</td>
<td>19 1/4</td>
</tr>
<tr>
<td>3 1/3</td>
<td>1 1/3</td>
<td>11 2/3</td>
</tr>
<tr>
<td>3 5/8</td>
<td>1 1/2</td>
<td>2 1/10</td>
</tr>
</tbody>
</table>
2. To make green paint, students mixed yellow paint with blue paint. The table below shows how many yellow and blue drops from a dropper several students used to make the same shade of green paint.
   a. Complete the table.

<table>
<thead>
<tr>
<th>Yellow (Y) (ml)</th>
<th>Blue (B) (ml)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ½</td>
<td>5 ¼</td>
<td>8 ¼</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4 ½</td>
<td>6 ¾</td>
<td>11 ¼</td>
</tr>
<tr>
<td>6 ½</td>
<td>9 ¾</td>
<td>16 ¼</td>
</tr>
</tbody>
</table>

   b. Write an equation to represent the relationship between the amount of yellow paint and blue paint.

   \[ B = 1.5Y \]

3. a. Complete the following table.

<table>
<thead>
<tr>
<th>Distance Ran (miles)</th>
<th>Distance Biked (miles)</th>
<th>Total Amount of Exercise (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3 1/2</td>
<td>7</td>
<td>10 1/2</td>
</tr>
<tr>
<td>2 1/4</td>
<td>5 1/2</td>
<td>8 1/4</td>
</tr>
<tr>
<td>2 1/8</td>
<td>4 1/2</td>
<td>6 3/8</td>
</tr>
<tr>
<td>1 2/3</td>
<td>3 1/3</td>
<td>5</td>
</tr>
</tbody>
</table>

   b. What is the relationship between distances biked and distances ran?

   *Distance biked is twice the distance ran.*

4. The following table shows the number of cups of milk and flour that are needed to make biscuits. Complete the table.

<table>
<thead>
<tr>
<th>Milk (cups)</th>
<th>Flour (cups)</th>
<th>Total (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>9</td>
<td>16.5</td>
</tr>
<tr>
<td>8 1/4</td>
<td>10.5</td>
<td>19 1/4</td>
</tr>
<tr>
<td>12.5</td>
<td>15</td>
<td>27.5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>
Lesson 14: Multistep Ratio Problems

Student Outcomes

- Students will solve multi-step ratio problems including fractional markdowns, markups, commissions, fees, etc.

Lesson Notes

In this lesson, students will solve multi-step ratio problems including fractional markdowns, fractional commissions, fees, and discounts. Problems with similar context but applying percent concepts will be seen in Modules 2 and 4.

Classwork

Example 1 (30 minutes): Bargains

Begin this lesson by discussing advertising. Share with students that businesses will create an advertisement that will encourage consumers to come to their business in order to purchase their products. Many businesses subscribe to the idea that if a consumer thinks that he is saving money, then the consumer will be more motivated to purchase the product.

Have students verbalize how they would determine the sales prices with a discount rate of \( \frac{1}{2} \) off the original price of the shirt and \( \frac{1}{4} \) off the original price of the shoes.

Students should provide an idea that is similar to this: discount price = original price \( - \) rate times the original price.

A certain retail clothes store advertises its end of season clothes at a price that is \( \frac{1}{2} \) off the original price and shoes at \( \frac{1}{4} \) off the original price (called the discount rate).

a. If a pair of shoes cost $40 and is advertised at \( \frac{1}{4} \) off the original price, what is the sales price?

Method 1: Tape Diagram

<table>
<thead>
<tr>
<th>$10</th>
<th>$10</th>
<th>$10</th>
<th>$10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( 1/4 \) off the original price is $30.

Method 2: Subtracting \( \frac{1}{4} \) of the price from the original price

\[
40 \quad - \quad \frac{1}{4} (40) = \quad 40 \quad - \quad 10 = \quad 30
\]

Method 3: Finding the fractional part of the price being paid by subtracting \( \frac{1}{4} \) of the price from 1 whole

\[
\left( 1 - \frac{1}{4} \right) 40 = \quad \left( \frac{3}{4} \right) 40 = \quad 30
\]

Scaffolding:

- Consumer – person buying items
- Remind students that “of” in mathematics is an operational word for multiply.
- Note of caution: students may find the amount of the discount and forget to subtract it from the original price.
b. At Peter’s Pants Palace a pair of pants usually sells for $33.00. If Peter advertises that the store is having 1/3 off sale, what is the sale price of Peter’s pants? Use questioning to guide students to developing the methods below. The do not need to use all three methods but should have a working understanding of how and why they work in this problem.

**Method 1: Tape Diagram**

\[
\text{\$33} \div 3 = \text{\$11} \\
2(\text{\$11}) = \text{\$22}
\]

**Method 2: Use the given rate of discount, multiply by the price and then subtract from the original price.**

\[
33 - \frac{1}{3}(33) = 33 - 11 = \text{\$22}
\]

The consumer pays \(\frac{2}{3}\) of the original price.

**Method 3: Subtract the rate from 1 whole, then multiply that rate times the original price.**

\[
1 - \frac{1}{3} = \frac{2}{3} \\
\frac{2}{3}(33) = \text{\$22.00}
\]

---

**Example 2: Big Al’s Used Cars**

Have students generate an equation that would find the commission for the sales person.

A used car sales person receives a commission of 1/12 of the sales price of the car for each car he sells. What would the sales commission be on a car that sold for $21,999?

\[
\text{Commission} = 21999 \left(\frac{1}{12}\right) = \text{\$1833.25}
\]

---

**Example 3: Tax Time**

As part of a marketing ploy, some businesses mark up their prices before they advertise a sales event. Some companies use this practice as a way to entice customers into the store without sacrificing their profits. A furniture store wants to host a sales event to improve their profit margin and to reduce their tax liability before their inventory is taxed at the end of the year.

How much profit will be business make on the sale of a couch that is marked-up by 1/3 and then sold at a 1/5 off discount if the original price is $2400?

\[
\text{Mark up: } 2400 + 2400 \left(\frac{1}{3}\right) = 2400 \left(1\frac{1}{3}\right) = \text{\$3200}
\]

\[
\text{Markdown: } 3200 - 3200 \left(\frac{1}{5}\right) = 3200 \left(1\frac{4}{5}\right) = \text{\$2560}
\]

\[
\text{Profit} = \text{sales price} - \text{original price} = 2560 - 2400 = \text{\$160.00}
\]
Example 4: Born to Ride

A motorcycle dealer paid a certain price for a motorcycle and marked it up by $1/5$ of the price he paid. Later he sold it for $\$14,000$ what is the original price?

*Explain that a whole plus the fractional increase will give $1 + 1/5 = 6/5$ of the original price.*

Let $x$ = the original price

\[
x + \frac{1}{5}x = 14000
\]

\[
\frac{6}{5}x = 14000
\]

\[
14000 \cdot \frac{5}{6} = \frac{14000 \cdot 5}{6} = 11,666.67
\]

Closing (5 minutes)

- Name at least two methods used to find the solution to a fractional markdown problem.
  - Find the fractional part of the markdown and subtract it from the original price.
  - Use the tape diagram to determine how much value each part represents, then subtract the fractional part from the whole.
- Compare and contrast a commission and a discount price?
  - The commission and the discount price are both fractional parts of the whole. The difference between them is that commission is found by multiplying the commission rate times the sale, while the discount is the difference between 1 and the fractional discount multiplied by the original price.

Lesson Summary:

- Discount price = original price – rate $\times$ original price OR (1 - rate) $\times$ original price
- Commission = rate $\times$ total sales amount
- Markup price = original price + rate $\times$ original price OR (1 + rate) $\times$ original price

Exit Ticket (5 minutes)
Lesson 14: Multistep Ratio Problems

Exit Ticket

1. A bicycle shop advertised all mountain bikes priced at a 1/3 discount.
   a. What is the amount of the discount if the bicycle originally costs $327?

   b. What is the discount price of the bicycle?

   c. Explain how you found your solution to part b.

2. A hand-held digital music player was marked down by ¼ of the original price.
   a. If the sales price is $128.00, what is the original price?

   b. If the item was marked up by 1/3 before it was placed on the sales floor, what was the price that the store paid for the digital player?

   c. What is the difference between the discount price and the price that the store paid for the digital player?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. A bicycle shop advertised all mountain bike’s priced at a 1/3 discount.
   a. What is the amount of the discount if the bicycle originally costs $327?
      \[
      \frac{1}{3} (327) = 109 \text{ discount} 
      \]
   b. What is the discount price of the bicycle?
      \[
      \frac{2}{3} (327) = 218 \text{ discount price – methods will vary} 
      \]
   c. Explain how you found your solution to part b.
      Answers will vary

2. A hand-held digital music player was marked down by ¼ of the original price.
   a. If the sales price is $128.00, what is the original price?
      \[
      x - \frac{1}{4}x = 128 \\
      3 \times \frac{1}{4}x = 128 \\
      x = 170.67 
      \]
   b. If the item was marked up by 1/2 before it was placed on the sales floor, what was the price that the store paid for the digital player?
      \[
      x + \frac{1}{2}x = 170.67 \\
      3 \times \frac{1}{2}x = 170.67 \\
      x = 113.78 
      \]
   c. What is the difference between the discount price and the price that the store paid for the digital player?
      \[
      128 - 113.78 = 14.22 
      \]
1. What is 1/32 commission of sales totaling $24,000?

\[
\left(\frac{1}{32}\right) 24000 = \$750
\]

2. DeMarkus says that a store overcharged him on the price of the video game he bought. He thought that the price was marked ¼ of the original price, but it was really ¼ off the original price. He misread the advertisement. If the original price of the game was $48, then what was the difference between the price that DeMarkus thought he should pay and the price that the store charged him?

\[
\frac{1}{4} \text{ of } \$48 = \$12 \text{ (the price DeMarkus thought he should pay)}; \frac{1}{4} \text{ off } \$48 = \$36; \text{ Difference between prices } \$36 - \$12 = \$24
\]

3. What is the cost of a $1200 washing machine that was on sale for a 1/5 discount?

\[
\left(1 - \frac{1}{5}\right) 1200 = \$960 \text{ or } 1200 - \frac{1}{5}(1200) = \$960
\]

4. If a store advertised a sale that gave customers a ¼ discount, what is the fraction part of the original price that the customer will pay?

\[
1 - \frac{1}{4} = \frac{3}{4} \text{ of original price}
\]

5. Mark bought an electronic tablet on sale for ¼ off its original price of $825.00. He also wanted to use a coupon for a 1/5 off the sales price. Before taxes, how much did Mark pay for the tablet?

\[
825 \left(\frac{3}{4}\right) = 618.75 \text{ then } 618.75 \left(\frac{4}{5}\right) = \$495
\]

6. A car dealer paid a certain price for a car and marked it up by 7/5 of the price he paid. Later he sold it for $24,000 what is the original price?

\[
x + \frac{7}{5}x = 24000, \frac{12}{5}x = 24000, x = \$10,000
\]

7. Joanna ran a mile in physical education class. After resting for one hour, her heart rate was 60 beats per minute. If her heart rate decreased by 2/5, what was her heart rate immediately after she ran the mile?

\[
x - \frac{2}{5}x = 60, \frac{3}{5}x = 60, x = 100 \text{ beats per minute}
\]
Lesson 15: Equations of Graphs of Proportional Relationships Involving Fractions

Student Outcomes

- Students use equations and graphs to represent proportional relationships arising from ratios and rates involving fractions. They interpret what points on the graph of the relationship mean in terms of the situation or context of the problem.

Classwork

Review with students the meaning of unit rate, the meaning of an ordered pair in the proportional relationship context, the meaning of (0, 0) and the meaning of (1, \( \frac{r}{r} \)) from Lesson 10. The goal here is to help students see the relationship between the unit rate and the changes in \( x \) and \( y \).

Example 1 (10 minutes): Mom’s 10K Race

Use the table to determine the constant of proportionality and remind students how this was done in earlier lessons. Help students to understand what the constant of proportionality means in the context of this problem.

Pose the questions to the students as a whole group:

- Create a chart that will show how far Sam’s mom has run after each half hour from the start of the race. Discuss and model with students how to graph fractional coordinates so that the ordered pairs are as accurate as possible.

Equation: \( D = \frac{10}{3} H \)

Scaffolding:

- A 10K race has a length of 10 Kilometers (approximately 6.2 miles)
- Help students find ordered pairs from graphs that fall on coordinates that are easy to see.
- Have students use the coordinates to determine the constant of proportionality (unit rate).

Sam’s mom has entered a 10K race. Sam and his family want to show their support of their mother but they need to figure out where they should go along the race course. They also need to determine how long it will take her to run the race so that they will know when to meet her at the finish line. Previously, his mom ran a 5K race with a time of 1 ½ hours. Assume Sam’s mom ran the same rate as the previous race in order to complete the chart.
1. Create a chart that will show how far Sam’s mom has run after each half hour from the start of the race and graph it on the grid at the right.

<table>
<thead>
<tr>
<th>Hours (H)</th>
<th>$\frac{10}{3}H$</th>
<th>Distance (D) Run in km</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{10}{3} \cdot \frac{1}{2} = \frac{5}{3}$</td>
<td>$2$</td>
</tr>
<tr>
<td>$1$</td>
<td>$\frac{10}{3} \cdot 1 = \frac{10}{3}$</td>
<td>$\frac{10}{3}$</td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>$\frac{10}{3} \cdot \frac{3}{2} = 5$</td>
<td>$5$</td>
</tr>
<tr>
<td>$2$</td>
<td>$\frac{10}{3} \cdot 2 = \frac{20}{3}$</td>
<td>$\frac{20}{3}$</td>
</tr>
<tr>
<td>$\frac{5}{2}$</td>
<td>$\frac{10}{3} \cdot \frac{5}{2} = \frac{25}{3}$</td>
<td>$\frac{25}{3}$</td>
</tr>
<tr>
<td>$3$</td>
<td>$\frac{10}{3} \cdot 3 = 10$</td>
<td>$10$</td>
</tr>
</tbody>
</table>

Note to instructor: The middle column of the table does not appear in the student materials. It is included here to show the student work to complete the distance column.

2. What are some specific things you notice about this graph?
   Possible answers: It forms a straight line through the origin; it relates hours to km run; the straight line through the origin means that the values are proportional.

3. What is the connection between the table and the graph?
   Possible answers: The time in hours is on the horizontal axis and the km run is on the vertical axis; the coordinates of the points on the line are the same as the pairs of numbers in the table.

4. What does the point $(2, 6\frac{2}{3})$ represent?
   After 2 hours she has run $6\frac{2}{3}$ km

5. Discuss with your elbow partner: Can you find Sam’s mom’s average rate for the entire race based on her previous race time?
   $3\frac{1}{3}$ km/hr

6. Have students write the equation that models the data in the chart. Record the student responses so that they can see all responses.
   $D = 3\frac{1}{3} H$, where $D =$ distance and $H =$ hours.

Discuss the responses with the class and draw a conclusion.
Example 2 (10 minutes): Organic cooking

Students should write the equation from the data given and complete the ordered pairs in the table. Pose the questions to the students as a whole group, one question at a time:

After taking a cooking class, you decide to try out your new cooking skills by preparing a meal for your family. You have chosen a recipe that uses an organic mushroom mix as the main ingredient. Using the graph below, complete the table of values and answer the following questions.

<table>
<thead>
<tr>
<th>Weight in pounds</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>½</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>1 ½</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>2 ¼</td>
<td>18</td>
</tr>
</tbody>
</table>

1. Is this relationship proportional? How do you know from examining the graph?
   Proportional – straight line through the origin. \( k = \frac{16}{2} = 8 \)

2. What is the unit rate for cost per pound?
   Unit rate = 8

3. Write an equation to model this data.
   \( C = 8w \)

4. What ordered pair represents the unit rate and what does it mean?
   \( (1, 8) \) Unit rate is 8 which means that one pound of mushrooms cost $8.00.

5. What does the ordered pair \( (2, 16) \) mean in the context of this problem?
   \( (2, 16) \) means 2 pounds of mushrooms cost $16.00.

6. If you could spend $10.00 on mushrooms, how many pounds could you buy?
   \( c = 8w; \quad 8w = 10 \Rightarrow \frac{10}{8} = \frac{1}{4} \quad 8w; \quad 1 \frac{1}{4} = w; \quad You \ can \ buy \ 1.25 \ pounds \ with \$10.00. \)

7. What would be the cost of 30 pounds of mushrooms?
   \( C = 8W; \quad C = 8 \cdot 30; \quad C = \$240 \)

- Have students share out how they would find the cost for 3 lbs 4 ounces of mushrooms?
  - $26
Discuss the usefulness of equations as models that help determine very large or very small values that are difficult or impossible to see on a graph.

Students should complete these problems in cooperative groups and then be assigned one problem per group to present in a gallery walk. As groups of students walk around the room to view the work, have them write feedback on sticky notes about presentation, clarity of explanations, etc. Students should compare their answers and have a class discussion after the walk about any solutions in which groups disagreed or found incomplete.

Closing Questions (5 minutes)
After the gallery walk refer back to the graphs and charts that students presented.

- Are all graphs straight lines through the origin?
- Did each group write the equations that models the situation in their problem?
- Did each group find the correct constant of proportionality (unit rate) for their problem and describe its meaning in the context of the problem using appropriate units?

Lesson Summary:
Proportional relationships can be represented through the use of graphs, tables, equations, diagrams, and verbal descriptions.

In a proportional relationship arising from ratios and rates involving fractions, the graph gives a visual display of all values of the proportional relationship, especially the quantities that fall between integer values.

Exit Ticket (5 minutes)
Lesson 15: Equations of Graphs of Proportional Relationships Involving Fractions

Exit Ticket

Using the graph and its title:

1. Describe the relationship that the graph depicts.

2. Identify two points on the line and explain what they mean in the context of the problem.

3. What is the unit rate?

4. What point represents the unit rate?
Exit Ticket Sample Solutions

The following responses indicate an understanding of the objectives of this lesson:

1. Describe the relationship that the graph depicts.
   
   The graph shows that in 3 days the water was at 4 inches high. The water is rising at a constant rate. Therefore the water was \( \frac{1}{3} \) inches per day.

2. Identify two points on the line and explain what they mean in the context of the problem.
   
   \((6, 8)\) means that by the 6th day, the water had risen 8 inches; \((9, 12)\) means that by the 9th day the water had risen 12 inches.

3. What is the unit rate?
   
   The unit rate is \( \frac{4}{3} \) inches in 1 day.

4. What point represents the unit rate?
   
   The point that shows the unit rate is \((1 \frac{2}{3}, 1)\).

Problem Set Sample Solutions

1. Students are responsible for providing snacks and drinks for the Junior Beta Club Induction Reception. Susan and Myra were asked to provide the punch for the 100 students and family members who will attend the event. The chart below will help Susan and Myra determine the proportion of cranberry juice to sparkling water that will be needed to make the punch. Complete the chart, graph the data, and write the equation that models this proportional relationship.

<table>
<thead>
<tr>
<th>Sparkling water (cups)</th>
<th>Cranberry juice (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{4}{5} )</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>( \frac{62}{5} )</td>
</tr>
<tr>
<td>12</td>
<td>( \frac{93}{5} )</td>
</tr>
<tr>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>100</td>
<td>80</td>
</tr>
</tbody>
</table>

\( C = \frac{4}{5} S, \) where \( C = \text{Cups of Cranberry Juice} \) and \( S = \text{Cups of Sparkling water} \)

2. Jenny is a member of a summer swim team.
   
   a. How many calories does she burn in one minute?
      
      Jenny burns 100 calories every 15 minutes so she burns \( \frac{62}{3} \) calories each minute.
   
   b. Use the graph below to determine the equation that models how many calories Jenny burns within a certain number of minutes.
      
      \( C = \frac{62}{3} t, \) where \( C = \text{calories} \) and \( t = \text{time in minutes} \)
   
   c. How long will it take her to burn off a 480 calorie smoothie that she had for breakfast?
      
      It will take Jenny 72 minutes of swimming to burn off the smoothie she had for breakfast.
3. Students in a World Geography Class want to determine the distances between cities in Europe. The map has a European Publisher which gives all distances in kilometers. These students want to determine the number of miles between towns so that they can compare distances with a unit of measure that they are already familiar with. The graph below shows the relationship between a given number of kilometers to the corresponding number of miles.

a. Find the constant of proportionality or the rate of miles per kilometer for this problem and write the equation that models this relationship.

The constant of proportionality is $\frac{3}{5}$ km/mi. The equation that models this situation is $m = 1 \frac{3}{5} k$, where $m =$ miles and $k =$ kilometers.

b. What is the distance in kilometers between towns that are 5 miles apart?

The distance between towns that are 5 miles apart is $3 \frac{1}{8}$ km.

c. Describe the steps you would take to determine the distance in miles between two towns that are 200 kilometers apart?

Have students solve the equation $m = 1 \frac{3}{5} (200)$ to find the number of miles for 200 km the students should multiply $200$ by $1 \frac{3}{5}$.

During summer vacation, Lydie spent time with her grandmother picking blackberries. They decided to make blackberry jam for their family. Grandmother said that to make jam, you must cook the berries until they become juice and then combine the juice with the other ingredients to make the jam.

a. Use the table below to determine the constant of proportionality of cups of juice to cups of blackberries?

Each cup of juice is produced by 3 cups of blackberries.

b. Write an equation that will model the relationship between the number of cups of blackberries and the number of cups of juice?

$j = \frac{1}{3} b$ where $j =$ cups of juice and $b =$ cups of blackberries.

c. How many cups of juice were made from 12 cups of berries? How many cups of berries are needed to make 8 cups of juice?

4 cups of juice are made from 12 cups of berries
24 cups of berries are needed to make 8 cups of juice.